Competitive Procurement with Ex Post Moral Hazard[1]

Indranil Chakraborty
National University of Singapore

Fahad Khalil and Jacques Lawarree
University of Washington

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Abstract: The goal of competitive procurement is to ensure that the most efficient firm is selected and delivers the contracted quality after winning the contract. When quality is not directly observable, it necessitates combining an incentive scheme with the optimal auction mechanism. By explicitly modeling ex-post moral hazard, we identify a new effect: a less efficient firm may earn rent by pretending to be more efficient and then shirk. Increased competition exacerbates this problem and may hurt the principal's payoff. Thus, she may find it profitable to limit the number of firms. Alternatively, she can neutralize the negative effect of competition by using a random mechanism that sacrifices allocative efficiency. Then, increased competition does not affect the principal's payoff.

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1 Introduction

Government agencies and private firms routinely rely on competitive procurement to obtain goods, services, or to complete projects. The OECD estimates that its members spend 12.1% of their GDP on public procurement.[2] It has long been known that the benefit of competitive procurement is to ensure productive efficiency and low cost by selecting the most efficient firm and reducing information rent (see, e.g., Demsetz (1968)). While it is particularly effective for standard items like office supplies, many procurement projects also involve additional work after the competitive process is over.[3] Procurers often use incentive schemes to improve ex post performance of selected firms.[4] Such incentive schemes typically require offering ex post rent to the winning firm. However, this begs the question whether ex ante competitive procurement leaves enough rent to ensure ex post performance by the selected firm.[5]

In this paper, we study how the ex ante competitive process interferes with the ex post moral hazard problem.[6] We find that competition can be a mixed blessing for the procurer who insists on allocative efficiency, where the most efficient firm must be selected. Increased competition may hurt a procurer if allocative efficiency is a

[2]The number is even higher for developing countries and the World Bank estimates it to be at 14.5% of the GDP for low-income countries (Djankov, Saliola and Islam (2016)).

[3]Such expenditures form a large part of procurement spending. Expenditure on service and construction contracts often constitute a large fraction of government procurement, accounting for 273 out of 277 billion dollars spent on the top 10 non-defense spending categories in the 2013 U.S. federal budget. http://www.govexec.com/contracting/2015/01/10-categories-where-federal-agencies-spend-most-contracting/102498/

[4]Lewis and Bajari (2014) measure the improvement in completion times in highway procurement due to ex post incentive schemes provided by the Department of Transportation in California.

[5]There is evidence that it may not. In the U.S., the introduction of an experimental competitive bidding program by Medicare had a negative impact on the quality of distribution service for diabetic products (Puckrein et al. (2015)). Reporting on competitive procurement for elderly care by the U.K. National Health Service, The Guardian newspaper notes that companies "bid low to win contracts and then cut back on quality to meet their profit targets." (Leys, "NHS contracting has been a disaster," April 22, 2014, *The Guardian.*) The extent of the loss can be significant. Decarolis (2014), for instance, found that switching to competitive first price auctions to procure public works projects in Italy resulted in a nearly 28% delay in completion time of those projects.

[6]Surprisingly, this crucial aspect of procurement has not been received much attention in the literature. In their survey on public contracting, Armstrong and Sappington (2007) stress the importance of unobservable quality when they discuss competitive procurement and note that relatively little work has been done in the topic. We discuss the literature in more detail below.

requirement.[7] If the procurer cannot limit the number of bidders, she must give up on allocative efficiency to neutralize the negative impact of competition by relying on a scheme that randomly allocates the project to a less efficient firm. There is evidence of such procedures in the U.S. and abroad.[8]

To study the tension between ex ante rent extraction and ex post performance, we consider a model of competitive procurement with ex post moral hazard. In the initial stage, each agent or firm is asked to report its cost of production (cost of effort in our model). Suppose that the procurer then selects the most efficient firm based on the reports. The selected firm must then exert costly effort to complete the project. Both effort and the cost of effort are private information of the firm. Thus, we study a mixed model with both adverse selection and moral hazard.

Ex post moral hazard introduces a new element that restricts the effectiveness of competition. Both downward and upward incentive constraints can be binding: instead of the standard problem of firms wanting to overstate cost, firms may now also want to understate cost and shirk.[9] While competition is known to be an effective tool to address the problem of overstating cost, we show that it can exacerbate the problem of understating cost while shirking. This negative effect of competition can overtake its benefits if the impact of moral hazard is strong enough. We find that high levels of competition may not be beneficial because the goal of rent extraction has given way to the need to sustain high ex post effort.

Limiting the number of competing firms can then be optimal for the procurer. When that is not possible,[10] we show that the procurer can effectively neutralize the negative impact of competition with a random allocation rule. That is, the procurer may not

[7]Indeed, allocative efficiency is a key feature in many procurement settings. For instance, FCC Chairman William E. Kennard (1999) notes that efficiency in the FCC spectrum auctions means that spectrum ends up in the hands of those who value it most highly

[8]Citing examples from various countries, Eun (2017) studies a Korean procurement mechanism that has a stochastic cutoff rule to eliminate the lowest bids. With a counterfactual analysis, he also shows that this rule lowers social cost by 7% relative to a standard first-price auction. Bajari et al. (2014) found that contracts were not allocated to the lowest bidder in nearly 4% of the California Department of Transportation first-price auctions.

[9]With a continuum of types types, in section 7, the global incentive constraint preventing the highest cost-type from mimicking the least cost-type and shirking is binding along with standard constraints to prevent over–stating cost. In our base-case binary-type setting, both incentive constraints are binding.

[10]For example, because it may appear as corruption.

allocate the project to the most efficient firm with probability one. We find that giving up on allocative efficiency relaxes the incentive compatibility constraints of less efficient firms who now have a higher chance to win the project by telling the truth. Since those constraints are what restricts the effectiveness of competition in extracting rent, a random allocation rule can remove the negative impact of competition. Specifically, we show that a random allocation rule allows the principal to mimic a mechanism with an efficient allocation rule where she can choose the number of firms. Then, increased competition ceases to affect her payoff.[11]

The literature on competitive bidding for procurement contracts goes back to the late eighties when a set of influential papers analyzed properties of incentive schemes that governed ex post incentives of the selected firm.[12] Highlighting a separation property, they showed how standard auction formats can be used to extract rent while providing second-best incentives at the same time. In these models, competition has no negative effect. The driving force behind the results in these papers is adverse selection rather than moral hazard. In Riordan-Sappington (1987), the quality is observable, so there is no moral hazard. While McAfee-McMillan (1987) and Laffont-Tirole (1987) have an unobservable effort, agents are risk neutral with unlimited liability, and the principal can deduce the effort once the agent has revealed his type. In the terminology of Laffont-Martimort (2002), these are "false moral hazard" models, where upward incentive constraints are not binding: high-cost firms do not want to pretend to be low cost. An increase in competition can only benefit the principal.

McAfee-McMillan (1986) have a true mixed model with risk averse agents and both adverse selection and moral hazard. However, they do not study the optimal contract but rather a linear contract that balances the cost-plus contract and the fixed-price contract. The linear cost-sharing parameter is assumed to be independent of the agent's type. Thus, the optimal choice of effort is independent of types, which implies that the upward incentive constraints are not binding. Our contribution is to solve the optimal auction in a tractable mixed model, and to show that insisting on allocative efficiency

[11] Thus, when competition is beneficial, we show that insisting on allocative efficiency, and therefore, on a deterministic mechanism, is without loss of generality.

[12] See, e.g., Riordan-Sappington (1987), McAfee-McMillan (1986, 1987) or Laffont-Tirole (1987).

can lead to competition being harmful.[13]

In a recent paper, but in a contest setting, Che-Iossa-Rey (2017) find that it may not always be optimal to allocate the project to the most efficient firm ex post in order to provide incentive to exert effort ex ante.[14]. Firms have private information on implementation cost and effort comes *before* being selected. In a procurement setting, authors also have highlighted allocative inefficiency in models where quality is exogenous. In a procurement setting, Manelli-Vincent (1995), firms have private information about the exogenous quality they can produce, leading to a lemons problem where the least cost agent also generates the least value to the principal. Burguet-Ganuza-Hauk (2012) provide a related analysis where firms have private information about their financial status. In Chillemi-Mezzetti (2014), the winning bidder privately discovers the value of the cost overruns during the project's completion. All these papers find allocative inefficiency to be optimal in models with exogenous quality or ex ante effort. Our objective, instead, is to focus on the effect of competition in the presence of ex post moral hazard that are observed in many private and public procurement situations including the examples mentioned above.

The paper is organized as follows. We present the model in section 2 and the principal's problem in section 3. We derive the optimal mechanism in section 4 and our main result on the impact of competition in section 5. In section 6, we show that a random allocation mechanism is optimal. We show our key results hold when there is continuous effort in section 7 and under continuous types in section 8.

2 The Model

A principal (she) must select one of n agents (he) to complete an indivisible project. The success of the project depends on the selected agent's effort, and agents have different costs of effort. The selected agent privately chooses effort $e \in \{0, 1\}$. With

[13] Piccione and Tan (1996) have shown the separation property, mentioned above, may not hold when bidders make an ex ante investment. Their focus was on the role of the R&D technology (in particular whether it exhibits decreasing returns to scale or not) in the implementation of the project.

[14] Taylor (1995) and Fullerton-McAfee (1999) have shown that admitting too many contestants in a research tournament reduces the ex ante effort of each contestant because their probability of winning becomes too low. Again, effort comes before being selected.

probability π_e, the output is high, h, and the principal receives $V > 0$, while, with probability $(1-\pi_e)$, the output is low, l, and she receives zero.[15] The output is publicly observed, and we assume that $0 < \pi_0 < \pi_1 \leq 1$.[16] Cost of effort is privately known to the agent, and an agent can be one of two types, $x \in \{g, b\}$. It is commonly known that the probability that an agent is of type g is given by $q \in (0, 1)$. With both effort and cost of effort private information of an agent, we have a model with both moral hazard and adverse selection. While our base model has binary efforts and types for ease of exposition, we later show our key results hold if effort or types are continuous.

Denoting the cost of effort by ψ_e^x, we assume the following conditions about the cost of effort.

Condition L. (i) $\psi_1^b > \psi_1^g$ and $\psi_0^b > \psi_0^g > 0$, (ii) $\psi_1^b - \psi_0^b > \psi_1^g - \psi_0^g > 0$, and (iii) $\psi_1^g \geq \frac{\pi_1}{\pi_0}\psi_0^b$.

The first condition ranks the cost of effort and determines that "g" is a good type with a lower cost of effort. The second condition is akin to a standard single-crossing property that the marginal cost of effort decreases with type. The third condition simplifies the exposition and captures the intensity with which ex post moral hazard interferes with rent extraction. Specifically, it creates an incentive for a bad type to mimic a good type by shirking. We discuss the implications of relaxing condition L after proposition 2.

Agents are assumed to have a zero outside option and also limited liability, such that the transfers from the principal are non-negative.[17] The principal's ex post payoff is the output net of paid transfers. The ex post payoff for an agent is the transfer from the principal net of effort cost.

We derive the optimal symmetric procurement mechanism where similar agents are treated similarly in terms of transfers and probability of being selected. By the Revelation Principle (see Myerson (1981)), we can restrict ourselves to truth-telling

[15] The low output may capture a variety of outcomes ranging from poor quality, costly delays, etc.

[16] We assume type-independent probabilities to focus on the effect of competition to screen the agents and extract rent. In our base model, there is no possible screening if there is one agent.

[17] Limited liability makes our moral hazard problem relevant with risk neutral agents. Alternatively, we could have assumed risk aversion without limited liability.

direct mechanisms. In the game that follows, we restrict ourselves to symmetric Perfect Bayes-Nash equilibria. The mechanism proceeds along the following timeline:

Stage 1. The principal announces the mechanism:

$$\left\{t_l^b(n), t_l^g(n), t_h^b(n), t_h^g(n), \phi_r(n), e_g(n), e_b(n)\right\},$$

where t_l^x and t_h^x are the output-based transfers for type $x \in \{g, b\}$, the allocation rule is $\phi_r(n)$, which we define as the probability of allocating the contract to a type g agent if r agents report type g, and e_x is effort for type $x \in \{g, b\}$.[18] To save on notation, we will suppress n when presenting terms in the mechanism.

Stage 2. The agents report their types and the contract is allocated to an agent according to the allocation rules set in stage 1.

Stage 3. The selected agent chooses the privately observable effort.

Stage 4. The output is realized and payments are made accordingly.

The mechanism has therefore two parts: (i) first an allocation rule that selects the agent based on the type announcements; (ii) an incentive contract that gives incentives to the selected agent to exert effort in completing the project.

Allocative efficiency and the allocation rule: Allocative efficiency requires that the principal allocate the contract to an agent who reports to be a good type. If r agents report type g, allocative efficiency requires that $\phi_r = 1$. An efficient allocation rule is therefore deterministic. However, as we will show later, an efficient allocation rule may not be optimal. We study the case of possibly inefficient allocation rules in section 6 by allowing for a random allocation rule ($\phi_r \in [0, 1]$) and show when it can be optimal.

When there are n agents and the agents report their types truthfully, the allocation rule by the principal pins down the probability that an agent will win the contract upon reporting type x, which we denote by γ_n^x. To see how γ_n^x is computed, consider the case when the efficient allocation rule when r agents reported type g. If every agent reports

[18] In Appendix I, we show it is without loss of generality to (i) restrict the transfers and efforts to depend only on an agent's own report; (ii) assume that only the winning bidder is paid in the mechanism.

truthfully, then we have,

$$\gamma_n^g = \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{1}{r}$$
$$= \frac{1}{nq} \sum_{r=1}^{n} \frac{n!}{(n-r)!r!} q^r (1-q)^{n-r}$$
$$= \frac{1}{nq} \left(1 - (1-q)^n\right)$$

$$\gamma_n^b = \frac{1}{n} (1-q)^{n-1}.$$

Often, we will use the ratio γ_n^g/γ_n^b of the two probabilities and denote it by γ_n. We can write

$$\gamma_n \equiv \frac{\gamma_n^g}{\gamma_n^b} = \frac{(1-(1-q)^n)}{q(1-q)^{n-1}} = \frac{1}{q(1-q)^{n-1}} - \frac{1}{q} + 1.$$

We denote by δ_n the probability that a good type is selected. For instance, if agents report their types truthfully and a good type is always selected whenever there is one,

$$\delta_n = 1 - (1-q)^n.$$

Note that, under such a rule, both γ_n and δ_n increase as n increases.

3 The principal's problem

Our goal is to explore the effect of competition between agents on the principal's payoff, and we start by assuming that the principal wants to implement high effort levels for both types of the agent ($e_g = 1, e_b = 1$) for expositional reasons. Later we specify conditions under which inducing high efforts $e_g = 1, e_b = 1$ are indeed optimal.[19] In the extension with continuous effort we again show that our key results continue to hold when the principal can induce different efforts for each type, which allows screening of types using effort.

We begin our analysis by clarifying the feasible set of contracts starting with the constraints that induce high effort by the selected agent. Given truthful reports of

[19] Given our analysis, it is straightforward to show that our key results also hold when it is optimal to have $e_g = 1$, and $e_b = 0$.

types, the optimal contract has to satisfy the following moral hazard constraints for each type of agent:

$$\pi_1 t_h^g + (1-\pi_1)t_l^g - \psi_1^g \geq \pi_0 t_h^g + (1-\pi_0)t_l^g - \psi_0^g \tag{MH_g}$$

$$\pi_1 t_h^b + (1-\pi_1)t_l^b - \psi_1^b \geq \pi_0 t_h^b + (1-\pi_0)t_l^b - \psi_0^b. \tag{MH_b}$$

To induce truth telling, the optimal contract must satisfy incentive compatibility constraints that account for the possibility of each type working (high effort) or shirking if they misreport their type. Thus, we write two incentive compatibility constraints for each type of agent:

$$\gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1)t_l^g - \psi_1^g\right) \geq \gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1)t_l^b - \psi_1^g\right) \tag{IC_1^g}$$

$$\gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1)t_l^g - \psi_1^g\right) \geq \gamma_n^b \left(\pi_0 t_h^b + (1-\pi_0)t_l^b - \psi_0^g\right) \tag{IC_0^g}$$

$$\gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1)t_l^b - \psi_1^b\right) \geq \gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1)t_l^g - \psi_1^b\right) \tag{IC_1^b}$$

$$\gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1)t_l^b - \psi_1^b\right) \geq \gamma_n^g \left(\pi_0 t_h^g + (1-\pi_0)t_l^g - \psi_0^b\right), \tag{IC_0^b}$$

where (IC_1^x) prevents misreporting while working and (IC_0^x) prevents misreporting while shirking, with $x \in \{g, b\}$. Note that the level of competition, n, affects payoffs on each side of the constraints.

Finally, the optimal contract must satisfy the following IR constraints to induce each type of agent to participate:

$$\gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1)t_l^g - \psi_1^g\right) \geq 0 \tag{IR^g}$$

$$\gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1)t_l^b - \psi_1^b\right) \geq 0. \tag{IR^b}$$

The principal's problem in this case is to choose the contract $\{t_0^b, t_1^b, t_0^g, t_1^g\}$ to maximize the expected payoff,

$$\Pi_{11}(n) = \delta_n \left(\pi_1 V - \pi_1 t_h^g - (1-\pi_1)\, t_l^g\right) + (1-\delta_n)\left(\pi_1 V - \pi_1 t_h^b - (1-\pi_1)\, t_l^b\right)$$

subject to the above eight constraints and the non-negativity conditions on all four transfers.

Next, we simplify the principal's problem. Since high effort is induced for both types, it is not optimal to reward either type after a low outcome. This allows us to reduce the number of transfers that we need to consider. The following lemma shows that we can set $t_l^g = t_l^b = 0$ w.l.o.g.

Lemma. Given any transfer vector $(t_h^g, t_l^g, t_h^b, t_l^b)$ that satisfy the IC, MH and IR constraints, there is a transfer vector $(\bar{t}_h^g, \bar{t}_l^g, \bar{t}_h^b, \bar{t}_l^b)$ with $\bar{t}_l^g = 0, \bar{t}_l^b = 0$ that gives the same payoff to the principal and satisfies the IC, MH and IR constraints.

Proof. See Appendix I. ■

We can simplify the problem further by eliminating some constraints. First, the rent from moral hazard, coupled with $L(iii)$, implies that the $(MH_g)$and (MH_b)make the $(IR_g)$and (IR_b)redundant. Second, the moral hazard rent given to the bad type induces the good type to work rather than shirk when misreporting, which means (IC_0^g) is redundant. Technically, (IC_0^g) is implied by (IC_1^g) and (MH_b). The bad type's incentive constraint cannot be ignored, which is unlike what is standard in models of contracting under adverse selection. In our setting, we anticipate that the bad type will have an incentive to claim to be a good type, and he will pursue this option by putting in low effort rather than high effort, i.e., we expect (IC_0^b) to be relevant. Indeed, (IC_0^b) will play an important role in our analysis. Finally, we will ignore (IC_1^b) and can verify later that the optimal contract satisfies this constraint.

The Reduced Problem

Defining the principal's payoff when she induces high effort by both types by

$$\Pi_{11}(n) = \pi_1 \left[V - \left(\delta_n t_h^g + (1 - \delta_n)\, t_h^b\right)\right]$$

we can rewrite the principal's problem in a simpler form, where she chooses the two transfers $\{t_h^b, t_h^g\}$ to solve:[20]

$$\max \Pi_{11}(n) \tag{1}$$

[20]Note that maximizing $\pi_{11}(n)$ is equivalent to minimizing the expected payment.

subject to

$$\gamma_n \left(\pi_1 t_h^g - \psi_1^g\right) \geq \pi_1 t_h^b - \psi_1^g \qquad (IC_1^g)$$

$$t_h^g \geq \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \qquad (MH_g)$$

$$\pi_1 t_h^b - \psi_1^b \geq \gamma_n \left(\pi_0 t_h^g - \psi_0^b\right) \qquad (IC_0^b)$$

$$t_h^b \geq \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0}. \qquad (MH_b)$$

It is useful to briefly consider a benchmark case of contracting with a single agent ($n = 1$, $\gamma_n^g = 1 = \gamma_n^b$), which is depicted in Figure 1 below. Since the principal wants to induce $e_g = e_b = 1$, the pair of transfers (t_h^g, t_h^b) must satisfy the moral hazard constraint for each type. However, there can be no screening since each type will choose the higher transfer given type-independent probabilities of success. Technically, we can see that $t_h^g = t_h^b$ from (IC_1^g) and (IC_1^b). Then (IC_0^b) reduces to (MH_b), so the optimal transfers are given by $t_h^g = t_h^b = \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0}$, making (MH_g) slack. In sum, when $n = 1$, and the constraints (IC_1^g), (IC_0^b) and (MH_b) all hold as equalities. Thus, the bad type receives a moral hazard rent but no adverse selection rent. The good type earns an adverse selection rent which is strictly higher than the rent needed to induce high effort.[21]

Our model allows us to focus on the effect of competition to screen the agents, which we study next.

4 Competitive Procurement with Multiple Agents ($n > 1$)

Competition is a critical part of the incentive mechanism as the principal uses it to screen the agents. This is reflected in the presence of γ_n^g and γ_n^b in the (IC) constraints. On the other hand, the (MH) constraints are not affected directly by competition because the effort decision occurs once the agent has been selected.

[21] To see this, note that, if the principal could observe types, the bad type would receive the same transfer, earning a MH rent. The good type, on the other hand, would receive less ($t_h^g = \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0}$).

A standard effect of competition is that it increases the cost of lying for a good type, which allows the principal to reduce his rent. By favoring a good type in the allocation rule, she gives him a greater chance of being selected if he tells the truth, $\gamma_n^g > \gamma_n^b$ when $n > 1$. This relaxes the good type's incentive constraint (IC_1^g) and allows the principal to reduce the transfer to the good type.[22] We call this the *good-type transfer effect* of increased competition. This is a standard effect of competition in adverse selection models with many agents.

Ex post moral hazard introduces a new element that restricts the effectiveness of competition as the bad type's incentive constraint (IC_0^b) is also binding. This constraint is typically not binding in pure adverse selection settings.[23] With unobservable effort, a bad type can misreport his type *and* exert low effort. This yields an additional rent to the bad type. Competition exacerbates this problem.[24] In other words, to induce truth-telling, the bad type has to be given a higher transfer as his chance of being selected decreases with competition. We refer to this as the *bad-type transfer effect* of increased competition, which is a new cost of competition due to the ex post moral hazard problem.

Technically, it is useful to first recall that both (IC_1^g) and (IC_0^b) were satisfied as equalities when $n = 1$. With multiple agents, the (IC_1^g) will become slack and (IC_0^b) will be violated unless the transfers are adjusted. In the optimal contract, the principal adjusts the transfers to both types. With (IC_1^g) slack, she definitely reduces t_h^g. This is the benefit of increased competition in reducing the good type's rent. However, the decrease in t_h^g is not enough to satisfy (IC_0^b) unless t_h^b is increased to remove the bad type's incentive to pretend to be good.[25]

Two possible cases emerge depending on the intensity of competition. For low n, we are in case (I). The good type's rent is reduced but it is still large enough to induce

[22] As n increases, the ratio γ_n^g/γ_n^b increases and this continues to relax (IC_1^g).

[23] This effect of shirking is absent in earlier models of competitive procurement, with false moral hazard as in Laffont-Tirole (1987) and McAfee-McMillan (1987), or if effort is observable as in Riordan-Sappington (1987).

[24] Again, since the ratio γ_n^g/γ_n^b increases with n, it tightens (IC_0^b).

[25] More precisely, the fact that t_h^b increases with n is implied by condition $L(iii)$, which reflects a strong moral hazard problem. Later, we relax condition $L(iii)$ to show that (IC_0^b) can still be binding and competition can hurt even under a less restrictive condition.

high effort by the good type, and the good type's moral hazard constraint (MH_g) is still slack. In proposition 1 below, for low n, the solution is given by the binding (IC_0^b) and (IC_1^g) and we call this case I:

$$t_h^g = \frac{\gamma_n \psi_1^g - \psi_1^g - \gamma_n \psi_0^b + \psi_1^b}{(\pi_1 - \pi_0)\,\gamma} \tag{I}$$
$$t_h^b = \frac{\gamma_n \left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) + \pi_1 \psi_1^b - \pi_0 \psi_1^g}{\pi_1 (\pi_1 - \pi_0)}.$$

For higher n, we are in case II. As the principal decreases t_h^g, the good type's moral hazard constraint (MH_g) will bind eventually. At that point, a larger n does not allow the principal to decrease t_h^g further, and (IC_1^g) becomes slack – the moral hazard rent is high enough to induce truth-telling by the good type. In proposition 1 below, for high enough n, the solution is given by the binding (IC_0^b) and (MH_g) and we call this case II:

$$t_h^g = \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \tag{II}$$
$$t_h^b = \frac{\gamma_n \pi_0}{\pi_1} \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \frac{\left(\gamma_n \psi_0^b - \psi_1^b\right)}{\pi_1}.$$

These results are illustrated in Figure 1 and summarized in the following proposition. In Figure 1, as n increases from $n = 1$, the solution moves north-west as t_h^g decreases and t_h^b increases. This is case I. Once the solution reaches the (MH_g) line, t_h^g cannot be further reduced. This is case II. The solution is given by the intersection of (MH_g) and (IC_0^b) and it moves up vertically with n. It is important to note that, in both cases (I) and (II), the bad-type transfer effect is present, i.e., t_h^b keeps increasing with n.

——— INSERT FIGURE 1 HERE (figure.pdf) ———

Proposition 1 *The solution to the principal's problem entails: (i) the constraint* (IC_0^b) *is binding for all* γ_n*, (ii) for* $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$*, the constraint* (IC_1^g) *is binding and the transfers are given by* (I)*, and (iii) for* $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$*, the constraint* (MH_g) *is also binding, and the transfers are given by* (II)*.*

Proof. See Appendix I. ■

Recall that we have assumed that it is optimal to induce both types of agents to choose the high effort. Intuitively, the principal will always want to induce high effort if the project is very valuable, i.e., V is large enough. Our next proposition gives a formal proof. We discuss in section 5 the case when the principal may want to induce different efforts for different types.

Proposition 2 *If V is high enough, $e_g = e_b = 1$ is optimal.*

Proof. See Appendix I. ■

Before discussing whether competition hurts or helps the principal, we briefly discuss the role of the assumptions $L(ii)$ and $L(iii)$. First consider $L(iii)$ $\psi_1^g \geq \frac{\pi_1}{\pi_0}\psi_0^b$. The main reason for imposing $L(iii)$ is that it simplifies the analysis. Indeed, condition $L(iii)$ implies that (IC_0^b) is binding for all n, i.e., for both cases I and II of proposition 1. While a binding (IC_0^b) in itself does not guarantee that t_h^b will increase with n, condition $L(iii)$ is necessary and sufficient to ensure that t_h^b increases in case I, and sufficient for t_h^b to increase in n for case II.[26] If condition $L(iii)$ does not hold, there are several cases to consider, but the bad-type transfer effect will continue to hold under weaker conditions.[27]

Next consider condition $L(ii)$ $\psi_1^g - \psi_0^g < \psi_1^b - \psi_0^b$. Without $L(ii)$, the good-type transfer effect is absent, and the bad-type transfer effect comes into play immediately (for any $n > 1$).[28] Since this condition is akin to a standard single-crossing property, relaxing it implies that the moral hazard of the good type is now more serious than the moral hazard of the bad type.

[26] In case II, a weaker condition $L(iv)$ is necessary and sufficient for t_h^b to increase with n. *Condition* $L(iv) : \psi_1^g > \frac{\pi_1}{\pi_0}\psi_0^b - \left(\psi_0^b - \psi_0^g\right)$.

[27] Without $L(iii)$, we cannot ignore (IR^g) since it is no longer implied by (MH^g). If we assume that (MH^g) implies (IR^g), and $L(iv)$ holds, the solution will be given by (IC_0^b) and (MH^g) and t_h^b is increasing in n for a high enough n. If (IR^g) is more restrictive than (MH^g) when lowering t_h^g, then a weaker condition than $L(iv)$ would be enough to obtain the bad-type transfer effect for high enough n.

[28] For $n = 1$, we still have $t_h^b = t_h^g$ but it is the good type's moral hazard constraint (MH_g) that is binding. As n increases, because the (MH_g) remains binding, t_h^g is constant at (MH_g).

5 Does competition help?

We say that a given level of competition n hurts the principal if her expected payoff is higher with fewer bidders.[29] The main finding from our analysis so far is that increased competition generates a trade-off between the rents of the good type and the bad type, and we now show that there may indeed be too much competition. Recall from proposition 1 that the optimal mechanism moves from case I to case II as n increases. We now discuss how the trade-off determines whether competition helps or hurts the principal in each of the two cases. Note that along with the good and bad-type transfer effects of increased competition mentioned already, there is also a *selection effect* of increased competition due to a change in δ_n in the principal's payoff $\Pi_{11}(n)$. Thus, there are three potential effects of increased competition:

- *selection effect*: an increase in n increases the probability, δ_n, of awarding the contract to a good type. This effect on the principal's payoff (1) is positive since $t_h^g < t_h^b$.
- *good-type transfer effect*: an increase in n decreases the transfer t_h^g. This effect is again positive for the principal.
- *bad-type transfer effect*: an increase in n increases the transfer t_h^b. This effect is negative for the principal.

In case I, all three effects are present. The first two are standard effects due to adverse selection, while the third is new due to ex post moral hazard. The net effect of competition on the principal's payoff depends on the combined impact of the three effects. The principal benefits from competition when the two positive effects (the selection effect and the good-type transfer effect) dominate the negative effect (the bad-type transfer effect).

The good-type transfer effect is limited by the need to provide the good type with a moral hazard rent. Hence, as competition increases the good type's transfer cannot be reduced indefinitely: t_h^g cannot be reduced further when (MH_g) is binding, and we are

[29] We say that increasing competition always helps if her expected payoff increases as n increases.

in case II. In that case the good-type transfer effect vanishes and only the selection and bad-type transfer effects remain. The principal benefits from competition when the remaining positive effect (the selection effect) dominates the negative effect (the bad-type transfer effect).

Therefore, it is possible that competition (i) never hurts, (ii) hurts in cases I and II, or (iii) hurts only in case II. This last case (iii) is particularly interesting. Intuitively, as competition intensifies, the principal lowers the transfer to the good type, but at some point the transfer becomes so low that the agent would no longer exert effort if the transfer kept decreasing. The principal cannot lower the transfer to the good type any further. Thus, the good-type transfer effect disappears, which weakens the overall benefit of competition. The precise conditions when competition hurts the principal in each case are given in the next proposition.

Proposition 3 *If allocative efficiency is a requirement, competition hurts if and only if*

$$\textit{Case } I: \quad \pi_1 q \left(\psi_1^b - \psi_1^g\right) < (1-q)\left[\pi_0\psi_1^g - \pi_1\psi_0^b\right] \qquad \textit{when } \gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right],$$

$$\begin{aligned} \textit{Case } II: \quad & \pi_1 q \left(\psi_1^b - \psi_1^g\right) < (1-q)\left[\pi_0\psi_1^g - \pi_1\psi_0^b\right] \\ & + (1-q)\left(\psi_0^b - \psi_0^g\right)\pi_0 + q\left(\pi_0\psi_1^b - \pi_1\psi_0^g\right) \quad \textit{when } \gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right). \end{aligned}$$

Proof. See Appendix I. ■

The proposition formalizes the relative impact of the positive and negative effects of competition for the principal. Consider case I first. Given condition $L(iii)$, the right hand side of the first inequality positive. However, for competition to hurt in case I, we also need a relatively large bad-type transfer effect. For instance, if ψ_1^b is close to ψ_1^g, adverse selection is not a serious problem when inducing high effort ($e = 1$). However, the moral hazard concern remains. We still have the issue of the bad type mimicking the high type while shirking. So the bad type transfer effect remains and, if it dominates, the competition will hurt. In case II, the good type transfer effect is absent, which implies that the condition for competition to hurt is less strict. This can be seen by the two extra positive terms on the right hand side of the inequality, making it easier to satisfy than the condition for case I. Therefore, if competition hurts the principal for low n it cannot help her when n is large. This is reflected in the following Corollary.

Corollary *If $\Pi_{11}(n)$ increases for $\gamma_n \in \left[\frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}, \infty\right)$ then it also increases for $\gamma_n \in \left[1, \frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}\right]$.*

Proof. See Appendix I. ■

Up to now we assumed that the principal takes n as given. We see that competition may not hurt when the number of participants is low but it may when the number is large. Therefore, we examine next the case when the principal could choose the number of participants.

Denoting by n^* the number of participants for which the principal's payoff is the highest, the proposition and the corollary imply that there are three possible outcomes of competition depending on the parameter values:

(i) $n^* = \infty$. Competition always helps the principal, in which case the principal would like as many participants as possible. This is the standard result due to competition.

(ii) $n^* = 1$. Competition always hurts the principal and contracting with a single agent is better. This would be the extreme implication of ex post moral hazard in a competitive setting.

(iii) $1 < n^* < \infty$. Low levels of competition help the principal, but higher levels of competition hurt the principal.[30] There is an optimal number of participants, and the principal would prefer to limit the number of participants to that level. The optimal n^* is given by $\gamma_{n^*} \approx \frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}$.[31] This may explain why some existing procurement mechanisms try to restrict the number of participants (see Eun (2017) for examples). This may also explain the criticisms in the earlier examples of NHS and Medicare procurement that too much competition adversely affects ex post quality of service. Next, we study

[30] Note that if we assumed $\psi_0^g = \psi_0^b$, case II disappears and MH_g never binds. In that case, either competition always helps or competition never helps.

[31] We write $\gamma_n \approx \frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}$ instead of $\gamma_n = \frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}$ because with discrete n there need not be a n for which the equality holds.

an alternative to restricting the number of participants by using random allocation mechanisms.

6 An inefficient allocation rule can be optimal: using a random allocation rule

Earlier, we explained that allocative efficiency implied a deterministic allocation rule where the principal committed to allocating the contract to a good type with probability one given when at least one agent reported to be a good type, $\phi_r = 1$. Here, we consider a random allocation rule, $\phi_r \in [0, 1]$, that gives a chance for a bad type to be selected even when another agent has reported to be a good type. We show that the principal can in fact entirely neutralize the negative effect of competition on her payoff. Thus, if a given level of competition under an efficient allocation rule hurts her payoff, optimality requires that she give up on allocative efficiency by randomly assigning the contract. Essentially, without actually limiting the number of agents, a random rule allows the principal to mimic a mechanism with an efficient allocation rule and with the optimal number of agents.

Proposition 4 *A random allocation rule is optimal whenever competition hurts (i.e., $n^* < n$) under an efficient allocation rule. Furthermore, the principal can use a random allocation rule to obtain the same payoff as under an efficient allocation rule with $n = n^*$.*

Proof. See Appendix I. ■

To see the intuition for this result, note that a bad type's incentive to mimic the good type with zero effort can be lowered in two different ways. We have already discussed one method, increasing his transfer t_h^b, which leads to the the bad-type transfer effect. An alternative approach is to give the bad type a positive probability of being selected despite the presence of good types. By randomly assigning the contract to a good type with probability less than 1, the principal increases the probability for a bad type to be selected, which used to be zero under an efficient allocation rule even if there were only one good type present. This relaxes (IC_0^b) and benefits the principal.

We now show how a random allocation achieves the same effect as limiting the number of agents. To distinguish the random allocation rule from the efficient allocation rule, we use a "tilde" on γ. Thus, we use $\tilde{\gamma}_n$ and $\tilde{\delta}_n$ to denote the relevant variables under the random allocation rule, and γ_n and δ_n are special cases of $\tilde{\gamma}_n$ and $\tilde{\delta}_n$ when $\phi_r = 1$ for $r \in \{1, 2, ..n\}$. In this case, if an agent reports g his probability of receiving the contract is

$$\tilde{\gamma}_n^g = \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{\phi_r}{r}$$

and if he reports b, his probability of receiving the contract is

$$\tilde{\gamma}_n^b = \sum_{r=0}^{n-1} \binom{n-1}{r} q^r (1-q)^{n-r-1} \frac{1-\phi_r}{n-r}.$$

Similarly as before, we define the ratio of these probabilities as $\tilde{\gamma}_n \equiv \tilde{\gamma}_n^g / \tilde{\gamma}_n^b$. Note that $\tilde{\gamma}_n$ now depends on n as well as ϕ_r. We show the derivation of $\tilde{\delta}_n$ in the Appendix I.

Except for replacing γ_n by $\tilde{\gamma}_n$, and δ_n by $\tilde{\delta}_n$, in the reduced problem of the principal, the expressions for the payoffs of the principal and the agents remain the same as before. By definition, if all agents report bad type, then the contract is allocated to an agent who reported bad type with probability one, i.e., $\phi_0 = 0$. Similarly, if all agents report good type, we have $\phi_n = 1$.

The principal's objective function and constraints depend on n only through $\tilde{\gamma}_n$ and $\tilde{\delta}_n$, and we prove that, by appropriately choosing ϕ_r, the principal can replicate her payoff from a selection mechanism with fewer agents. For instance, if the principal obtains a higher payoff from having $n^* < n$, then she can appropriately choose ϕ_r to obtain the same payoff with n agents. More specifically, suppose competition hurts in case I. Then the principal would want to set $n = 1$, that is $\tilde{\gamma}_n = 1$. By appropriately choosing ϕ_r, she can indeed set $\tilde{\gamma}_n = 1$, i.e., $\tilde{\gamma}_n^g = \tilde{\gamma}_n^b$. Thus, the principal can offer a mechanism where her payoff is identical to the optimal contract when there is only one agent. Similarly, if competition is beneficial in case I but not in case II, the principal would like to choose n such that $\tilde{\gamma}_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_1^g}$. If she can't restrict the number of agents directly, she can again choose $\phi_r \leq 1$ to neutralize the negative effect of competition and obtain a payoff identical to that under $\tilde{\gamma}_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_1^g}$.[32] Finally, if competition is beneficial for all n, then restricting attention to an efficient allocation rule is without loss of generality.

7 Screening with effort

So far we have considered the case where effort was binary and the principal found it optimal to induce high effort by both types ($e_b = e_g = 1$). As a result, the principal did not use effort as a screening instrument and the agents were restricted to only one alternative effort level when shirking. In this section, we relax that restriction by letting the agent adjust effort $e \geq 0$ continuously. We find that, although some amount of competition is always beneficial, being able to adjust effort continuously is not enough for the principal to guarantee that high levels of competition are beneficial. On the one hand, by adjusting effort continuously, the principal can screen the agent better. On the other hand, the bad type agent has now more options in choosing effort to shirk when mimicking the good type. We find that, this latter effect prevails and that (IC^b) is binding for all n. Except that competition is always beneficial when n is small, the rest of the results are largely analogous to those from the binary model. We relegate details to Appendix II and outline the model and key arguments here

We consider a model where the agent privately chooses effort $e \in [0, 1]$. We focus on interior solutions such that effort adjusts continuously as competition (n) increases. Accordingly, we will assume that V is not too large.[33] Let $\psi_x(e) = xe^2$ be the cost of effort to type $x \in \{b, g\}$, where $0 < g < b$. We assume that the probability of high outcome given effort e is $\pi(e)$, and $\pi(e) = e$.[34] We restrict attention to interior solutions

[32] Note that under the general allocation rule the optimal choice of $\tilde{\gamma}_n$ treats $\tilde{\gamma}_n$ as a continuous argument. Thus, if under deterministic allocation if n^* is the optimal number of agents, it is possible that the optimal $\tilde{\gamma}_n$ satisfies $\gamma_{n^*-1} < \tilde{\gamma}_n < \gamma_{n^*}$ or $\gamma_{n^*} < \tilde{\gamma}_n < \gamma_{n^*+1}$. In that case, when $n > n^*$ the principal is strictly better off under the optimal random allocation rule than limiting the number of bidders to n^*.

[33] Our binary model presents a corner solution where V is so large that $e = 1$ is always optimal.

When effort is binary a smaller V would result in effort choices $(e_g = 1, e_b = 0)$ or $(e_g = 0, e_b = 0)$. In the former case the solution is given by (IC_0^b) and (MH_g), but our key results continue to hold.

[34] Alternatively, we could assume the agent chooses the probability of high outcome π at a cost $x\pi^2$,

for effort on and off the equilibrium path and provide a sufficient condition for that in Appendix II. As in the main section we first consider efficient allocation rules where the contract is awarded to the good type whenever there is one, i.e., $\phi_r = 1$ for all $r \geq 1$, and then we consider random allocation rules by allowing $\phi_r \leq 1$.

We present some preliminary steps and notation before writing the (IC) constraints. The optimal effort by a type x agent who reports type y is denoted by

$$e_{xy} \in \arg\max_{e} \left\{ \pi(e) t_h^y + (1 - \pi(e)) t_l^y - \psi_x(e) \right\}, \tag{2}$$

and $e_x \equiv e_{xx}$. The first order conditions determining effort e_{xy} in an interior solution are:

$$t_h^y - t_l^y = 2x e_{xy}.$$

We see that rent extraction can be limited by moral hazard. If the principal tries to extract rent from the good type by lowering t_h^g, it will result in a lower equilibrium effort by the good type. The two (IC) constraints are:

$$\gamma_n \left(e_g t_h^g + (1 - e_g) t_l^g - \psi_g(e_g) \right) \geq e_{gb} t_h^b + (1 - e_{gb}) t_l^b - \psi_g(e_{gb}) \tag{IC^g}$$

$$e_b t_h^b + (1 - e_b) t_l^b - \psi_b(e_b) \geq \gamma_n \left(e_{bg} t_h^g + (1 - e_{bg}) t_l^g - \psi_b(e_{bg}) \right). \tag{IC^b}$$

The principal's problem is to choose the transfers $\{t_l^b, t_h^b, t_l^g, t_h^g\}$ to maximize her objective function:

$$(\delta_n e_g + (1 - \delta_n) e_b) V - \delta_n \left(e_g t_h^g + (1 - e_g) t_l^g \right) - (1 - \delta_n) \left(e_b t_h^b + (1 - e_b) t_l^b \right),$$

subject to $(IC^g), (IC^b)$, and $t_h^x, t_l^x \geq 0$. The participation constraints are automatically satisfied as $t_h^x - t_l^x \geq 0$.

We again find that (IC^b) is binding for all n, and that competition will hurt the principal if the ex post moral hazard problem plays a significant enough role. As in the binary effort model, we again have two cases I and II depending on whether (IC^g) is binding. The (IC^g) is binding for low n, i.e., for $\gamma_n < \left(2 + \frac{g}{b} \frac{(1-q)}{q}\right)^2$, when rent

with $\pi \in [0, 1]$.

extraction from the good type is important. For high n, i.e., for $\gamma_n \geq \left(2 + \frac{g}{b}\frac{(1-q)}{q}\right)^2$, the (IC^g) ceases to bind. As before, we denote by n^* the number of participants for which the principal's payoff is the highest.

Proposition 5 *When effort is continuous,*

(i) The bad type's incentive constraint (IC^b) *is binding for all* n*, but the good type's incentive constraint* (IC^g) *binds if and only if* $\gamma_n < \left(2 + \frac{g}{b}\frac{(1-q)}{q}\right)^2$.

(ii) If type g *does not have a large cost advantage over type* b*, i.e., if* $(b-g)^2/g^2 < 1/q$*, competition hurts the principal if and only if* $\gamma_n > \left(\frac{b}{g}\right)^2$*, i.e.,* $n > n^*$.

Proof. See Appendix II. ■

With continuous effort the agent has more options when shirking, and the (IC^b) is binding for all n without the need for parameter restrictions. However, for competition to hurt, we do need restrictions on parameters that are reminiscent of results from the binary model. Part (ii) of the proposition determines when competition hurts. For competition to hurt, the cost of (high) effort for the good type cannot be too small (similarly to condition $L(iii)$), and the good type cannot have a large cost advantage over the bad type such that adverse selection is not a serious problem (similarly to propositions 1 and 3). On the other hand, if $(b-g)^2/g^2 > 1/q$, competition can only help. As before, we can derive the optimal n^* when competition hurts. The optimal n^* is given by $\gamma_n \approx \left(\frac{b}{g}\right)^2$.[35] Since $\frac{b}{g} > 1$, this suggests that some degree of competition is always beneficial.[36]

[35] We write $\gamma_{n^*} \approx \left(\frac{b}{g}\right)^2$ instead of $\gamma_{n^*} = \left(\frac{b}{g}\right)^2$ because with discrete n there need not be a n for which the equality holds.

Furthermore, in case I, it is shown in the appendix that competition hurts the principal if and only if $\gamma_n > \left(\frac{b}{g}\right)^2$. This means that if competition hurts in case I, it must also hurt in case II. In case I, there is a possibility of competition to be helpful for low levels of n but harmful for larger n. In that case, the optimal level of competition is determined by n^* such that $\gamma_{n^*} \approx \left(\frac{b}{g}\right)^2$ if competition hurts. If $q > \frac{g^2}{(b-g)^2}$, increased competition can only help even though (IC^b) is binding.

[36] Note that, since n is an integer, the optimal n could turn out to be 1 if $\gamma_{n^*} = \left(\frac{b}{g}\right)^2$ implies that $n^* \in (1, 2)$.

As in the binary effort case, we can again show that a random allocation rule with $\phi_r \leq 1$, with strict inequality for some r, is optimal whenever the principal's payoff is decreasing in n under a deterministic rule. Furthermore, we can clearly show that the principal prefers to have some competition. That is, if he can choose the number of participants, he will pick at least two agents to participate in the mechanism. For example, if the solution to the optimality condition $\gamma_{n^*} = \left(\frac{b}{g}\right)^2$ implies that $n^* \in (1, 2)$, then the principal would prefer to have two participants and use a random allocation rule ($\phi_r < 1$) to obtain her maximum payoff. [37] We present the result below without a separate proof since we follow similar steps as before.

Proposition 6 *Whenever competition hurts under an efficient allocation rule, it is optimal to have a random allocation rule, and some amount of competition is always beneficial, i.e.,* $n^* > 1$.

Two other notable results in the continuous effort case are for high n : (i) the bad type's effort can be higher than the good type's effort ($e_b > e_g$) and (ii) the bad type can receive a positive transfer even when the outcome is low ($t_l^b > 0$). To keep inducing truth-telling by the bad type as his chance of winning decreases due to an increase in n, he must be given a higher rent conditional on winning. This can push up e_b above e_g. The principal can also make $t_l^b > 0$, which occurs in case II when only the (IC^b) is binding. These results are formally derived in Appendix II.

8 Continuum of Types

ln this section, we consider a model with a continuum of types to show that our key results do not depend on the binary type modeling choice. Increasing the number of types offers more options for each type to misreport and shirk, which greatly complicates the model. We find that local ICs do not necessarily imply the global ones given ex post moral hazard. We again show that the incentive to understate cost while shirking

[37] Again, the incentive constraints depend on n only through γ_n and the principal's payoff depends on n only through δ_n.

remains a relevant problem, and that competition makes this problem worse. We also show that high levels of competition can be harmful under an efficient allocation rule under similar conditions as before. We discuss these further after presenting the model, but proofs and detailed arguments are relegated to Appendix III.

We assume that the agent's type θ is distributed over $[g, b]$, where we now refer to type-g as the best and type-b as the worst type. The distribution and density functions are given by $F(\theta)$ and $f(\theta)$ respectively. As in the main section, suppose that the agent chooses one of two effort levels $e \in \{0, 1\}$, and the probability of high output from effort e is π_e, with $0 < \pi_0 < \pi_1 \leq 1$. To keep our analysis simple, we assume that, for each type θ, the cost of zero effort is 0 and the cost of effort 1 is $\psi(\theta) > 0$, with $\psi'(\theta) > 0$, and that the principal uses a deterministic allocation rule and induces $e(\theta) = 1$.

The transfers to the selected agent if his type is θ is $t_h(\theta)$ when the output is high. Again, w.l.o.g. we can set low-output transfer $t_l(\theta) = 0$. The probability that an agent will win the contract upon reporting type θ in an efficient mechanism is $\gamma_n(\theta) = (1 - F(\theta))^{n-1}$. For notational convenience, define an "expected transfer function" $T(\theta)$ for each type as follows:

$$T(\theta) \equiv \gamma_n(\theta)\, t_h(\theta).$$

Then, the incentive constraints are given by

$$\pi_1 T(\theta) - \gamma_n(\theta)\,\psi(\theta) \geq \pi_1 T\left(\hat{\theta}\right) - \gamma_n\left(\hat{\theta}\right)\psi(\theta) \qquad (IC_1)$$

$$\pi_1 T(\theta) - \gamma_n(\theta)\,\psi(\theta) \geq \pi_0 T\left(\hat{\theta}\right). \qquad (IC_0)$$

where $\hat{\theta}$ is the agent's report in $[g, b]$. The (IC_1) ensure that no type has an incentive to misreport while working, (IC_0) ensure that types do not misreport while shirking, and the participation constraint is satisfied whenever (IC_0) is satisfied. Note that (IC_0) is identical to the moral hazard constraints when $\theta = \hat{\theta}$.

The incentive constraint while shirking (IC_0) is the more interesting one, and we find that local ICs are not enough to guarantee global incentives. However, we show that (IC_0) can be simply presented as one global constraint which prevents the worst type from mimicking the best type while shirking. For this, we use (IC_1) to characterize the expected transfer $T(\theta)$ and show in Appendix III that $T(\theta)$ is non-increasing.[38] This implies that the best type receives the highest expected transfer. Thus, from the RHS of (IC_0), while considering to misreport, each type will prefer to claim to be the best type when shirking. Since this incentive is the greatest for the worst type who has the lowest payoff, we can replace (IC_0) as one global constraint:[39]

$$\pi_1 T(b) - \gamma_n(b)\psi(b) \geq \pi_0 T(g). \qquad (IC_0')$$

More importantly for our analysis, this global $(IC_0^{'})$ captures the incentive to understate cost while shirking, and we find that it is always binding. If not, the expected transfer $T(\theta)$ can be decreased by the same amount for each θ without violating either IC constraint.[40]

Next, we study the impact of competition on the principal's payoff. The following proposition shows that high levels of competition hurts the principal in general.

Proposition 7 *Given an efficient allocation rule, there can be too much competition. Specifically, there is a n^* such that the principal is worse off with an additional agent whenever $n \geq n^*$.*

Proof. See Appendix III. ■

Recall from our previous sections that under the efficient allocation rule the effect of competition on the principal's payoff depended on the extent of cost advantage of the good type over the bad type. Using a uniform distribution for θ for tractability, we find a similar effect with continuous types as well which is relevant for lower levels of competition. In particular, if the best type's cost advantage over the worst type is large, some competition is desirable. Of course, as noted more generally in the above proposition there can be too much competition.

[38] In fact, we show that $\pi_1 T^{'}(\theta) = \gamma_n^{'}(\theta)\psi(\theta)$.

[39] As is standard, (IC_1) also implies that the agent's payoff is non-increasing in θ.

[40] From (IC_1), the expected transfer function can be characterized up to a constant $T(b)$:

$$T(\theta) = T(b) - \frac{1}{\pi_1}\int_\theta^b \gamma_n'(x)\psi(x)\,dx,$$

and $T(b)$ can be decreased if $(IC_0^{'})$ is not binding.

Proposition 8 *Given an efficient allocation rule, suppose that the cost type θ is distributed uniformly over $[g, b]$ where $b > g > 0$ and $\psi(\theta) = \theta$.*

(i) If the best type's cost advantage over the worst type is not large, i.e., $(b - g)/g < \frac{3\pi_0}{\pi_1 - \pi_0}$, increased competition always hurts the principal.

(ii) If the best type's cost advantage over the worst type is large, i.e., $(b - g)/g > \frac{3\pi_0}{\pi_1 - \pi_0}$, there is an $n^* \geq 2$ such that increased competition helps the principal when $n < n^*$ and hurts the principal when $n \geq n^*$.

Proof. See Appendix III. ■

9 Conclusion

As noted in the introduction, there is widespread concern that competitive bidding can lead to poor quality ex post. This connection has been largely ignored in the theoretical literature that has focused on the adverse selection problem to emphasize ex ante rent extraction. Procuring a project requires not only to select the most efficient firm (adverse selection), but also to make sure that the selected firm has the correct incentives to implement the project (moral hazard). Our analysis highlights the interaction between the two and explains how competition for the project results in a trade-off that may hurt the procurer. While competition is typically expected to be beneficial in dissipating the rent due to adverse selection, the presence of moral hazard can significantly interfere with rent extraction. Introducing the option to shirk allows a high-cost firm to mimic a low-cost firm and put in low effort, which results in an additional rent for the high-cost firm. Attempts to use increased competition to extract a low-cost firm's rent may lead to an increased rent to a high-cost firm. As a consequence, the procurer may find it optimal to limit the number of potential firms.

We show that insisting on allocative efficiency is costly from an incentive point of view and gives a strong incentive for a bad type to claim to be a good type as it lowers the probability that a bad type will be awarded the contract – it is zero as soon as there is only one good type present. By randomly assigning the contract to a bad type even when a good type is present, the procurer can lower the cost of inducing truth-telling significantly. Remarkably, we show that the procurer can use a random allocation mechanism to mimic a deterministic mechanism with the optimal number of agents without actually limiting the number of agents.

The framework presented here captures an important element of procurement auction. At the same time, it is highly tractable which should allow exploration of a variety of relevant interesting questions in competitive procurement. For instance, suppose that the procurer can use an audit technology to verify ex post the efficiency of the selected firm (adverse selection) or its effort (moral hazard). Which one should she concentrate her resources on? We have also ignored the cost of suppliers to participate in the procurement process. What if it is costly to prepare a submission? Similarly, the procurer could also decide to impose a fee to participate in the mechanism. We could study the role of endogenous entry instead of assuming that there is a fixed number of participants. The model presented here is simple enough that it would be possible to explore these and other related questions in procurement with moral hazard.

10 Appendix

10.1 Appendix I: Binary effort

First, we show that the restrictions imposed on the transfers are without loss of generality.

Restricting transfers to be functions of agent's own report only is w.l.o.g. Consider general transfers $\bar{t}_h^{\hat{\theta}_i,m}$, $\bar{t}_l^{\hat{\theta}_i,m}$, and $\beta\left(\hat{\theta}_i,m\right)$ the probability of receiving the contract by agent i reporting $\hat{\theta}_i$ when m of the other $n-1$ agents report type g. The expected utility to the agent if he chooses effort e is given by

$$\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i,m\right)\left(\pi_e \bar{t}_h^{\hat{\theta}_i,m} + (1-\pi_e)\,\bar{t}_l^{\hat{\theta}_i,m} - \psi_e^{\theta_i}\right)$$

where p_m is the probability that m of the other bidders are of type g. Define

$$\eta_n^{\hat{\theta}_i} \equiv \sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right) \text{ and } t_h^{\hat{\theta}_i} \equiv \frac{\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right) \bar{t}_h^{\hat{\theta}_i, m}}{\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right)}, t_l^{\hat{\theta}_i} \equiv \frac{\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right) \bar{t}_l^{\hat{\theta}_i, m}}{\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right)}$$

Then the expected utility under the new transfers $t_h^{\hat{\theta}_i, m}$ and $t_l^{\hat{\theta}_i, m} = 0$ with effort e is

$$\begin{aligned}
&\eta_n^{\hat{\theta}_i} \left(\pi_e t_h^{\hat{\theta}_i} + (1-\pi_e)\, t_l^{\hat{\theta}_i} - \psi_e^{\theta_i}\right) \\
&= \left(\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right)\right) \left(\pi_e \frac{\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right) \bar{t}_h^{\hat{\theta}_i, m}}{\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right)} + (1-\pi_e) \frac{\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right) \bar{t}_l^{\hat{\theta}_i, m}}{\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right)} - \psi_e^{\theta_i}\right) \\
&= \left(\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right) \left(\pi_e \bar{t}_h^{\hat{\theta}_i, m} + (1-\pi_e)\, \bar{t}_l^{\hat{\theta}_i, m}\right) - \sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right) \psi_e^{\theta_i}\right) \\
&= \left(\sum_{m=0}^{n-1} p_m \beta\left(\hat{\theta}_i, m\right) \left(\pi_e \bar{t}_h^{\hat{\theta}_i, m} + (1-\pi_e)\, \bar{t}_l^{\hat{\theta}_i, m} - \psi_e^{\theta_i}\right)\right)
\end{aligned}$$

which is the same as the expected payoff under the original transfers $\bar{t}_h^{\hat{\theta}_i, m}$, $\bar{t}_l^{\hat{\theta}_i, m}$. Hence, if the old transfers satisfy incentive constraints so do the new transfers. Furthermore, since the expected payment is the same to each bidder of each type under the old and new transfers the principal's expected payment are equal. Thus, the restriction of transfers to be functions of an agent's own report only is w.l.o.g.

The principal cannot be better off making a payment to a losing bidder

Let us consider a mechanism $\left\{(f^g, t_h^g, t_l^g, \phi_r), \left(f^b, t_h^b, t_l^b, \phi_r\right)\right\}$ where allocation is potentially random that satisfies the incentive and participation constraints:

$$\pi_1 t_h^g + (1-\pi_1)\, t_l^g - \psi_1^g \geq \pi_0 t_h^g + (1-\pi_0)\, t_l^g - \psi_0^g \qquad (MH_g)$$

$$\pi_1 t_h^b + (1-\pi_1)\, t_l^b - \psi_1^b \geq \pi_0 t_h^b + (1-\pi_0)\, t_l^b - \psi_0^b \qquad (MH_b)$$

$$(1-\gamma_n^g)\, f^g + \gamma_n^g \pi_1 t_h^g + \gamma_n^g (1-\pi_1)\, t_l^g - \gamma_n^g \psi_1^g \geq \left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_1 t_h^b + \gamma_n^b (1-\pi_1)\, t_l^b - \gamma_n^b \psi_1^g \qquad (IC_1^g)$$

$$(1-\gamma_n^g)\, f^g + \gamma_n^g \pi_1 t_h^g + \gamma_n^g (1-\pi_1)\, t_l^g - \gamma_n^g \psi_1^g \geq \left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_0 t_h^b + \gamma_n^b (1-\pi_0)\, t_l^b - \gamma_n^b \psi_0^g \qquad (IC_0^g)$$

$$\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b-\gamma_n^b \psi_1^b \geq\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^b\left(1-\pi_0\right) t_l^b-\gamma_n^g \psi_1^b \tag{IC_1^b}$$

$$\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b-\gamma_n^b \psi_1^b \geq\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_0 t_h^g+\gamma_n^b\left(1-\pi_0\right) t_l^b-\gamma_n^g \psi_0^b \tag{IC_0^b}$$

$$\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g-\gamma_n^g \psi_1^g \geq 0 \tag{IR^g}$$

$$\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b-\gamma_n^b \psi_1^b \geq 0 \tag{IR^b}$$

Now construct payments

$$\hat{t}_h^g=\frac{\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g}{\gamma_n^g \pi_1}, \hat{t}_l^g=0$$

$$\hat{t}_h^b=\frac{\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b}{\gamma_n^b \pi_1}, \hat{t}_l^b=0$$

and consider the mechanism $\left\{\left(\hat{t}_h^g, \phi_r\right),\left(\hat{t}_h^b, \phi_r\right)\right\}$ without the payments to a losing bidder.

$$\frac{\pi_0}{\pi_1}\left(1-\pi_1\right)<\left(1-\pi_0\right)$$

$$\pi_0<\pi_1$$

Substituting these transfers in the constraints we have

$$\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g-\gamma_n^g \psi_1^g \geq \frac{\pi_0}{\pi_1}\left[\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g\right]-\gamma_n^g \psi_0^g \tag{$MH_g{}'$}$$

$$\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b-\gamma_n^b \psi_1^b \geq \frac{\pi_0}{\pi_1}\left[\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b\right]-\gamma_n^b \psi_0^b \tag{$MH_b{}'$}$$

$$\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g-\gamma_n^g \psi_1^g \geq\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b-\gamma_n^b \psi_1^g \tag{$IC_1^{g\prime}$}$$

$$\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g-\gamma_n^g \psi_1^g \geq \frac{\pi_0}{\pi_1}\left[\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b\right]-\gamma_n^b \psi_0^g \tag{$IC_0^{g\prime}$}$$

$$\left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_1 t_h^b - \gamma_n^b \psi_1^b \geq \left(1-\gamma_n^g\right) f^g + \gamma_n^g \pi_1 t_h^g - \gamma_n^g \psi_1^b \qquad (IC_1^{b\prime})$$

$$\left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_1 t_h^b - \gamma_n^b \psi_1^b \geq \frac{\pi_0}{\pi_1}\left[\left(1-\gamma_n^g\right) f^g + \gamma_n^g \pi_1 t_h^g\right] - \gamma_n^g \psi_0^b \qquad (IC_0^{b\prime})$$

$$\left(1-\gamma_n^g\right) f^g + \gamma_n^g \pi_1 t_h^g + \gamma_n^g \left(1-\pi_1\right) t_l^g - \gamma_n^g \psi_1^g \geq 0 \qquad (IR^{g\prime})$$

$$\left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_1 t_h^b + \gamma_n^b \left(1-\pi_1\right) t_l^b - \gamma_n^b \psi_1^b \geq 0 \qquad (IR^{b\prime})$$

$\pi_0 < \pi_1$ implies $\frac{\pi_0}{\pi_1} < 1$ and $\frac{\pi_0}{\pi_1}\left(1-\pi_1\right) < \left(1-\pi_0\right)$. Hence, when the original mechanism $\left\{\left(f^g, t_h^g, t_l^g, \phi_r\right), \left(f^b, t_h^b, t_l^b, \phi_r\right)\right\}$ satisfies the MH, IC and IR constraints so do the conditions ($IC_1^{g\prime}$), ($IC_1^{b\prime}$), ($IR^{g\prime}$) and ($IR^{b\prime}$) (which are identical), as well as the weaker conditions ($MH_g{}'$), ($MH_b{}'$), ($IC_0^{g\prime}$), and ($IC_0^{b\prime}$).

Now we give the formal proofs of our Lemma, Propositions and Corollary.

Proof of Lemma. Suppose $\left(t_h^g, t_l^g, t_h^b, t_l^b\right)$ satisfy the IC, MH and IR constraints and consider the alternative transfers $\left(\bar{t}_h^g, \bar{t}_l^g, \bar{t}_h^b, \bar{t}_l^b\right)$ that keep the principal's payoff unchanged:

$$\bar{t}_h^g = \frac{\pi_1 t_h^g + (1-\pi_1) t_l^g}{\pi_1}, \bar{t}_h^b = \frac{\pi_1 t_h^b + (1-\pi_1) t_l^b}{\pi_1}, \bar{t}_l^g = \bar{t}_l^b = 0.$$

Substituting these transfers in the IC and MH constraints for $\left(\bar{t}_h^g, \bar{t}_l^g, \bar{t}_h^b, \bar{t}_l^b\right)$ we need to verify that the following inequalities hold:

$$\begin{aligned}
IC_1^g &: \gamma_n^g \left(\pi_1 \bar{t}_h^g + (1-\pi_1)\bar{t}_l^g - \psi_1^g\right) = \gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1) t_l^g - \psi_1^g\right) \\
&\geq \gamma_n^b \left(\pi_1 \bar{t}_h^b + (1-\pi_1)\bar{t}_l^b - \psi_1^g\right) = \gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1) t_l^b - \psi_1^g\right) \\
IC_0^g &: \gamma_n^g \left(\pi_1 \bar{t}_h^g + (1-\pi_1)\bar{t}_l^g - \psi_1^g\right) = \gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1) t_l^g - \psi_1^g\right) \\
&\geq \gamma_n^b \left(\pi_0 \bar{t}_h^b + (1-\pi_0)\bar{t}_l^b - \psi_0^g\right) = \gamma_n^b \left(\frac{\pi_0}{\pi_1}\left(\pi_1 t_h^b + (1-\pi_1) t_l^b\right) - \psi_0^g\right)
\end{aligned}$$

$$\begin{aligned}
MH_g &: \gamma_n^g \left(\pi_1 \bar{t}_h^g + (1-\pi_1)\bar{t}_l^g - \psi_1^g\right) = \gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1) t_l^g - \psi_1^g\right) \\
&\geq \gamma_n^g \left(\pi_0 \bar{t}_h^g + (1-\pi_0)\bar{t}_l^g - \psi_0^g\right) = \gamma_n^g \left(\frac{\pi_0}{\pi_1}\left(\pi_1 t_h^g + (1-\pi_1) t_l^g\right) - \psi_0^g\right)
\end{aligned}$$

$$IC_1^b : \gamma_n^b \left(\pi_1 \bar{t}_h^b + (1-\pi_1)\bar{t}_l^b - \psi_1^b\right) = \gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1) t_l^b - \psi_1^b\right)$$
$$\geq \gamma_n^g \left(\pi_1 \bar{t}_h^g + (1-\pi_1)\bar{t}_l^g - \psi_1^b\right) = \gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1) t_l^g - \psi_1^b\right)$$
$$IC_0^b : \gamma_n^b \left(\pi_1 \bar{t}_h^b + (1-\pi_1)\bar{t}_l^b - \psi_1^b\right) = \gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1) t_l^b - \psi_1^b\right)$$
$$\geq \gamma_n^g \left(\pi_0 \bar{t}_h^g + (1-\pi_0)\bar{t}_l^g - \psi_0^b\right) = \gamma_n^g \left(\frac{\pi_0}{\pi_1}\left(\pi_1 t_h^g + (1-\pi_1) t_l^g\right) - \psi_0^b\right)$$

$$MH_b : \gamma_n^b \left(\pi_1 \bar{t}_h^b + (1-\pi_1)\bar{t}_l^b - \psi_1^b\right) = \gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1) t_l^b - \psi_1^b\right)$$
$$\geq \gamma_n^b \left(\pi_0 \bar{t}_h^b + (1-\pi_0)\bar{t}_l^b - \psi_0^b\right) = \gamma_n^b \left(\frac{\pi_0}{\pi_1}\left(\pi_1 t_h^b + (1-\pi_1) t_l^b\right) - \psi_0^b\right)$$

$\frac{\pi_0}{\pi_1} < 1$ imply that the above inequalities are satisfied. The IR constraints are automatically satisfied. And, by construction, the principal's payoff remains unchanged. ■

Proof of Proposition 1. First note that under a deterministic rule the principal can either (i) allocate the contract to a good type over a bad type whenever there is one, or (ii) allocate the contract to a bad type over a good type whenever there is one. If the contract is awarded to a bad type with probability 1, whenever there is one, for $n > 1$, we have $\gamma_n < 1$. In this case, the IC_1^g and IC_1^b become

$$IC_1^g : \gamma_n \pi_1 t_h^g - \gamma_n \psi_1^g + \psi_1^g \geq \pi_1 t_h^b$$
$$IC_1^b : \pi_1 t_h^b \geq \gamma_n \pi_1 t_h^g - \gamma_n \psi_1^b + \psi_1^b$$

Putting these together we have

$$\gamma_n \left(\psi_1^b - \psi_1^g\right) \geq \psi_1^b - \psi_1^g.$$

Thus IC_1^g and IC_1^b cannot hold together when $\gamma_n < 1$, i.e., the solution necessarily involves awarding the contract to a good type when there is one.

Now consider the reduced problem given in Section 3. The principal chooses the two transfers $\{t_h^b, t_h^g\}$ to solve

$$\max \Pi_{11}(n) = \pi_1 \left[V - \left(\delta_n t_h^g + (1-\delta_n)\, t_h^b\right)\right]$$

subject to,

$$\gamma_n \left(\pi_1 t_h^g - \psi_1^g\right) \geq \pi_1 t_h^b - \psi_1^g \tag{IC_1^g}$$

$$t_h^g \geq \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \tag{MH^g}$$

$$\pi_1 t_h^b - \psi_1^b \geq \gamma_n \left(\pi_0 t_h^g - \psi_0^b\right) \tag{IC_0^b}$$

$$t_h^b \geq \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0}. \tag{MH^b}$$

First, consider the restricted problem of maximizing

$$\pi_1 V - \left[\left(1 - (1-q)^n\right) \pi_1 t_h^g + (1-q)^n \pi_1 t_h^b\right]$$

subject only to the IC_1^g and IC_0^b constraints. Since the principal's payoff is decreasing in t_h^g and t_h^b, the inequality and the fact that the LHS of IC_1^g and the RHS of IC_0^b above are both increasing in t_h^g together imply that the solution of the reduced problem is given by

$$t_h^g = \frac{\gamma_n \left(\psi_1^g - \psi_0^b\right) + \left(\psi_1^b - \psi_1^g\right)}{\gamma_n \left(\pi_1 - \pi_0\right)}$$

$$t_h^b = \frac{\gamma_n \pi_0}{\pi_1} t_h^g - \frac{\left(\gamma_n \psi_0^b - \psi_1^b\right)}{\pi_1} = \frac{\gamma_n \left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) + \pi_1 \psi_1^b - \pi_0 \psi_1^g}{\pi_1 \left(\pi_1 - \pi_0\right)}$$

We will now show that for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$ the remaining constraints are satisfied: Substituting for t_h^g we have

$$\begin{aligned} MH_g : t_h^g - \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} &= \frac{\psi_1^b - \psi_1^g - \gamma_n \left(\psi_0^b - \psi_0^g\right)}{\left(\pi_1 - \pi_0\right) \gamma_n} \\ &\geq \frac{\psi_1^b - \psi_1^g - \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g} \left(\psi_0^b - \psi_0^g\right)}{\left(\pi_1 - \pi_0\right) \gamma_n} \\ &= 0 \end{aligned}$$

Substituting for t_h^b

$$MH_b : t_h^b - \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0} = \left(\gamma_n - 1\right) \frac{\pi_0 \psi_1^g - \pi_1 \psi_0^b}{\pi_1 \left(\pi_1 - \pi_0\right)} > 0$$

and under our assumption.

$$IC_1^b : \left(\pi_1 t_h^b - \psi_1^b\right) - \gamma_n \left(\pi_1 t_h^g - \psi_1^b\right) = \left(\gamma_n + 1\right)\psi_1^g + \left(\gamma_n - 1\right)\psi_1^b > 0.$$

Next for $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$ consider the restricted problem of maximizing $\pi_1 V - \left[\left(1 - (1-q)^n\right)\pi_1 t_h^g + (1-q)^n \pi_1 t_h^b\right]$ subject only to the MH_g and IC_0^b constraints. Following similar arguments as above, the solution is given by

$$\begin{aligned} t_h^g &= \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \\ t_h^b &= \frac{\gamma_n \pi_0}{\pi_1}\frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \frac{\gamma_n \psi_0^b - \psi_1^b}{\pi_1} \end{aligned}$$

We will now show that all the remaining constraints are satisfied by this solution:

$$\begin{aligned} IC_1^g : \frac{1}{\pi_1}\left(\gamma_n\left(\pi_1 t_h^g - \psi_1^g\right) + \psi_1^g\right) - t_h^b \\ = \frac{\psi_1^g - \psi_1^b + \gamma_n\left(\psi_0^b - \psi_0^g\right)}{\pi_1} \\ > \frac{\psi_1^g - \psi_1^b + \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\left(\psi_0^b - \psi_0^g\right)}{\pi_1} \\ = \frac{\psi_1^g - \psi_1^b + \psi_1^b - \psi_1^g}{\pi_1} \\ = 0 \end{aligned}$$

$$\begin{aligned} IC_1^b &: \left(\pi_1 t_h^b - \psi_1^b\right) - \gamma_n\left(\pi_1 t_h^g - \psi_1^b\right) \\ &= \gamma_n \pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \gamma_n \psi_0^b - \gamma_n \pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + \gamma_n \psi_1^b \\ &= \frac{\gamma_n}{\pi_1 - \pi_0}\left(\pi_1 - \pi_0\right)\left[\left(\psi_1^b - \psi_0^b\right) - \left(\psi_1^g - \psi_0^g\right)\right] \\ &> 0 \text{ by L(ii)} \end{aligned}$$

$$\begin{aligned} MH_b &: t_h^b - \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0} \\ &= \frac{1}{\pi_1 - \pi_0}\left[\gamma_n\left[\pi_0\psi_1^g - \pi_1\psi_0^b + \pi_0\left(\psi_0^b - \psi_0^g\right)\right] + \psi_1^b\left(\pi_1 - \pi_0\right) - \pi_1\left(\psi_1^b - \psi_0^b\right)\right] \end{aligned}$$

(using L(iii) the coefficient of γ_n is positive, hence replacing γ_n by its minimum value)

$$\geq \frac{1}{(\pi_1-\pi_0)\left(\psi_0^b-\psi_0^g\right)}\left[\begin{array}{c}\left(\psi_1^b-\psi_1^g\right)\left[\pi_0\psi_1^g-\pi_1\psi_0^b+\pi_0\left(\psi_0^b-\psi_0^g\right)\right] \\ +\psi_1^b(\pi_1-\pi_0)\left(\psi_0^b-\psi_0^g\right)-\pi_1\left(\psi_1^b-\psi_0^b\right)\left(\psi_0^b-\psi_0^g\right)\end{array}\right]$$

$$= \frac{1}{(\pi_1-\pi_0)\left(\psi_0^b-\psi_0^g\right)}\left[\left(\psi_1^b-\psi_0^b\right)-\left(\psi_1^g-\psi_0^g\right)\right]\left(\pi_0\psi_1^g-\pi_1\psi_0^b\right)$$

> 0 by L(ii) and L(iii)

■

Proof of Proposition 2 The expected payoff to the principal under a given contract is given by

$$\delta_n\left(\pi_{e_g}V-\pi_{e_g}t_h^g-\left(1-\pi_{e_g}\right)t_l^g\right)+(1-\delta_n)\,\pi_{e_b}V$$

which can be rewritten as

$$\delta_n\pi_{e_g}V+(1-\delta_n)\,\pi_{e_b}V-\left[\delta_n\left(\pi_{e_g}t_h^g+\left(1-\pi_{e_g}\right)t_l^g\right)+(1-\delta_n)\left(\pi_{e_b}t_h^b+(1-\pi_{e_b})\,t_l^b\right)\right]$$

First, notice that given a good type is favored in an auction and the effort levels assigned to the two types, the additive separability of V and the transfers imply that the principals expected payoff maximization problem is equivalent to her expected transfer minimization problem. Second in the minimization problem subject to the relevant IC, MH and IR constraints the minimum is well defined. Let the minimum expected transfer be defined for a given effort assignment e_g and e_b by $\tau(e_g, e_b)$.

Proof. The principal's problem is then to solve

$$\max_{e_g,e_b}\ \delta_n\pi_{e_g}V+(1-\delta_n)\,\pi_{e_b}V-\tau(e_g,e_b).$$

Specifically, we need to identify the maximum among

$$\pi_1V-\tau(1,1),\ \ (\delta_n\pi_1+(1-\delta_n)\,\pi_0)\,V-\tau(1,0),\ \text{and}\ \pi_0V-\tau(0,0)$$

It follows that for V large enough the maximum is given by setting $e_g=1, e_b=1$ since $\pi_1>(\delta_n\pi_1+(1-\delta_n)\,\pi_0)>\pi_0$ and for large enough V we have

$$\pi_1V-(\delta_n\pi_1+(1-\delta_n)\,\pi_0)\,V>\tau(1,1)-\tau(1,0)$$

and

$$\left(\delta_n \pi_1 + (1-\delta_n)\,\pi_0\right) V - \pi_0 V > \tau(1,0) - \tau(0,0).$$

■

Proof of Proposition 3. The expected payoff to the principal is

$$\begin{aligned} &(1-(1-q)^n)\,\pi_1\,(V - t_h^g) + (1-q)^n\,\pi_1\left(V - t_h^b\right) \\ &= \pi_1 V - \left[(1-(1-q)^n)\,\pi_1 t_h^g + (1-q)^n\,\pi_1 t_h^b\right] \end{aligned}$$

For $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$ the payoff at the optimal transfers is

$$\Pi(n) = \pi_1 V - \left(\begin{array}{c} (1-(1-q)^n)\,\pi_1 \frac{\left(\gamma_n \psi_g - \psi_1^g - \gamma_n \psi_0^b + \psi_1^b\right)}{(\pi_1 - \pi_0)\gamma_n} \\ +\,(1-q)^n\,\pi_1 \frac{\gamma_n\left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) + \pi_1 \psi_1^b - \pi_0 \psi_1^g}{\pi_1(\pi_1 - \pi_0)} \end{array} \right)$$

and for $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$ it is

$$\Pi(n) = \pi_1 V - \left((1-(1-q)^n)\,\pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + (1-q)^n \left(\gamma_n \pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \gamma_n \psi_0^b + \psi_1^b \right) \right)$$

We have for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$

$$\Pi(n+1) - \Pi(n) = \frac{(1-q)^{n-1}}{\pi_1 - \pi_0} \left(\pi_1 \left(\psi_0^b\,(1-q) + \left(\psi_1^b - \psi_1^g\right) q \right) - \pi_0 \psi_1^g\,(1-q) \right)$$

Thus, $\Pi(n)$ is *decreasing* for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$ if and only if

$$\pi_1 \left(\psi_0^b\,(1-q) + \left(\psi_1^b - \psi_1^g\right) q \right) - \pi_0 \psi_1^g\,(1-q) < 0$$

or,

$$\pi_1 q \left(\psi_1^b - \psi_1^g\right) < (1-q)\left[\pi_0 \psi_1^g - \pi_1 \psi_0^b\right]$$

For $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$ we have

$$\Pi(n+1) - \Pi(n) = \frac{(1-q)^n}{\pi_1 - \pi_0} \left(\begin{array}{c} \pi_1 \left(\psi_0^b - \psi_0^b q + \left(\psi_0^g + \psi_1^b - \psi_1^g \right) q \right) \\ -\pi_0 \left(\psi_0^b + \psi_1^g + \psi_0^g\,(-1+q) - \psi_0^b q + \psi_1^b q - \psi_1^g q \right) \end{array} \right)$$

In this case $\Pi(n)$ is *decreasing* if and only if

$$\left(\pi_1\left(\psi_0^b-\psi_0^b q+\left(\psi_0^g+\psi_1^b-\psi_1^g\right)q\right)-\pi_0\left(\psi_0^b+\psi_1^g+\psi_0^g\left(-1+q\right)-\psi_0^b q+\psi_1^b q-\psi_1^g q\right)\right)<0$$

or, simplifying,

$$\pi_1 q\left(\psi_1^b-\psi_1^g\right)<(1-q)\left[\pi_0\psi_1^g-\pi_1\psi_0^b\right]+(1-q)\left(\psi_0^b-\psi_0^g\right)\pi_0+q\left(\pi_0\psi_1^b-\pi_1\psi_0^g\right)$$

■

Proof of Corollary. The proof follows from Proposition 3 upon observing that condition L implies $\pi_0\psi_1^b-\pi_1\psi_0^g\geq 0$ and $\psi_0^b>\psi_0^g$, so

$$(1-q)\left(\psi_0^b-\psi_0^g\right)\pi_0+q\left(\pi_0\psi_1^b-\pi_1\psi_0^g\right)\geq 0.$$

In this case, if

$$\pi_1 q\left(\psi_1^b-\psi_1^g\right)>(1-q)\left[\pi_0\psi_1^g-\pi_1\psi_0^b\right]+(1-q)\left(\psi_0^b-\psi_0^g\right)\pi_0+q\left(\pi_0\psi_1^b-\pi_1\psi_0^g\right)$$

holds, so does

$$\pi_1 q\left(\psi_1^b-\psi_1^g\right)>(1-q)\left(\pi_0\psi_1^g-\pi_1\psi_0^b\right).$$

■

Proof of Proposition 4. In this proof, we use $\tilde{\gamma}_n$ and $\tilde{\delta}_n$ to denote the relevant variables under the random allocation rule and γ_n and δ_n under the deterministic allocation rule. Thus γ_n and δ_n are special cases of $\tilde{\gamma}_n$ and $\tilde{\delta}_n$ when $\phi_r=1$ for $r\in\{1,2,..n\}$.

Fix $n^*<n$. Let

$$\tilde{\gamma}_n\equiv\frac{\sum_{r=1}^{n}\binom{n-1}{r-1}q^{r-1}\left(1-q\right)^{n-r}\frac{\phi_r}{r}}{\sum_{r=0}^{n-1}\binom{n-1}{r}q^{r}\left(1-q\right)^{n-1-r}\frac{1-\phi_r}{n-r}}$$

the γ generated by the random allocation rule when there are n bidders. We have $\tilde{\gamma}_n=1$ if and only if

$$\begin{aligned}&\sum_{r=1}^{n}\binom{n-1}{r-1}q^{r-1}\left(1-q\right)^{n-r}\frac{\phi_r}{r}\\&=\frac{1}{n\left(1-q\right)}\sum_{r=1}^{n}\frac{n!}{\left(n-r+1\right)!\left(r-1\right)!}q^{r-1}\left(1-q\right)^{n-r+1}\left(1-\phi_{r-1}\right)\end{aligned}$$

or,

$$\sum_{r=1}^{n}\frac{(n-1)!}{(n-r)!r!}q^{r-1}(1-q)^{n-r}\left[\phi_r-\frac{r}{(n-r+1)}(1-\phi_{r-1})\right]=0.$$

The difference equation $\phi_r=\frac{r}{(n-r+1)}(1-\phi_{r-1})$ is solved by $\phi_r=\frac{r}{n}$. Hence for $\phi_r=\frac{r}{n}$ we have $\tilde{\gamma}_n=1$. Now $\tilde{\gamma}_n$ is continuously increasing in ϕ_r. For $\phi_r=\frac{r}{n}$ we have $\tilde{\gamma}_n=1$ and for $\phi_r\equiv 1$ we have $\tilde{\gamma}_n=\gamma_n$. Since $1\le n^*<n$ implies $1\le\gamma_{n^*}<\gamma_n$, by continuity there exist $\phi_r^*\ge\frac{r}{n}$, $r=1,..,n-1$, such that $\tilde{\gamma}_n=\gamma_{n^*}$. In what follows we consider this $\tilde{\gamma}_n$ evaluated at ϕ_r^*.

We have

$$\tilde{\delta}_n=\sum_{r=1}^{n}\binom{n}{r}q^r(1-q)^{n-r}\phi_r^*$$

and

$$1-\tilde{\delta}_n=\sum_{r=0}^{n-1}\binom{n}{r}q^r(1-q)^{n-r}(1-\phi_r^*)$$

Also, we can rewrite

$$\tilde{\gamma}_n^g=\frac{1}{nq}\sum_{r=1}^{n}\frac{n!}{(n-r)!r!}q^r(1-q)^{n-r}\phi_r^*=\frac{1}{nq}\tilde{\delta}_n$$

and

$$\tilde{\gamma}_n^b=\frac{1}{n(1-q)}\sum_{r=0}^{n-1}\frac{n!}{(n-r)!r!}q^r(1-q)^{n-r}(1-\phi_r^*)=\frac{1}{n(1-q)}\left(1-\tilde{\delta}_n\right)$$

so that

$$\tilde{\gamma}_n=\frac{1-q}{q}\frac{\tilde{\delta}_n}{1-\tilde{\delta}_n}\tag{3}$$

Next consider the above $n^*<n$ and the γ_{n^*} that is generated through a deterministic allocation strategy when there are n^* bidders. The expected payoff to the principal is given by

$$\begin{aligned}\Pi_{11}(n^*)&=\pi_1\left[V-\left(\delta_{n^*}t_h^g+(1-\delta_{n^*})\,t_h^b\right)\right]\\&=\pi_1\left[V-\left(\frac{q\gamma_{n^*}}{1-q+q\gamma_{n^*}}t_h^g+\left(1-\frac{q\gamma_{n^*}}{1-q+q\gamma_{n^*}}\right)t_h^b\right)\right]\\&=\pi_1\left[V-\left(\frac{q\tilde{\gamma}_n}{1-q+q\tilde{\gamma}_n}t_h^g+\left(1-\frac{q\tilde{\gamma}_n}{1-q+q\tilde{\gamma}_n}\right)t_h^b\right)\right]\\&=\pi_1\left[V-\left(\tilde{\delta}_nt_h^g+\left(1-\tilde{\delta}_n\right)t_h^b\right)\right]\end{aligned}$$

where the last equality is obtained using (3). Recall that the incentive constraints depend on n only through γ_n. Hence, the optimal transfers obtained under the deterministic rule with n^* bidders also satisfy the IC and MH constraints for n bidders under random allocation with $\phi_r = \phi_r^*$. Thus the optimal payoff to the principal under the random allocation rule ϕ_r^* with n bidders is equal to the optimal payoff under the deterministic allocation rule with n^* bidders.

Finally, suppose competition hurts so that the payoff to the principal is higher when there are $n^* < n$ bidders, i.e., $\Pi_{11}(n^*) > \Pi_{11}(n)$. The above derivation implies that there are ϕ_r^*-s with at least some $\phi_r^* < 1$ $(r \geq 1)$ so that the payoff to the principal is equal to $\Pi_{11}(n^*)$ under the random allocation rule. Thus the optimal allocation in this case assigns bad types with probability $1 - \phi_r^* > 0$ when r agents with good types are present for some $r \geq 1$. ■

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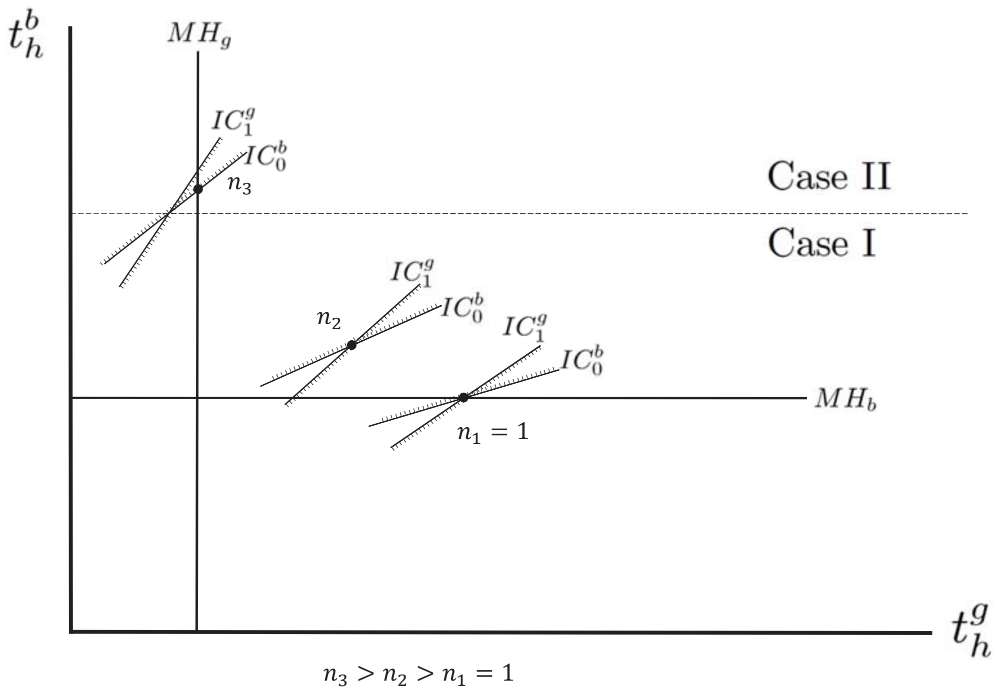

Figure 1: Optimal mechanism