Competition for Local Public Good Provision*

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Abstract

The public good literature claims that lotteries are theoretically and empirically superior to VCM in funding public goods. In reality, however, the two mechanisms coexist. Why is that? One possible explanation is that the current research assumes that individuals are eligible to participate only in one mechanism. This paper develops a three-stage model where two local communities choose a fund-raising mechanism (VCM or lottery) at stage 0, all individuals select a local community at stage 1, and make their public good contributions at stage 2. We characterize Subgame Perfect Equilibria in the model. Our main message is that the lottery mechanism is the dominant choice of the richer community and the VCM mechanism is the dominant choice of the poorer community.

1 Introduction

Studies on public good provision mechanisms have been well documented, with main efforts on looking for a mechanism that generates the most efficient level of public good provision. Different mechanisms have been designed, examined, and compared, among which the voluntary contribution mechanism (VCM) and the *lottery* mechanism are the most frequently discussed ones. Most of the literature compares the relative performance of the VCM and *lottery* mechanisms and reaches the following conclusions. Theoretically, the VCM mechanism suffers from the free-rider problem and leads to under provision of public goods in an equilibrium. On the contrary, theory suggests that the *lottery* mechanism overcomes the free-rider problem and generates almost efficient level of public good provision (see Morgan, 2000). Experimental observations of the VCM mechanism report more contributions than the equilibrium predictions, but these

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contributions usually decay over time when the game is played repeatedly (see Isaac and Walker, 1988; Andreoni 1988, 1995; and reviews of the literature in Chaudhuri, 2011). Experimental studies show that the *lottery* mechanism generates higher public good provision than the VCM mechanism (see Morgan and Sefton, 2000; Lange, List, and Price, 2007; Corazzini, Faravelli, and Stanca, 2010). However, a common assumption both in theory and experiments, is that individuals are only eligible to one mechanism (either VCM or *lottery*) at any time with no other choices. Remarkably, not much has been done to compare the two mechanisms when they are eligible to individuals at the same time. This has raised an intriguing question: if the lottery mechanism is more efficient than the VCM one in terms of public good provision, why we observe the coexistence of the two mechanisms in reality? In this paper, we develop a three-stage model where local communities choose a fund-raising mechanism (VCM or *lottery*) at stage 0, all individuals select local communities at stage 1, and make their public good contributions at stage 2.

Note that stage 2 of our model is either a standard VCM game or a *lottery* game. A sequential stage 1 and 2 game is new in the literature, because individuals can choose the public good mechanism (VCM or *Lottery*) first, before making their contributions in the public good game at stage 2. Finally, we are able to analyze how local communities make their decisions at stage 0.

Our solution concept is a SPE. First, we show that a SPE always exists in our model. Then, we describe all SPEs. It turns out that there are three types of SPEs. In the two types of SPEs: VCM and *Lottery*, all individuals select the same mechanism; and in the $Mixed$ SPE some individuals choose the VCM mechanism and the others select the *Lottery* mechanism. We provide conditions for uniqueness of each type of SPE. It turns out that our model also allows coexistence of multiple SPEs.

Our main message is about local communities decisions at stage 0. It turns out that the *lottery* mechanism is the dominant choice of the richer local community and the VCM mechanism is the dominant choice of the poorer local community.

The rest of the paper is organized as follows. In section 2 we present the model. A sequential stage 1 and 2 game is analyzed in sections 3 and 4. Section 5 studies stage 0 and delivers the main message. All proofs are in Appendix.

2 Model

Suppose that two local communities are seeking for funds to finance local public goods (for example, a city park, a fire department, or a school). Their goal is to maximize the local public good provision. Each community i has a budget, $B_i > 0$, for raising public funds. There are two fund-raising mechanisms that communities could choose from: the Voluntary Contribution Mechanism (VCM) and the Lottery. Communities' budgets are used either as the seed money in the VCM or as the prize in the Lottery.

There are N risk-neutral individuals. Each individual has to join one of the communities in order to benefit from the local public good. Once in the community, an individual decides about her contribution to the local public good mechanism.

The time line is as follows:

- Stage 0: each community simultaneously chooses either VCM or *Lottery* to raise public good funds.
- Stage 1: each individual observes the communities' mechanisms and chooses one to participate.
- Stage 2: each individual observes the number of individuals in her chosen mechanism and makes her contribution decision.

In other words, at stage 0, communities play the following simultaneous-move game

	V	L
V	$E\pi_1(V,V), E\pi_2(V,V)$	$E\pi_1(V,L), E\pi_2(V,L)$
L	$E\pi_1(L,V), E\pi_2(L,V)$	$E\pi_1(L,L), E\pi_2(L,L)$

FIGURE 1: Community game

where, V means VCM choice and L means *Lottery* choice, and, for example, $E\pi_i(V,V)$ is the expected payoff of community i when both communities choose the VCM mechanism. Communities choices of mechanisms at stage 0 depend on their expectations about individuals' choices at stages 1 and 2. At the same time, individuals' choices at stage 1 are determined by their expected payoffs at stage 2. Therefore, we solve the model using backward induction, starting from stage 2. Stage 2 was extensively analyzed in the literature and we provide a brief summary below.

2.1 VCM mechanism

Suppose that there are $k \geq 1$ individuals in the VCM mechanism at stage 2. Each individual i has to decide how much to contribute, $x_i \geq 0$, for the public good provision in order to maximize her payoff:

$$\max_{x_i} u_i(x_i, x_{-i}) = \beta(M + x_i + x_{-i}) - x_i, \tag{1}$$

where $M \geq 0$ is seed money, x_{-i} is total contribution of all individuals but individual i in the VCM mechanism, and $0 < \beta < 1$ is a marginal per capita return (MPCR). The dominant action for each individual is to contribute nothing, which is independent from the number of individuals in the VCM mechanism. Therefore,

$$x^*(k) = 0 \text{ for any } k \geq 1. \tag{2}$$

Hence, each individual gets the same payoff in the VCM mechanism which is

$$u_{VCM} = \beta M. \tag{3}$$

2.2 *Lottery* mechanism

Suppose that there are $l \geq 1$ individuals in the *Lottery* mechanism at stage 2. Each individual has to decide how much to contribute, $y_i \geq 0$, in order to maximize her expected payoff:

$$\max_{y_i} v_i\left(y_i, y_{-i}\right) = \frac{y_i}{y_i + y_{-i}} V + \beta\left(y_i + y_{-i}\right) - y_i, \tag{4}$$

where $V \geq 0$ is the lottery prize and y_{-i} is total contribution of all individuals but individual i in the *Lottery* mechanism. We assume that if $y_i + y_{-i} = 0$, then each individual in the *Lottery* mechanism has the same probability $\frac{1}{l}$ to win the prize V. It is well known that the equilibrium action for each individual is[1]

$$y^*\left(l\right) = \frac{l-1}{l^2} \frac{V}{\left(1-\beta\right)} \tag{5}$$

and the expected individual payoff is

$$EL\left(l\right) = \frac{1}{l} V + \left(\beta l - 1\right) \frac{l-1}{l^2} \frac{V}{\left(1-\beta\right)}. \tag{6}$$

Note that the equilibrium action and the expected individual payoff depend on the number of individuals in (5) and (6).

Define l^* such that

$$\beta l^* = 1. \tag{7}$$

Public good provision is desirable if the number of individuals in the *Lottery* mechanism is $l \geq l^* = 1/\beta$. The following lemma describes the expected equilibrium payoff as a function of the number of individuals in the *Lottery* mechanism.

Lemma 1 *The expected equilibrium payoff function $EL\left(l\right)$ has the following properties on the interval $[1, +\infty)$:*
(1) $EL\left(l\right)$ is decreasing on the interval $[1, 1/\beta]$;
(2) $EL\left(l\right)$ is increasing on the interval $[1/\beta, \infty)$;
(3) $EL\left(l\right)$ is convex on the interval $\left[1, \frac{3}{2\beta}\right]$;
(4) $EL\left(l\right)$ is concave on the interval $\left[\frac{3}{2\beta}, \infty\right)$;
(5)

$$\min_{l \in [1, +\infty)} EL(l) = EL(l^*) = EL(1/\beta) = \beta V;$$

(6)

$$EL\left(l^*\right) = \beta V < V = EL\left(1\right);$$

(7)

$$\lim_{l \to \infty} EL\left(l\right) = \frac{\beta}{\left(1-\beta\right)} V; \tag{8}$$

[1] Morgan (2000) was the first to analyze the Lottery game.

(8)

$$\sup_{l\in[1,+\infty)} EL(l) = \max\left\{V, \frac{\beta}{(1-\beta)}V\right\}.$$

Lemma 1 shows that if the lottery public good provision is desirable, or $\beta l \geq 1$, then more individuals lead to higher expected individual payoffs. However, if the lottery public good provision is undesirable, or $\beta l < 1$, then an addition of an individual might lead to lower expected individual payoffs. Moreover, the minimal expected individual payoff is reached at exactly $l = l^*$. The following Example 1 illustrates Lemma 1.

Example 1. Suppose that $V = 1$. There are two shapes of the expected payoff function, $EL(l)$.

Case A. If $0 < \beta \leq 0.5$, then $\sup_{l\in[1,+\infty)} EL(l) = EL(1) = V$.

Case B. If $0.5 < \beta < 1$, then $\sup_{l\in[1,+\infty)} EL(l) = \frac{\beta}{(1-\beta)}V$.

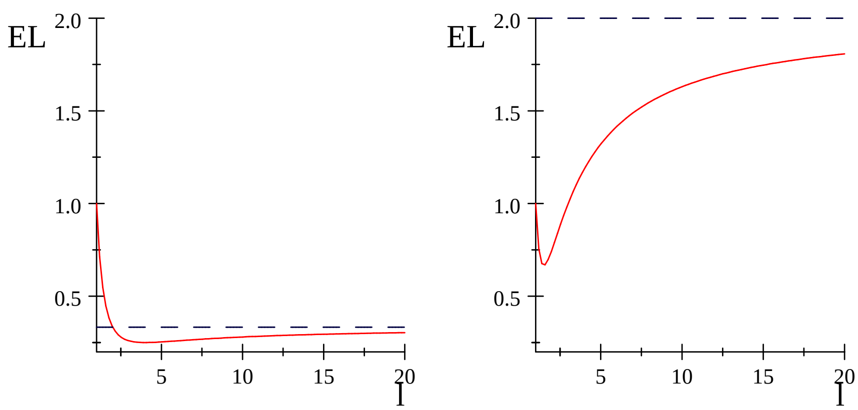

FIGURE 2: $EL(l)$. Left Panel: $\beta = .25$, $l^* = 4$. Right Panel: $\beta = 2/3$, $l^* = 3/2$.

In the following sections, we will find SPEs for the N-player two-stage (stages 1 and 2) public good games. Since communities choose either VCM or *Lottery* at stage 0, we will consider VCM vs *Lottery*, *Lottery* vs *Lottery*, and VCM vs VCM public good games. In each case, we first consider stage 2 and find optimal individuals' actions. Then, we move backward to stage 1 and discover individuals' optimal mechanisms.

VCM vs VCM public good game is easy to analyze. If both communities choose the VCM mechanism, then the equilibrium actions and expected payoffs are described in (2) and (3). Therefore, all individuals select the richer community, or the community with more seed money.

VCM vs *Lottery* and *Lottery* vs *Lottery* public good games are more complicated.

3 VCM vs Lottery

Suppose that one community chooses the VCM mechanism and the other chooses the *Lottery* mechanism. Based on the previous section, we can characterize SPEs. It turns out that there are three types of SPEs. In all these SPEs, the second-stage behavior is uniquely determined in (2) and (5). At stage 1, all individuals can either select the VCM mechanism or the *Lottery* mechanism depending on the parameters of the model. Moreover, there might exist a $Mixed$ SPE where some individuals chooses the *Lottery* mechanism and the others select the VCM mechanism. The following theorem characterizes SPEs of the VCM vs *Lottery* public good game.

Theorem 2 *In any SPE, individuals choose actions $x^*(k)$ and $y^*(l)$ from (2) and (5) at the corresponding mechanisms at stage 2.*
(VCM) If

$$V \leq \beta M, \tag{9}$$

then there exists a SPE where all individuals choose the VCM mechanism at stage 1.
(Lottery) If

$$EL(N) \geq \beta M, \tag{10}$$

then there exists a SPE where all individuals choose the Lottery mechanism at stage 1.
($Mixed$) If there exists $1 \leq l^M < N$ such that

$$EL\left(l^M + 1\right) \leq \beta M \leq EL\left(l^M\right), \tag{11}$$

then there exists a $Mixed$ SPE, where l^M individuals choose the Lottery mechanism and $\left(N - l^M\right)$ individuals choose the VCM mechanism at stage 1. In this SPE, the public good provision for the Lottery is undesirable, or less than the value of the lottery prize, V.

It is important to note that in a $Mixed$ SPE $l^M > 0$ individuals choose the *Lottery* mechanism and the other individuals choose the VCM mechanism at stage 1. From Lemma 1, it follows that this can only occur when it is *undesirable* to produce public good with l^M individuals in the *Lottery* mechanism. This is an unusual situation for the lottery public good games. Typically in the literature, the number of individuals is fixed in a lottery public good game and is such that public good provision is desirable.[2] Our model predicts endogenously undesirable situation in the *Lottery* mechanism in any $Mixed$ SPE.

The following example illustrates Theorem 2.

Example 2. Suppose that $\beta = 0.25$ and $N = 10$.
Then, $l^* = 1/\beta = 4$. Note that

$$EL(l) = \frac{1}{l}V + (0.25l - 1)\frac{l-1}{l^2}\frac{V}{.75},$$

[2] See, for example, Morgan (2000), Morgan and Sefton (2000), and Orzen (2008).

$EL\,(10) = 0.28V$, and condition (11) becomes

$$\frac{1}{(l^M+1)}V + \left(0.25\left(l^M+1\right)-1\right)\frac{l^M}{(l^M+1)^2}\frac{V}{0.75} \leq$$

$$0.25M \leq \frac{1}{l^M}V + \left(0.25l^M - 1\right)\frac{l^M-1}{(l^M)^2}\frac{V}{0.75},$$

where $l^M < l^* = 4$. For example, if $l^M = 1$, then condition (11) becomes

$$\frac{V}{3} \leq 0.25M \leq V.$$

If $l^M = 2$, then condition (11) becomes

$$\frac{7}{9}\frac{V}{3} \leq 0.25M \leq \frac{V}{3}.$$

Finally, if $l^M = 3$, then condition (11) becomes

$$\frac{1}{4}V \leq \frac{1}{4}M \leq \frac{7}{9}\frac{V}{3}.$$

Figure 3 demonstrates different SPEs for different values of M and V.

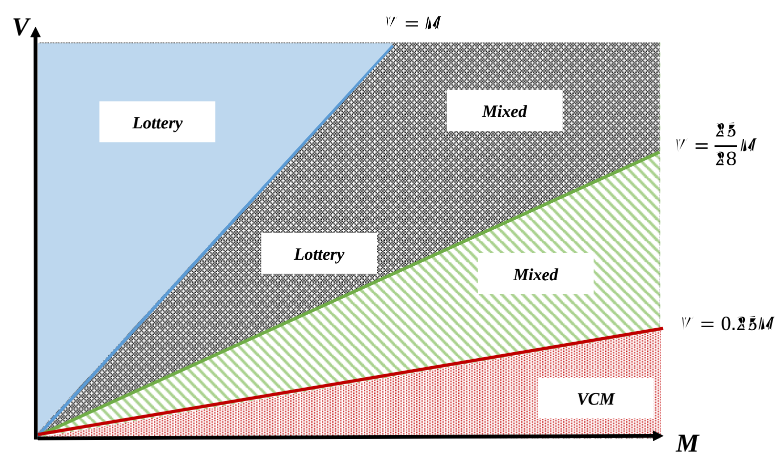

FIGURE 3: $N = 10$ and $\beta = 0.25$.

3.1 Mixed SPE

Theorem 2 gives condition (11) for a *Mixed* SPE where l^M individuals choose the *Lottery* mechanism and $\left(N - l^M\right)$ individuals choose the *VCM* mechanism. The following Lemma provides a necessary condition for existence of a *Mixed* SPE.

Lemma 3 *If there exists a Mixed SPE, then*

$$\beta V \leq \beta M \leq V. \tag{12}$$

Condition (12) can be used to establish existence of a *Mixed* SPE almost all the time, because, from Lemma 1, it can be rewritten in the following way

$$EL(l^*) \leq u_{VCM} \leq EL(1).$$

For example, if

$$1/\beta \in \mathbb{N}, \tag{13}$$

then there exist values of M and V such that in a *Mixed* SPE, $l^M = 1$ individuals choose the *Lottery* mechanism and $(N-1)$ individuals choose the VCM mechanism at stage 1.

The following example illustrates how to find l^M.

Example 3. Suppose that $\beta = 0.5$ (which means that condition (13) holds) and condition (12) is satisfied:

$$0.5V = \beta V \leq 0.5M = \beta M \leq V. \tag{14}$$

Then, $l^* = 2$, $EL(1) = V$, $EL(2) = V/2$. Note that from (14):

$$EL(2) = V/2 \leq 0.5M = \beta M \leq V = EL(1)$$

and, from Theorem 2, there exists a *Mixed* SPE where $l^M = 1$ individual chooses the *Lottery* mechanism and $(N-1)$ individuals choose the VCM mechanism for any $N \geq 2$.

Figure 4 presents the expected payoff function, $EL(l)$, and the constant payoff in the VCM mechanism.

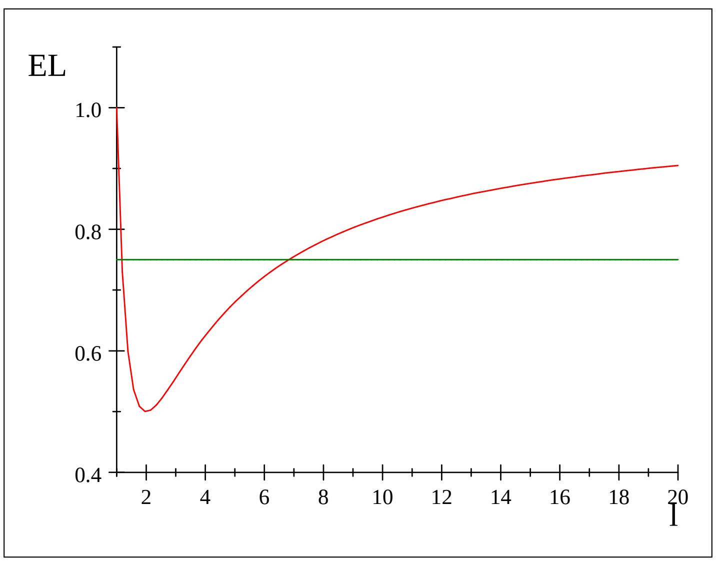

FIGURE 4: $EL(l)$ and βM for $V = 1,\ M = 3/2$, and $\beta = 1/2$.

If the population size, N, goes to infinity, then, using (8) from Lemma 1, Theorem 2 can be illustrated by Figures 5 and 6, where we replace condition (11) with condition (12). Figures 5 and 6 summarize how different sets of parameters of the model lead to different SPEs. For particular values of β, M, and V, a SPE (*Lottery*, VCM, or $Mixed$) might be unique, or the model might have multiple SPE (*Lottery* and $Mixed$, or *Lottery* and VCM). It is interesting to note that β, a marginal per capita return, plays an important role in Figures 5 and 6. We have already seen that in Figure 2.

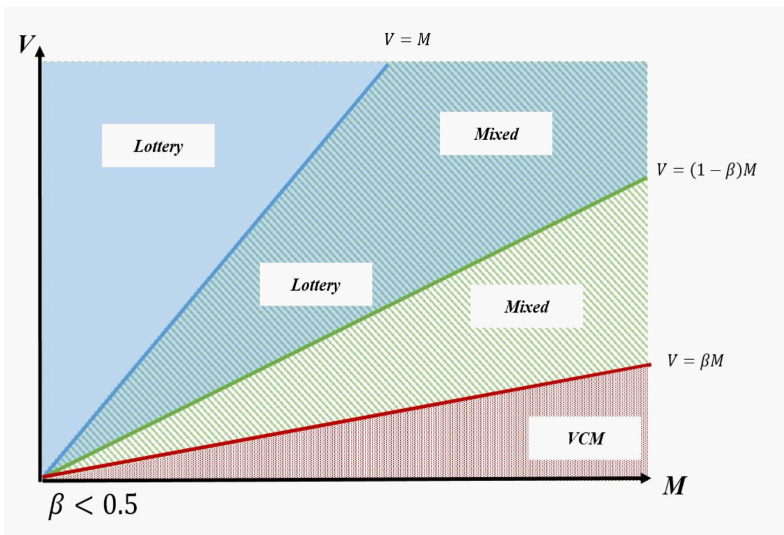

FIGURE 5: $0 < \beta < 0.5$

However, a $Mixed$ SPE might not exist even if condition (12) is satisfied because of the integer problem. The following example illustrates.

Example 4. Suppose that $V = 1$, $M = 1.00144$, and $\beta = \frac{1}{4.4}$.
Then, condition (12) holds. However, a $Mixed$ SPE does not exists because

$$l^* = 4.4$$

and

$$\beta M < EL\,(4) = .22794 < EL\,(5) = 0.22824,$$

which means that

$$\beta M < EL\,(l) \text{ for any } l \in \mathbb{N}.$$

If there exists a $Mixed$ SPE, then there also exists a symmetric mixed-strategy $Mixed$ SPE, where all individuals randomize between two mechanisms. The following example shows how to find a symmetric mixed-strategy $Mixed$ SPE for $N = 2$.

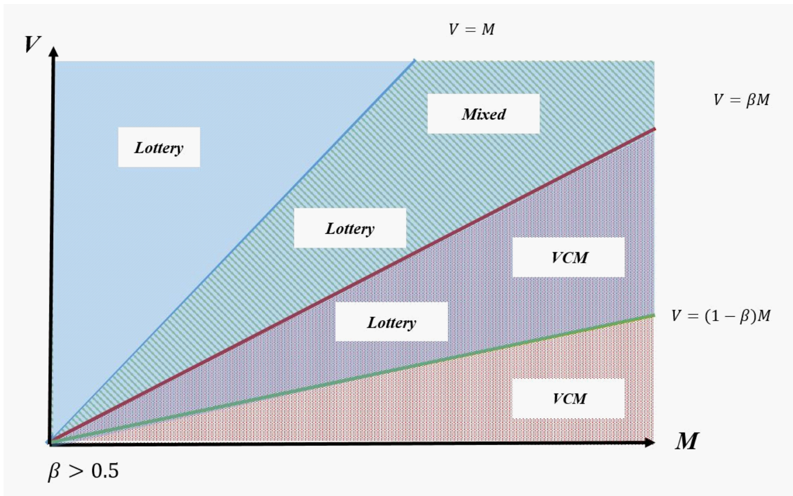

FIGURE 6: $0.5 < \beta < 1$.

Example 5. Suppose that $N = 2$, $\beta = 0.5$, and condition (12) is satisfied:

$$0.5V \leq 0.5M \leq V.$$

Then, $l^* = 2$, $EL(1) = V$, and $EL(2) = V/2$. There exists a *Mixed* SPE where $l^M = 1$ individual chooses the *Lottery* mechanism and 1 individual chooses the *VCM* mechanism (see Example 3). Let us find a symmetric mixed-strategy *Mixed* SPE, where both individuals randomize between the *VCM* and *Lottery* mechanisms at stage 1.

Suppose that each individual chooses the *Lottery* mechanism with the same probability $0 < p < 1$. Then, if an individual chooses the *VCM* mechanism, she gets payoff

$$\beta M = \frac{M}{2}.$$

If an individual chooses the *Lottery* mechanism, she gets the following expected payoff

$$(1-p)\,EL(1) + pEL(2) = (1-p)\,V + p\frac{V}{2} = \left(1 - \frac{1}{2}p\right)V.$$

In the symmetric mixed-strategy *Mixed* SPE, the expected payoffs on both mechanisms have to be the same, or

$$\frac{M}{2} = \left(1 - \frac{1}{2}p\right)V,$$

or

$$p = \frac{2V - M}{V} = 2 - \frac{M}{V}. \tag{15}$$

This means that in the symmetric mixed-strategy $Mixed$ SPE, all individuals choose the *Lottery* mechanism with the same probability (15) at stage 1. Moreover, since $0 \leq p \leq 1$, then the following condition has to hold for the existence of a symmetric mixed-strategy $Mixed$ SPE:

$$V \leq M \leq 2V.$$

Expression (15) is the equilibrium probability only for $N = 2$. In general, the equilibrium probability depends on the number of individuals, N. For $N > 3$, calculations for a symmetric mixed-strategy $Mixed$ SPE become intractable. However, it is always easy to calculate a $Mixed$ SPE, if it exists.

3.2 Unique SPE

Based on Theorem 2 we can describe conditions for uniqueness of a SPE. A SPE is unique if only one out of three conditions (9), (10), and (11) holds. Moreover, all inequalities have to be strict.

Proposition 4 *Suppose that the number of individuals, N, seed money, M, prize value, V, and β are given. Then, a VCM SPE is a unique SPE if*

$$\beta M > \max\left\{V, EL\left(N\right)\right\}.$$

A Lottery SPE is a unique SPE if

$$M < V.$$

A $Mixed$ SPE is a unique SPE if

$$\min\left\{\beta V, EL\left(N\right)\right\} < \beta M < V.$$

If the population size, N, goes to infinity, then Proposition 4 can be illustrated by Figures 5 and 6. Next, we provide conditions for SPEs uniqueness for any N.

Proposition 5 *Suppose that seed money, M, prize value, V, and β are given. Then, a VCM SPE is a unique SPE for any N if*

$$M > \max\left\{\frac{V}{\beta}, \frac{V}{(1-\beta)}\right\}.$$

A Lottery SPE is a unique SPE for any N if

$$M < V.$$

A $Mixed$ SPE is a unique SPE for any N if $\beta < 0.5$ and

$$\frac{V}{1-\beta} < M < \frac{V}{\beta}.$$

3.3 Multiple SPEs

Note that for some parameter values, the two-stage VCM vs *Lottery* public good game has multiple SPEs. Consider these cases.

Proposition 6 *If*

$$V \leq \beta M \leq EL(N),$$

then, there are two SPEs: VCM *and Lottery.*
If

$$\beta V \leq \beta M \leq \min\{V, EL(N)\},$$

then, there are two SPEs: Lottery and $Mixed$.

Note that, from Theorem 2 and Lemma 1, the VCM and $Mixed$ SPEs cannot coexist.

If the population size, N, goes to infinity, then Proposition 6 can be illustrated by Figures 5 and 6. Next, we provide conditions for multiple SPEs for any N.

Proposition 7 *If* $\beta > 0.5$ *and*

$$(1-\beta) M < V < \beta M,$$

then, there are two SPEs: VCM *and Lottery.*
If

$$\max\{\beta M, (1-\beta) M\} < V < M, \tag{16}$$

then, there are two SPEs: Lottery and $Mixed$.

4 Lottery vs Lottery

Suppose that both communities choose the *Lottery* mechanism at stage 1. Denote the equilibrium action for an individual in Lottery i with l individuals at stage 2 as

$$y_i^*(l) = \frac{l-1}{l^2}\frac{V_i}{(1-\beta)}, \tag{17}$$

and the corresponding expected payoff as[3]

$$EL_i(l) = \frac{1}{l}V_i + (\beta l - 1)\frac{l-1}{l^2}\frac{V_i}{(1-\beta)}. \tag{18}$$

The following theorem characterizes SPEs of the Lottery vs Lottery public good game. Theorem 8 provides conditions for three types of SPEs.

[3] See Section 2.2 for details.

Theorem 8 *In any SPE, individuals choose action* $y_i^*(l)$ *in* (17) *at stage 2.*
(L_i) If

$$V_j \leq EL_i(N), \tag{19}$$

then there exists a SPE where all individuals choose Lottery i *at stage* 1*, where* $i \neq j$*, and* i*,* $j \in \{1, 2\}$*.*
(Mixed) If there exists $1 \leq l^{ML} < N$ *such that*

$$EL_2\left(N - l^{ML} + 1\right) \leq EL_1\left(l^{ML}\right) \tag{20}$$

and

$$EL_1\left(l^{ML} + 1\right) \leq EL_2\left(N - l^{ML}\right), \tag{21}$$

then there exists a Mixed SPE, where l^{ML} *individuals choose Lottery* 1 *and* $\left(N - l^{ML}\right)$ *individuals choose Lottery* 2 *at stage* 1*.*

The following example illustrates Theorem 8.

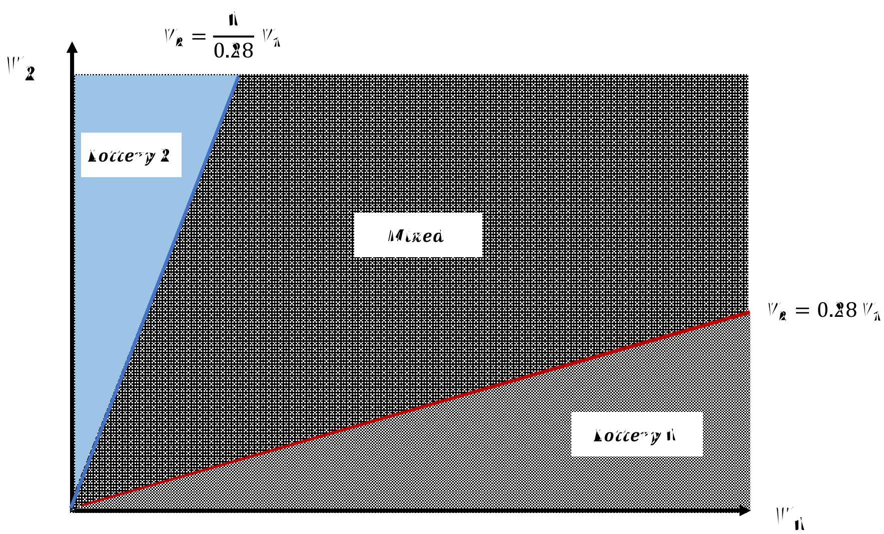

FIGURE 7: $N = 10$ and $\beta = 0.25$.

Example 6. Suppose that $\beta = 0.25$ and $N = 10$.
Then, $l^* = 1/\beta = 4$. Note that

$$EL_i(l) = \frac{1}{l}V_i + (0.25l - 1)\frac{l-1}{l^2}\frac{V_i}{.75}, \text{ for } i = 1, 2,$$

and $EL_i(10) = 0.28V_i$. For example, if $l^{ML} = 1$, then conditions (20) and (21) become

$$0.28V_2 \leq V_1$$

and

$$V_1 \leq \frac{67}{81} V_2,$$

or

$$0.28 V_2 \leq V_1 \leq \frac{67}{81} V_2.$$

And so on (similar to Example 2). Finally, if $l^{ML} = 9$, then conditions (20) and (21) become

$$0.28 V_1 \leq V_2 \leq \frac{67}{81} V_1.$$

Figure 7 demonstrates different SPEs for different values of V_1 and V_2.

Small N might lead to an inefficient *Mixed* SPE where individuals choose both lotteries, but there are undesirable number of individuals in each lottery. The following example illustrates.

Example 7. Suppose that $V_1 = 1$, $V_2 = 2$, $N = 4$, and $\beta = .25$. Then, $l^* = 4$ and

$$EL_i(l) = \frac{V_i}{l} + (.25l - 1) \frac{l-1}{l^2} \frac{V_i}{(1 - .25)}, \text{ for } i = 1, 2.$$

Note that $l^{ML} = 1$ and $\left(N - l^{ML}\right) = 3$ in the *Mixed* SPE. Since $l_1 < l^*$ and $l_2 < l^*$, both lotteries are undesirable. Figure 8 illustrates the expected lottery payoffs. Dash (red) line - the expected payoff $EL_1(l)$. Solid (green) line - the expected payoff $EL_2(l)$.

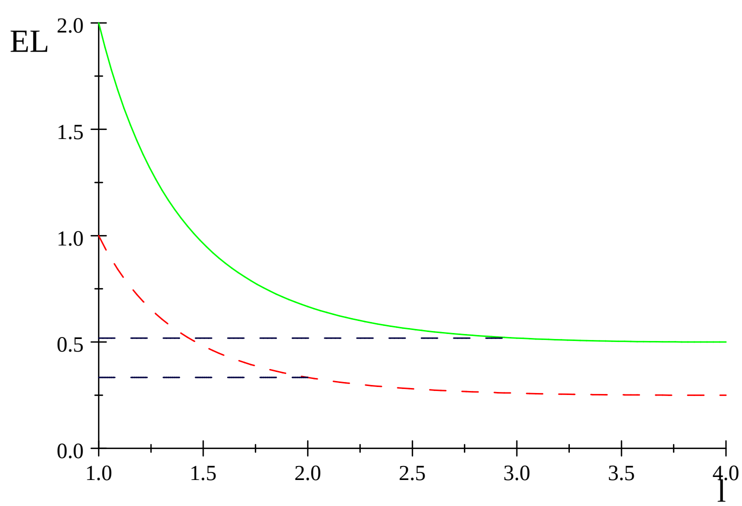

FIGURE 8: $EL(l)$, $l^* = 4$.

4.1 $0 < \beta < 0.5$

Based on Theorem 8 and Lemma 1 we can characterize SPEs. If one lottery always gives higher expected individual payoffs than the other lottery, then all individuals choose the first lottery.

Proposition 9 *If* $0 < \beta < 0.5$ *and*

$$V_j < EL_i(N),$$

where $i \neq j$*, and* $i,\ j \in \{1,2\}$*, then* L_i *SPE is a unique SPE.*

A *Mixed* SPE might also be unique.

Proposition 10 *If* $0 < \beta < 0.5$ *and*

$$EL_i(N) < V_j < V_i,$$

where $i \neq j$*, and* $i,\ j \in \{1,2\}$*, then there exists a unique Mixed SPE.*

Examples 6 and 7 illustrate these Propositions.

4.2 $0.5 < \beta < 1$

Note that from Lemma 1, there exists

$$\overline{N} = \left\lceil \frac{1}{(2\beta - 1)} \right\rceil,$$

where $\lceil x \rceil$ is the smallest integer greater or equal to x, such that

$$V_i < EL_i(N), \text{ for any } N > \overline{N}.$$

As we saw above, if one lottery always gives higher expected individual payoffs than the other lottery, then all individuals choose the first lottery.

Proposition 11 *If* $0.5 < \beta < 1$*,* $N > \overline{N}$*, and*

$$EL_i(l^*) > EL_j(N),$$

where $i \neq j$*, and* $i,\ j \in \{1,2\}$*, then* L_i *SPE is a unique SPE.*

There might be multiple SPEs.

Proposition 12 *If* $0.5 < \beta < 1$*,* $N > \overline{N}$*, and*

$$EL_i(l^*) < EL_j(N) < V_i,$$

where $i \neq j$*, and* $i,\ j \in \{1,2\}$*, then there exists* L_i *SPE and there might be a Mixed SPE.*

The following example illustrates the proposition.

Example 8. Suppose that $V_1 = 1$, $V_2 = 2$, $N = 10$, and $\beta = .65$. Then,

$$EL_i\,(2) = 0.71429 V_i,\ \text{for } i = 1, 2,$$

and

$$EL_1\,(9) = 1.48.$$

There are two SPEs: L_1 SPE and a *Mixed* SPE where one player chooses *Lottery* 1 and the other nine players select *Lottery* 2. Figure 9 shows the expected payoffs in both lotteries.

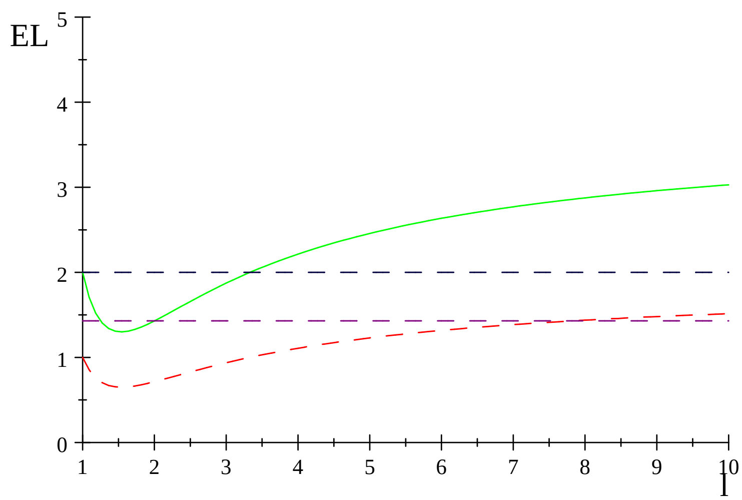

FIGURE 9: $EL_1\,(l)$ and $EL_2\,(l)$.

L_1 SPE and L_2 SPE can coexist.

Proposition 13 *If* $0.5 < \beta < 1$, $N > \overline{N}$, *and*

$$\min\left\{EL_1\,(N)\,, EL_2\,(N)\right\} > \max\left\{V_1, V_2\right\},$$

then there are two SPEs: L_1 *and* L_2.

5 Competition for Public Good Provision

Based on our analysis in Sections 2 - 4, we can characterize the simultaneous-move game between two local communities at stage 0. We assume that each local community maximizes its public good provision. Each community i can either use its budget B_i as a seed money in the VCM mechanism, or as a prize in the *Lottery* mechanism. Both communities make their decisions simultaneously.

First, note that if community i chooses the VCM mechanism, then, from (2) and (3), its public good provision is B_i, or

	VCM	*Lottery*
VCM	B_1, B_2	B_1,
Lottery	, B_2	,

TABLE 1: Local Community Game

Second, if richer local community 1 chooses the VCM mechanism and poorer local community 2 chooses the *Lottery* mechanism at stage 0, then the budget of community 1 becomes the seed money in the VCM, and the budget of community 2 becomes the lottery prize ($V = B_2$, $M = B_1$). From Theorem 2 and Lemma 1, if

$$EL\left(N\right) < \beta M,$$

or

$$EL\left(N\right) < \frac{\beta}{(1-\beta)} V < \beta M,$$

or

$$B_2 < (1-\beta)\, B_1, \tag{22}$$

then there exists either a SPE where all individuals choose the VCM mechanism at stage 1, or a *Mixed* SPE, where a few individuals choose the (undesirable) *Lottery* mechanism and the other individuals choose the VCM mechanism at stage 1.

Third, if richer local community 1 chooses the *Lottery* mechanism and poorer local community 2 chooses the VCM mechanism at stage 0, then the budget of community 1 becomes the lottery prize and the budget of community 2 becomes the seed money in the VCM ($V = B_1$, $M = B_2$). From Proposition 4, the *Lottery* SPE is a unique SPE, where all the individuals choose the *Lottery* mechanism, in this case.

Finally, if both communities choose the *Lottery* mechanism, then their budgets become the lottery prizes ($V_1 = B_1$, $V_2 = B_2$). From Propositions 9 and 10, there exists either L_1 SPE where all individuals choose *Lottery* 1 mechanism at stage 1, or a *Mixed* SPE, where a few individuals choose the (undesirable) *Lottery* 2 mechanism and the other individuals choose the *Lottery* 1 mechanism at stage 1.

Consider the game in Table 2.

	VCM	*Lottery*
VCM	B_1, B_2	B_1, T_v
Lottery	$\frac{N-1}{N}\frac{B_1}{(1-\beta)}$, B_2	H, T_l

TABLE 2: Local Community Game

where $T_v < B_2$ and $T_l < B_2$ correspond to the best-case $Mixed$ SPEs (see Theorems 2 and 8, and Proposition 10) outcomes for community 2, and $H > B_1$ correspond to the worst-case $Mixed$ SPE (see Theorem 8 and Propositions 9 and 10) outcome for community 1. Note that in a $Mixed$ SPE, *Lottery* is undesirable for community 2 which means that the public good provision in community 2 is less than its lottery prize. Moreover, in the L_1 SPE, *Lottery* 2 attracts no individuals and provides no public good in community 2.

It is easy to see that both local communities have dominant strategies in the Local Community Game. The richer community 1's dominant strategy is to use the *Lottery* mechanism. The poorer community 2's dominant strategy is to use the VCM mechanism. Formally,

Theorem 14 *Suppose that*

$$0 < \beta < 0.5$$

and condition (22) *holds. For sufficiently large population size N, there exists a unique Nash equilibrium in the Local Community Game. In this equilibrium, the richer community's dominant strategy is to use the Lottery mechanism, and the poorer community's dominant strategy is to use the VCM mechanism.*

6 Conclusion

Most of the literature on public good provision analyze the voluntary contribution mechanism (VCM) and/or the *lottery* mechanism. These two mechanisms have always been considered separately. In this study, we develop a three-stage public good game to analyze a competition between local public good providers. There are several findings in our study. First, we describe a competition between two local providers who use different mechanisms: VCM and *Lottery*. Then, we consider a competition between two local *Lotteries*. We characterized all SPEs in these cases. Based on our analysis, we are able to answer the main question about local competition for public good provision. We demonstrate that both local providers have dominant strategies: richer provider should use the *Lottery* and poorer provider should use the VCM. This result is consistent with the every-day observations when we see large state lotteries and small VCMs for schools and churches.

7 Appendix A: Proofs

Proof of Lemma 1. Consider a twice continuously differentiable function

$$EL(x) = \frac{1}{x}V + (\beta x - 1)\frac{x-1}{x^2}\frac{V}{(1-\beta)}$$

on the interval $x \in [1, +\infty)$. Then,

$$EL'(x) = \frac{V}{1-\beta}\frac{2}{x^3}(x\beta - 1).$$

Therefore, $EL'(l^*) = 0$. Note that $EL'(x) < 0$ if $x < l^*$ and $EL'(x) > 0$ if $x > l^*$, which prove statements (1), (2), and (5).

It is straightforward that

$$EL''(x) = -\frac{4V}{x^4(1-\beta)}\left(x\beta - \frac{3}{2}\right).$$

Hence, $EL''(x) < 0$ if $x > \frac{3}{2\beta}$ and $EL''(x) > 0$ if $x < \frac{3}{2\beta}$, which prove statements (3) and (4). Statement (6) follows from statement (1). Statement (7) is straightforward. Statement (8) follows from statements (1), (2), (6), and (7). ■

Proof of Theorem 2.

First, note that in a SPE individuals have to play a Nash equilibrium at stage 2. This means that each individual plays action $x^*(k) = 0$ from (2) in the VCM mechanism with k individuals, and action $y^*(l)$ from (5) in the *Lottery* mechanism with l individuals.

Consider stage 1. Suppose that there exists a SPE where l_1 individuals choose the *Lottery* mechanism and l_2 individuals choose the VCM mechanism at stage 1. Then, the following conditions have to hold.

An individual in the VCM mechanism does not benefit from moving to the *Lottery* mechanism, or

$$\beta M = u_{VCM} \geq EL(l_1 + 1). \tag{23}$$

An individual in the *Lottery* mechanism does not benefit from moving to the VCM mechanism, or

$$EL(l_1) \geq u_{VCM} = \beta M. \tag{24}$$

Suppose that all individuals choose the VCM mechanism at stage 1. Then, in a SPE, only condition (23) has to be satisfied. This condition becomes

$$\beta M \geq EL(1) = V,$$

which proves statement (VCM).

Suppose that all individuals choose the *Lottery* mechanism at stage 1. Then, in a SPE, only condition (24) has to be satisfied. This condition becomes

$$EL(N) \geq \beta M,$$

which proves statement (*Lottery*).

Suppose that $l_1 > 0$ individuals choose the *Lottery* mechanism and $l_2 > 0$ individuals choose the VCM mechanism at stage 1. Then, in a SPE, both conditions (23) and (24) have to be satisfied. These conditions can be rewritten as

$$EL(l_1 + 1) \leq \beta M \leq EL(l_1). \tag{25}$$

■

Proof of Lemma 3. From Lemma 1, inequalities (25) mean that $l_1 < l^*$. Therefore, from statements (1) and (5) of Lemma 1, it follows

$$EL(l^*) \leq \beta M \leq EL(1),$$

or condition (12). ■

Proof of Proposition 4. In the unique VCM SPE, condition (9) has to hold and conditions (10) and (11) have to be violated, or

$$V < \beta M$$

and

$$EL(N) < \beta M.$$

Therefore,

$$\beta M > \max\{V, EL(N)\}.$$

In the unique *Lottery* SPE, condition (10) has to hold and conditions (9) and (11) have to be violated, or

$$EL(N) > \beta M, \tag{26}$$

$$V > \beta M, \tag{27}$$

and

$$M < V. \tag{28}$$

All inequalities (26)-(28) are satisfied if condition (28) holds.

In the unique $Mixed$ SPE, condition (11) has to hold and conditions (9) and (10) have to be violated, or

$$\beta V < \beta M < V$$

and

$$EL(N) < \beta M.$$

Therefore,

$$\min\{\beta V, EL(N)\} < \beta M < V.$$

■

Proof of Proposition 5. The statements follow from Proposition 4 and Lemma 1. ■

Proof of Proposition 6. The statements follow from Theorem 2. ■

Proof of Proposition 7. For the coexistence of the VCM and *Lottery* SPEs, conditions (9) and (10) have to hold and condition (12) has to be violated. Then,

$$(1-\beta)\, M < V < \beta M.$$

Note that

$$(1-\beta) < \beta,$$

if $\beta > 0.5$.

For the coexistence of the *Lotter* and $Mixed$ SPEs, conditions (10) and (12) have to hold and condition (9) has to be violated. Then,

$$\max\left\{\beta M, (1-\beta)\, M\right\} < V < M.$$

■

Proof of Theorem 8. Suppose that there exists a SPE where l_1 individuals choose *Lottery* 1 and l_2 individuals choose *Lottery* 2 at stage 1. Then, the following conditions have to hold.

An individual in *Lottery* 1 does not benefit from moving to *Lottery* 2, or

$$EL_1\left(l_1\right) \geq EL_2\left(l_2+1\right). \tag{29}$$

An individual in *Lottery* 2 does not benefit from moving to *Lottery* 1, or

$$EL_2\left(l_2\right) \geq EL_1\left(l_1+1\right). \tag{30}$$

Suppose that all individuals choose *Lottery* 1 at stage 1. Then, in this SPE, only condition (29) has to be satisfied. This condition becomes

$$EL_1\left(N\right) \geq V_2 = EL_2\left(1\right),$$

which proves statement (19).

Suppose that $0 < l_1$ individuals choose *Lottery* 1 and $0 < N - l_1$ individuals choose *Lottery* 2 at stage 1. Then, in a SPE, both conditions (29) and (30) have to be satisfied. Then,

$$EL_1\left(l_1\right) \geq EL_2\left(N-l_1+1\right)$$

and

$$EL_2\left(N-l_1\right) \geq EL_1\left(l_1+1\right).$$

■

Proof of Proposition 9. If $0 < \beta < 0.5$ and

$$V_2 < EL_1\left(N\right),$$

then, from Lemma 1,

$$EL_2\left(N\right) < V_2 < EL_1\left(N\right) < V_1.$$

Therefore, from Theorem 8, L_1 SPE is a unique SPE.■

Proof of Proposition 10. If $0 < \beta < 0.5$ and

$$EL_i(N) < V_j < V_i,$$

then, from Theorem 8, there are no L_1 and L_2 SPEs. There exists a unique *Mixed* SPE.■

Proof of Proposition 11. If $0.5 < \beta < 1$, $N > \overline{N}$, and

$$EL_1(l^*) > EL_2(N),$$

Therefore, from Theorem 8 and Lemma 1, L_1 SPE is a unique SPE.■

Proof of Proposition 12. The statement follows from Theorem 8 and Lemma 1. ■

Proof of Proposition 13. The statement follows from Theorem 8 and Lemma 1. ■

Proof of Theorem 14. The statement of the theorem follows from Lemma 1, Theorems 2 and 8 and Propositions 4, 9, and 10. ■

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