Inertial Behavior and Generalized Partition[*]

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Abstract

Behavior is inertial if it does not react to the apparent arrival of relevant information. In a context where information is subjective, we formulate an axiom that captures inertial behavior, and provide a representation that suggests a rational decision maker who perceives a particular type of information structure we call a generalized partition. We characterize the learning processes that generate generalized partitions. We then investigate behavioral protocols that may lead individuals to perceive a generalized partition even when facing a different type of information structure: A cognitive bias referred to as cognitive inertia and a bound on rationality termed shortsightedness.

Key words: Inertial behavior, subjective learning, generalized partition, uniform cover, cognitive inertia, shortsighted rationality

1. Introduction

Individuals often seem reluctant to change their behavior in response to new information, unless that information is conclusive. Examples include managers who do not adapt their strategy to changes in the business environment (Hodgkinson 1997 and references therein), a "one advice suits all" approach among consultants, such as medical doctors or financial advisors dealing with a heterogeneous population (clients with different symptoms or different levels of risk attitude), or a bias in favor of renewing the previously selected health plan or the previously selected contribution to a retirement fund after a change in fundamentals. The last two phenomena are documented in field studies by Samuelson and Zeckhauser (1988), who also provide experimental evidence that confirms the tendency of subjects to stick to

their initial decision when choosing sequentially from a fixed set of alternatives.[1] We refer to this apparent bias as inertial behavior.

We contend that in most applied situations, including those mentioned above, the analyst may suspect inertial behavior even without being aware of the exact information available to the agent; behavior does not change although it is likely that some relevant information became available. Our goal in this paper is to formally define a notion of inertial behavior, and to analyze potential sources of such behavior, under the assumption that learning about the relevant state of the world is subjective (unobserved).

Since we are unable to directly observe whether or not behavior changes in response to the arrival of information, we instead follow the approach in Dillenberger, Lleras, Sadowski, and Takeoka (2014, henceforth DLST). Let S be a space of payoff-relevant states. Our primitive is a preference relation over sets (or menus) of actions, where each action (an act on S) results in a state-contingent payoff.[2] This allows us to make inference about the conclusive information the decision maker (DM) expects to receive, where a signal conveys conclusive information about some event $I \subseteq S$ if there is a state $s \in I$ under which that signal cannot be obtained.[3] Loosely speaking, we say that the DM displays inertial behavior if he plans to choose the initially preferred action unless he receives relevant conclusive information: if $I \subseteq S$ is an event the DM can foresee learning and f and g are two acts that may differ only on I ($f(s) \neq g(s)$ only if $s \in I$), then the ranking of these two acts will not change as long as no state in I has been ruled out.

Theorem 1 derives a *generalized-partition representation*, where a *generalized partition* is an information structure in which each signal rules out certain states without providing any information concerning the relative likelihood of states that are not ruled out (a property we term *non-shifting-weights*). Our representation can be interpreted as follows. The DM holds prior beliefs μ over S. He also perceives a generalized partition ρ on S that determines which event he will learn contingent on the true state. The function ρ has the feature that the conditional probability of learning event I is the same for any $s \in I$, so that it can be

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[1] Samuelson and Zeckhauser (1988) investigate both the effect of an exogenously given status quo (for example, a default option) and of an endogenous one, which is determined by subjects' previous choices. While the former has received significant attention in the subsequent literature on status quo bias, we are concerned here only with the latter case.

[2] The interpretation is that the DM initially chooses among menus and subsequently chooses an act from the menu. If the ultimate choice of an act takes place in the future, then the DM may expect information to arrive prior to this choice. Analyzing preferences over future choice situations allows one to identify the anticipated future choice behavior, without the need to observe ex post preferences. Throughout we measure payoffs in terms of utilities in order to simplify the exposition.

[3] In order to accommodate a DM who may ignore information, we do not assume that anticipated future choice between acts correctly takes into account any signal the DM may receive. We do, however, assume that this choice ignores states under which that signal cannot be obtained. In that sense, menu choice allows inference about conclusive information.

written as $\rho(I)$, and the total probability of learning I is $\mu(I)\rho(I)$. Upon learning an event, the DM calculates posterior beliefs using Bayes' law, which leads him to exclude all states that are not in that event, keeping the relative likelihood of the remaining states fixed. The DM then chooses from the menu the act that maximizes the corresponding expected utility. The value of a menu of acts F is then given by

$$V(F) = \sum_{I \in 2^S} \max_{f \in F} \left[\sum_{s \in I} f(s)\mu(s) \right] \rho(I).$$

Both parameters of this representation, namely the prior μ and the generalized partition ρ, can be uniquely identified from observable behavior.

We show that the class of generalized partitions can accommodate a variety of information structures that correspond to intuitive learning processes and to plausible behavior. As the name suggests, generalized partition extends the notion of set partition, according to which the DM learns which cell of a partition contains the true state. In the case of a set partition, signals are deterministic; that is, for each state there is only one possible event that contains it. Another example of a generalized partition is a random partition, where one of multiple partitions is randomly drawn and then an event in it is reported. A situation that may give rise to a random partition is an experiment with uncertainty about its precision. A sequential elimination of candidates, say during a recruiting process, where k out of n candidates are to be eliminated in the first stage (so that the resulting collection of events the DM might learn is the set of all $(n-k)$-tuples), can also be accommodated. Theorem 2 characterizes all types of learning that can be described by a generalized partition.

Thus far we argued that a rational Bayesian DM will display inertial behavior if and only if he faces a generalized partition. Now suppose the analyst does know the process according to which information is produced, and that it is actually not a generalized partition. Are there (non-standard) updating protocols that can reconcile this true information structure with the generalized partition the DM appears to perceive? While this is not a typical line of investigation in decision theoretic work, we believe that besides gaining insight into possible cognitive sources of inertial behavior, being aware of how the DM translates one information structure to another can also help predict the DM's behavior in different environments (i.e., when facing different underlying true information structures). In Section 4 we discuss two possible protocols and assess to what extent they lead the DM to perceive a generalized partition even if the true information structure is not. The first protocol is based on the psychological notion of cognitive inertia, and can sometimes lead the DM to falsely perceive a generalized partition. The second protocol is inspired by a concept from social psychology, called construal level theory, which contends that the closer in time a situation is, the more

easily and concretely it will be thought of. Applied to our context, we study a boundedly rational DM who is constrained in his ability to directly assess probabilities for the outcome of random variables that resolve in the distant future, and tries to behave rationally given this constraint. Theorem 3 establishes that following the second protocol will *always* lead the DM to perceive a generalized partition. Theorem 4 provides conditions under which the induced generalized partition is unique.

The remainder of the paper is organized as follows. Section 2 formalizes our notion of inertial behavior and provides our representation result. Section 3 characterizes the learning processes that generate a generalized partition. Section 4 provides the two protocols that lead to learning via a generalized partition even if the true underlying information structure does not belong to this class. Related literature is discussed in Section 5. Most proofs are relegated to the appendices.

2. Inertial Behavior and Generalized Partition

Our analysis is broken into three parts. First, we formulate an axiom that captures inertial behavior in terms of preferences over menus of acts when the information structure, as well as the ex post preferences, are unobservable. Second, we discuss which information structures give rise to inertial behavior under Bayesian updating. Finally, our main theorem establishes that, when information is subjective, inertial behavior of a forward looking DM can always be explained as that of a Bayesian who faces an appropriate information structure, namely a generalized partition.

2.1. Inertial Behavior

Let $S = \{s_1, ..., s_k\}$ be a finite state space. An act is a mapping $f : S \to [0, 1]$. We interpret payoffs in $[0, 1]$ to be in utils; that is, we look directly at utility acts (assuming that the cardinal utility function over outcomes is known and payoffs are stated in its units) instead of the corresponding Anscombe-Aumann (1963) acts. For any number $c \in [0, 1]$, we simply denote by c the constant act that yields c in every state. Let $\mathcal{F}$ be the set of all acts.

We model a DM who expects to receive a signal from some set Σ prior to choosing an act from some feasible set. Let $r : S \times \Sigma \to [0, 1]$ be the information structure he faces, where $r_s(\sigma)$ denotes the probability of learning signal $\sigma \in \Sigma$ given $s \in S$. Since we consider an environment where the information structure r is subjective and ex post preferences (after a signal has been received) are not observable, we adopt the menu choice approach, with the interpretation that preferences over menus of acts reflect the DM's anticipated, signal-contingent preferences. To accommodate a DM who may ignore information, we do not

assume that anticipated future choice between acts correctly takes into account any signal the DM may receive. We do, however, assume that this choice ignores states under which that signal cannot be obtained. Let $\mathcal{K}(\mathcal{F})$ be the set of all non-empty compact subsets of $\mathcal{F}$. Capital letters denote sets, or menus, and small letters denote acts. For example, a typical menu is $F = \{f, g, h, ...\} \in \mathcal{K}(\mathcal{F})$. Let $\succeq$ be a binary relation over $\mathcal{K}(\mathcal{F})$. The symmetric and asymmetric parts of $\succeq$ are denoted by $\sim$ and $\succ$, respectively. Let $fIg \in \mathcal{F}$ be the act that agrees with $f \in \mathcal{F}$ on event I and with $g \in \mathcal{F}$ outside I, that is,

$$fIg(s) = \begin{cases} f(s) & s \in I \\ g(s) & \text{otherwise} \end{cases}$$

In order to capture inertial behavior in terms of preferences over menus, we start by defining the class of *irreducible events*. According to Definition 1 below, event $I \in 2^S$ is irreducible if and only if a bet on I, $\{cI0\}$, cannot be perfectly substituted by the option to bet on any of its strict subsets, $\{cI'0 \,|\, I' \subset I\}$.[4] Intuitively, event I is irreducible if and only if the DM considers all states in I possible and can foresee learning at least one signal σ that does not rule out any of those states.

Definition 1. *Event $I \subseteq S$ is irreducible if for any $c > 0$,*

$$\{cI0\} \succ \{cI'0 \,|\, I' \subset I\}.$$

Note that contingent on any signal σ with $r_s(\sigma) = 0$, the act $c(I \backslash \{s\})0$ is as good as $cI0$. Consequently, if for every foreseen signal σ there is at least one state $s \in I$ with $r_s(\sigma) = 0$, then $\{cI0\} \sim \{cI'0 \,|\, I' \subset I\}$. That is, $\{cI'0 \,|\, I' \subset I\}$ is a perfect substitute for $\{cI0\}$. Conversely, if the DM considers all states in I possible and can foresee a signal σ with $r_s(\sigma) > 0$ for all $s \in I$,[5] then contingent on learning σ, the act $cI0$ does strictly better than any act in $\{cI'0 \,|\, I' \subset I\}$, and thus $\{cI0\} \succ \{cI'0 \,|\, I' \subset I\}$.

Let $\mathcal{F}_+ := \{f \in \mathcal{F} \,|\, f(s) > 0 \text{ for all } s \in S\}$ be the collection of acts with strictly positive payoffs in all states. Under very mild assumptions, it is sufficient to impose our main axiom only for acts in $\mathcal{F}_+$, since $\mathcal{F}_+$ is dense in $\mathcal{F}$ with respect to the Euclidean metric (viewing acts as vectors in $[0, 1]^{|S|}$). Confining attention to $\mathcal{F}_+$ is convenient in writing Definition 2 and Axiom 1 below.

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[4]Throughout the paper, $\subseteq$ (resp., $\subset$) denotes weak (resp., strict) set inclusion. The support of any function is denoted by $\text{supp}(\cdot)$.

[5]Formally, all states in I should not be null in the sense of Savage (1954), i.e., for every s it is not the case that $\{f\} \sim \{f\{s\}g\}$ for all f and g. Clearly, if $s \in I$ is null, then $c(I \backslash \{s\})0$ is as good as $cI0$, and hence $\{cI0\} \sim \{cI'0 \,|\, I' \subset I\}$, that is, $\{cI'0 \,|\, I' \subset I\}$ is a perfect substitute for $\{cI0\}$.

Definition 2. *For an irreducible event I and acts $f, g \in \mathcal{F}_+$ with $f(s) \neq g(s)$ only if $s \in I$, define $\succeq_I$ by*

$$f \succeq_I g \Leftrightarrow \{f, g\} \sim \{f\} \cup \{0\{s\} g \,|\, s \in I\}.$$

According to Definition 2, $f \succeq_I g$ describes the property that the DM is willing to commit to choose f over g as long as no state in I has been ruled out. To see this, note that $g \in \mathcal{F}_+$ and hence for any signal σ and $s \in I$ such that $r_s(\sigma) > 0$, g is better than $0\{s\} g$. Therefore, the DM will be indifferent between $\{f, g\}$ and $\{f\} \cup \{0\{s\} g \,|\, s \in I\}$ upon learning σ only if he expects to choose f from $\{f, g\}$. At the same time, for any signal σ such that $r_s(\sigma) = 0$ for some $s \in I$, the DM will be indifferent between choosing from $\{f, g\}$ and choosing from $\{f\} \cup \{0\{s\} g\}$.

Axiom 1 (Inertial behavior). *For any irreducible event I and acts $f, g \in \mathcal{F}_+$ with $f(s) \neq g(s)$ only if $s \in I$,*

$$\{f\} \succeq \{g\} \Rightarrow f \succeq_I g.$$

The axiom captures our interpretation of inertial behavior: the ranking of two acts that differ only on event I does not change as long as no state in I has been ruled out.

It is worth mentioning that Axiom 1 is consistent with the possibility that the DM has strict preference for flexibility, in the sense that it does not preclude the ranking $\{f, g\} \succ \{f\} \succeq \{g\}$. That is, while given I the DM would prefer f to g when no states in I have been ruled out, it may well be that for some σ with $r_s(\sigma) = 0$ for some $s \in I$, g is better than f.

2.2. Generalized Partition and the Non-Shifting Weights Property

Suppose our DM is an expected utility maximizer, who holds prior beliefs μ over S and, for any signal $\sigma \in \Sigma$ that is generated by the information structure r, forms a posterior $\Pr(\cdot \,|\, \sigma)$ that governs his ultimate choice of an act. The requirement that the ranking of any two acts does not change after learning a signal that does not rule out any of the states where they differ, suggests a relationship between μ, r, and the collection $(\Pr(\cdot \,|\, \sigma))_{\sigma \in \Sigma}$, which we now explore.

We say that the triple $(\mu, r, (\Pr(\cdot \,|\, \sigma))_{\sigma \in \Sigma})$ satisfies the *non-shifting weights property* if (i) $r_s(\sigma) = 0$ implies $\Pr(s \,|\, \sigma) = 0$, and (ii) the relative weight of any two states s, s' with $r_s(\sigma) > 0$ and $r_{s'}(\sigma) > 0$ remains intact when σ is learned, that is,

$$\frac{\mu(s)}{\mu(s')} = \frac{\Pr(s \,|\, \sigma)}{\Pr(s' \,|\, \sigma)}. \tag{1}$$

Consider $f,g \in \mathcal{F}$ and $\sigma \in \Sigma$ such that $f(s) \neq g(s)$ only if $r_s(\sigma) = 0$, and such that $\sum_{s \in S} f(s) \mu(s) \geq \sum_{s \in S} g(s) \mu(s)$. It follows immediately from (1) that for some s' with $r_{s'}(\sigma) > 0$,

$$\sum_s f(s) \Pr(s \mid \sigma) = \frac{\Pr(s' \mid \sigma)}{\mu(s')} \sum_{s : r_s(\sigma) > 0} f(s) \mu(s)$$

$$\geq \frac{\Pr(s' \mid \sigma)}{\mu(s')} \sum_{s : r_s(\sigma) > 0} g(s) \mu(s) = \sum_{s \in S} g(s) \Pr(s \mid \sigma)$$

That is, if the DM is not only an expected utility maximizer ex ante but also expects to maximize expected utility ex post, then the behavioral implication of non-shifting weights is precisely our notion of inertial behavior.

We now establish that non-shifting weights is consistent with Bayesian updating if and only if the information structure r can be described as what we term a *generalized partition*.

Definition 3. *A function $\rho : 2^{S'} \to [0,1]$ is a generalized partition of $S' \subseteq S$ if for any $s \in S'$, ρ_s defined by $\rho_s(I) = \begin{cases} \rho(I) & \text{if } s \in I \\ 0 & \text{if } s \notin I \end{cases}$ satisfies $\sum_{I \subseteq S'} \rho_s(I) = 1$.*

A generalized partition induces an information structure where $\rho_s(I)$ is the probability of event I being reported contingent on the state being s. (When there is no danger of confusion, we simply refer to ρ itself as an information structure.) The special case of a set partition corresponds to ρ taking only two values, zero and one. In that case, for every $s \in S'$ there exists a unique $I_s \in 2^{S'}$ with $s \in I_s$ and $\rho_s(I_s) = 1$. Furthermore, $s' \in I_s$ implies that $I_s = I_{s'}$, that is, $\rho_{s'}(I_s) = 1$ for all $s' \in I_s$.

The fact that $\rho_s(I)$ is independent of s (conditional on $s \in I$) implies that even a Bayesian DM cannot draw any inferences from learning an event other than that states outside that event were not realized – in this sense signals are appropriately described as events. Formally:

Proposition 1. *The triple $(\mu, r, (\Pr(\cdot \mid I))_{I \in \mathcal{I} \subseteq 2^S})$ satisfies the non-shifting weights property if and only if we can label signals as events, $\Sigma = \mathcal{I}$, and there is a generalized partition ρ such that $r_s \equiv \rho_s$.*

Proof. For generalized partition ρ, Bayes' law implies that for any $s, s' \in I$,

$$\frac{\Pr(s \mid I)}{\Pr(s' \mid I)} = \frac{\rho_s(I) \mu(s) / \mu(I)}{\rho_{s'}(I) \mu(s') / \mu(I)} = \frac{\mu(s)}{\mu(s')}$$

independently of I. Conversely, if the information structure r is not a generalized partition, then there exists an signal $\sigma \in \Sigma$ and two states s, s' with $r_s(\sigma) > 0$ and $r_{s'}(\sigma) > 0$, such that $r_s(\sigma) \neq r_{s'}(\sigma)$. For these two states, condition (1) fails. ∎

In the next sections we turn to describe how, with subjective information, inertial behavior can be explained as that of a Bayesian who perceives a generalized partition.

2.3. Generalized Partition Representation

The following utility function over menus incorporates the notion of a generalized partition.

Definition 4. *The pair* (μ, ρ) *is a generalized-partition representation if (i)* $\mu : S \to [0, 1]$ *is a probability measure; (ii)* $\rho : 2^{\text{supp}(\mu)} \to [0, 1]$ *is a generalized partition of* $\text{supp}(\mu)$*; and (iii)*

$$V(F) = \sum_{I \in 2^{\text{supp}(\mu)}} \max_{f \in F} \left[\sum_{s \in I} f(s) \mu(s) \right] \rho(I)$$

represents $\succeq$.

According to Definition 4, the DM behaves as if he (i) holds prior beliefs μ over the state space; and (ii) perceives a generalized partition that determines which event he will learn contingent on the true state. Upon learning event I, the DM calculates posterior beliefs using Bayes' law, which leads him not to shift weights; he excludes all states that are not in I and keeps the relative likelihood of the remaining states fixed. He then chooses from the menu the act that maximizes the corresponding expected utility.

The two parameters of the generalized partition representation, the prior μ on S and the generalized partition ρ, are independent, in the sense that the definition places no joint restriction on the two.[6] In other words, a generalized-partition representation accommodates any prior beliefs the DM might have about the objective state space, combined with any information structure that can be described as a generalized partition. It is evident from the description of the model that the information structure in Definition 4 is not objectively given. Instead, the generalized-partition should be derived from choice behavior.

Our representation theorem builds on the notion of a subjective-learning representation of $\succeq$, introduced in DLST

Definition 5. *A subjective-learning representation is a function* $V : \mathcal{K}(\mathcal{F}) \to \mathbb{R}$*, such that*

$$V(F) = \int_{\Delta(S)} \max_{f \in F} \left(\sum_{s \in S} f(s) \pi(s) \right) dp(\pi), \tag{2}$$

where $p(\cdot)$ *is a probability measure on* $\Delta(S)$*, the space of all probability measures on* S*.*

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[6]The only role of the restriction that ρ be a generalized partition of $\text{supp}(\mu)$ rather than S is to facilitate the statement of the identification result in Theorem 1: which event is reported contingent on states the DM considers impossible has no behavioral implications, and hence the value of ρ would not be identified on subsets of $S\backslash\text{supp}(\mu)$.

The interpretation of a subjective-learning representation is that the DM behaves as if he has beliefs over the possible posterior distributions that he might face at the time of choosing from the menu. For each posterior $\pi \in \Delta(S)$, he expects to choose from the menu the act that maximizes the corresponding expected utility. As established by DLST (Theorem 1 in their paper), the axioms that are equivalent to the existence of a subjective-learning representation are familiar from the literature on preferences over menus of lotteries – *Ranking, vNM Continuity, Nontriviality,* and *Independence* – adapted to the domain $\mathcal{K}(\mathcal{F})$, in addition to *Dominance*, which implies monotonicity in payoffs, and *Set Monotonicity*, which captures preference for flexibility. DLST further show that the function $p(\cdot)$ in (2) is unique.

Theorem 1. *Suppose that the relation $\succeq$ admits a subjective-learning representation (as in (2)). Then $\succeq$ satisfies Axiom 1 if and only if it has a generalized-partition representation, (μ, ρ). Furthermore, the pair (μ, ρ) is unique.*

Proof. See Appendix B. ∎

One immediate implication of a subjective-learning representation (2), is that a forward looking DM with *any* updating rule can be thought of as Bayesian, where the prior that governs the ranking of singletons is simply the expectation over posteriors. Non-shifting weights further requires this prior to agree with *each* posterior on the relative weights of all states that have not been ruled out. In the previous section we verified that this requirement implies that the information structure must be a generalized partition. That is, a forward looking DM who displays inertial behavior (as in Axiom 1) can always be thought of as being rational given some generalized partition.

Compared to the information structures permitted by a subjective-learning representation (as captured by the function $p(\cdot)$ in (2)) a generalized partition (Definition 3) rules out information structures in which two distinct states can generate the same signal but with different probabilities. For example, consider a defendant who is on a trial. Generalized partition is inconsistent with a setting in which there are two states of nature, guilty (G) or innocent (I), and two signals, guilty (g) and innocent (i), such that $\Pr(g\,|G) > \Pr(g\,|I) > 0$ and $\Pr(i\,|I) > \Pr(i\,|G) > 0$. Obviously, generalized partition is consistent with the familiar set-partition where signals are deterministic contingent on the state, but it also accommodates many other plausible learning processes. In the next section we provide examples, as well as a characterization, of those learning processes that give rise to generalized partitions.

In terms of behavior, the following is a simple example of a pattern which our axioms and representation accommodate, but would be precluded if anticipated signals were deterministic. Consider the state space $\{s_1, s_2\}$ and the menu $\{(1,0), (0,1), (1-\varepsilon, 1-\varepsilon)\}$, which

contains the option to bet on either state, as well as an insurance option that pays reasonably well in both states. A DM who is uncertain about the information he will receive by the time he has to choose from the menu may strictly prefer this menu to any of its subsets (for ε small enough). For instance, an investor may value the option to make a risky investment in case he understands the economy well, but also value the option to make a safe investment in case uncertainty remains unresolved at the time of making the investment choice. Our model accommodates this ranking. In contrast, such a ranking of menus is ruled out if signals are deterministic. If the DM expects to learn the true state, then preference for flexibility stems *exclusively* from the DM's prior uncertainty about the true state and the insurance option is irrelevant, that is, $\{(1,0),(0,1),(1-\varepsilon,1-\varepsilon)\} \sim \{(1,0),(0,1)\}$. And if the DM does not expect to learn the true state, then, for ε small enough, he anticipates choosing the insurance option with certainty, that is, $\{(1,0),(0,1),(1-\varepsilon,1-\varepsilon)\} \sim \{(1-\varepsilon,1-\varepsilon)\}$.[7]

3. A Characterization of Generalized Partitions

In this section we characterize the types of learning processes that can give rise to a generalized partition, by identifying all sets of events that support such an information structure. Formally, we characterize the set

$$\left\{\Psi \subseteq 2^{S'} \,\middle|\, \text{there is a generalized partition } \rho : 2^{S'} \to [0,1] \text{ with } \operatorname{supp}(\rho) = \Psi \right\}.$$

It is worth mentioning that the analysis in this section does not have any decision theoretical aspect and it also does not depend on the distinction between observable and unobservable information.

Definition 6. *A collection of events* $\Psi \subseteq 2^S$ *is a* uniform cover *of a set* $S' \subseteq S$, *if (i)* $S' = \bigcup_{I \in \Psi} I$; *and (ii) there is a function* $\beta : \Psi \to \mathbb{Z}_+$ *and a constant* $k \geq 1$, *such that* $\sum_{I \in \Psi | s \in I} \beta(I) = k$ *for all* $s \in S'$.

To better understand the notion of a uniform cover, consider the following example. Suppose $S = \{s_1, s_2, s_3\}$. Any partition of S, for example $\{\{s_1\},\{s_2,s_3\}\}$, is a uniform cover of S (with $k = 1$). A set that consists of multiple partitions, for example $\{\{s_1\},\{s_2,s_3\},\{s_1,s_2,s_3\}\}$, is a uniform cover of S (in this example with $k = 2$). The set $\Psi = \{\{s_2,s_3\},\{s_1,s_2,s_3\}\}$ is *not* a uniform cover of S, because $\sum_{I | s_1 \in I} \beta(I) < \sum_{I | s_2 \in I} \beta(I)$ for any $\beta : \Psi \to \mathbb{Z}_+$. The set

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[7]Appendix A provides a formal behavioral comparison between our model and one with deterministic signals.

$\{\{s_2, s_3\}, \{s_1\}, \{s_2\}, \{s_3\}\}$, however, is a uniform cover of S with

$$\beta(I) = \begin{cases} 2 & \text{if } I = \{s_1\} \\ 1 & \text{otherwise} \end{cases}.$$

Lastly, the set $\{\{s_1, s_2\}, \{s_2, s_3\}, \{s_1, s_3\}\}$ is a uniform cover of S (with $k = 2$), even though it does not contain a partition.

An empirical situation that gives rise to a uniform cover consisting of two partitions is an experiment that reveals the state of the world if it succeeds, and is completely uninformative otherwise. For a concrete example that gives rise to a uniform cover that does not contain a partition, consider the sequential elimination of n candidates, say during a recruiting process. If k candidates are to be eliminated in the first stage, then the resulting uniform cover is the set of all $(n - k)$-tuples.

Theorem 2. *A collection of events Ψ is a uniform cover of $S' \subseteq S$ if and only if there is a generalized partition $\rho : 2^{S'} \to [0, 1]$ with $\operatorname{supp}(\rho) = \Psi$.*[8]

Proof. See Appendix C ∎

The 'only if' part in the proof of Theorem 2 amounts to finding a solution to a system of linear equations, showing that any uniform cover can give rise to a generalized partition. To illustrate the idea, consider the collection $\{\{s_1\}, \{s_2, s_3\}, \{s_1, s_2, s_3\}\}$ discussed above, which consists of multiple partitions and is a uniform cover with $k = 2$. An information structure r that can be described as a generalized partition

$$\rho : \{\{s_1\}, \{s_2, s_3\}, \{s_1, s_2, s_3\}\} \to (0, 1]$$

should satisfy two conditions. First, $r_s(I)$ – the probability of receiving the signal I given state s – should be positive and identical for any $s \in I$, that is,

$$r_{s_1}(\{s_1\}) =: \rho(\{s_1\}) > 0$$
$$r_{s_2}(\{s_2, s_3\}) = r_{s_3}(\{s_2, s_3\}) =: \rho(\{s_2, s_3\}) > 0$$
$$r_{s_1}(\{s_1, s_2, s_3\}) = r_{s_2}(\{s_1, s_2, s_3\}) = r_{s_3}(\{s_1, s_2, s_3\}) =: \rho(\{s_1, s_2, s_3\}) > 0$$

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[8]The notion of uniform cover is closely related to that of balanced collection of weights, which Shapley (1967) introduces in the context of cooperative games. Similarly, a generalized partition formally resembles a cooperative game with unit Shapley value.

and 0 otherwise. Second, each $r_s(\cdot)$ must be a probability measure, that is,

$$r_{s_1}(\{s_1\}) + r_{s_1}(\{s_1, s_2, s_3\}) = 1$$
$$r_{s_2}(\{s_2, s_3\}) + r_{s_2}(\{s_1, s_2, s_3\}) = 1$$
$$r_{s_3}(\{s_2, s_3\}) + r_{s_3}(\{s_1, s_2, s_3\}) = 1$$

The solutions of this system are given by $\rho(\{s_1, s_2, s_3\}) = \alpha$ and $\rho(\{s_1\}) = \rho(\{s_2, s_3\}) = 1 - \alpha$, for any $\alpha \in (0, 1)$.

Theorem 2 characterizes the types of learning that can be accommodated by a generalized partition. To illustrate it, let us consider a specific example. An oil company is trying to learn whether there is oil in a particular location. Suppose the company can perform a test-drill to determine accurately whether there is oil, $s = 1$, or not, $s = 0$. In that case, the company learns the uniform cover $\Psi = \{\{0\}, \{1\}\}$ that consists of a partition of the state space, and $\rho(\{0\}) = \rho(\{1\}) = 1$ provides a generalized partition. Now suppose that there is a positive probability that the test may not be completed (for some exogenous reason, which is not indicative of whether there is oil or not). The company will either face the trivial partition $\{\{0, 1\}\}$, or the partition $\{\{0\}, \{1\}\}$, and hence $\Psi = \{\{0, 1\}, \{0\}, \{1\}\}$. Suppose the company believes that the experiment will succeed with probability q. Then $\rho(\{0, 1\}) = 1 - q$ and $\rho(\{0\}) = \rho(\{1\}) = q$ provides a generalized partition.

We can extend the previous example and suppose the company is trying to assess the size of an oil field by drilling in l proximate locations, which means that the state space is now $\{0, 1\}^l$. As before, any test may not be completed, independently of the other tests. This is an example of a situation where the state consists of l different attributes (i.e., the state space is a product space), and the DM may learn independently about any of them. Such learning about attributes also gives rise to a uniform cover that consists of multiple partitions and can be accommodated.

To find a generalized partition based on (i) a uniform cover Ψ of a state space S, for which there is a collection Π of partitions whose union is Ψ; and (ii) a probability distribution q on Π, one can set $\rho(I) = \sum_{\mathcal{P} \in \Pi | I \in \mathcal{P}} q(\mathcal{P})$. We refer to the pair (q, Π) as a random partition.

A different situation in which the DM effectively faces a random partition, (q, Π), is one where learning is governed by a given filtration of S, but the speed of learning is uncertain. In that case, distinct partitions in Π simply correspond to the information the DM might have at the time of choice, and should thus be ordered by fineness. In that case, $q(\mathcal{P})$ captures the probability of arriving at $\mathcal{P}$ by the time of choice.

Lastly, reconsider the example of sequential elimination of candidates outlined above. Suppose that one out of three candidates will be selected. Write the corresponding state space as $S = \{s_1, s_2, s_3\}$. If one candidate will be eliminated in the first round, then the

uniform cover of events the DM might learn is given by $\Psi = \{\{s_1, s_2\}, \{s_2, s_3\}, \{s_1, s_3\}\}$. Suppose that, contingent on person i being the best candidate, the DM considers any order of elimination of the other candidates as equally likely. This corresponds to the generalized partition with $\rho(I) = 0.5$ for all $I \in \Psi$ and $\rho(I) = 0$ otherwise.

Each of the generalized partitions discussed in this section can be coupled with any prior beliefs μ to generate a generalized-partition representation (μ, ρ).

4. Non-Bayesian Updating and Generalized Partition

So far we have assumed the analyst is completely unaware of the underlying information structure. However, in some applications, the analyst may have at least some knowledge of the true information structure, which may be inconsistent with the generalized partition that explains the observed behavior according to Theorem 1. To reconcile this gap, we now investigate two different protocols that may lead the DM to subjectively perceive a generalized partition (and to display inertial behavior) even if the true information structure is not.

The first protocol is based on the psychological notion of cognitive inertia. The second protocol is inspired by a concept from social psychology, called construal level theory, which contends that the closer in time a situation is, the more easily and concretely it will be thought of. Applied to our context, we study a boundedly rational agent who is constrained in his ability to directly assess probabilities for the outcome of random variables that resolve in the distant future, and tries to behave rationally given this constraint. Understanding how these possible cognitive sources of inertial behavior lead the DM to distort the true information structure can help predict the DM's behavior in different environments (i.e., in the presence of different underlying true information structures).

4.1. Cognitive Inertia

Behavioral psychologists use the notion of cognitive inertia to describe the "tendency for beliefs or sets of beliefs to endure once formed".[9] For instance, one explanation for the difficulties of implementing organizational change is that managers fail to update and revise their understanding of a situation even when the environment changes, say when new technologies emerge (Tripsas and Gavett 2000). This impedes their ability to adjust their actions in response to relevant information, and may lead to inertial behavior.

Theorem 1 does not preclude the interpretation that our DM 'suffers' from cognitive inertia, but implies that it is restrictive. In our formal environment, we interpret cognitive

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[9]https://en.wikipedia.org/wiki/Cognitive_inertia

inertia to mean that the only information the DM reacts to is which states have conclusively been ruled out, and that this information is taken into account following Bayes' law. Our result then establishes that, for a forward looking DM (for whom the expectation over posteriors coincides with the prior), cognitive inertia can explain behavior only if the *true* information structure is such that the *biased* one — which is derived from the true one by ignoring all information about relative weights — is a generalized partition. In particular, by Theorem 2, the collection of events supported by the possible signals under the true information structure must be a uniform cover. In this case it is unnecessary to model the DM as biased, even if he is subject to cognitive inertia; learning by generalized partition provides a rational *as if* model of his behavior.

4.2. Shortsighted Rationality

Throughout the paper we model a DM who receives information in two stages: the first stage — after the choice of a menu but before the choice from it — corresponds to receiving a signal, whereas the second stage, which happens after all decisions have been made, is the actual realization of the state. This section considers a DM who receives a partial (but accurate) description of the true information structure, namely for each event the probability that it will be ruled out in the first stage, and who interprets these probabilities as being generated by some generalized partition as well as a probability distribution over the state space.

While the constraint on the DM's knowledge could be exogenous, we will interpret it as a bound on the horizon up to which the DM can independently form beliefs about his uncertain environment. We call such a DM *shortsighted.*

The effect of this bound on the information that is available at each stage of choice will depend on the physical timing of the decision problem, rather than just the sequence of events. We consider the situation where partial commitments (i.e., menu choice) can be made before information arrives, after which a final decision must be made, where the quality of that decision will become known (i.e., the state will realize) only in the distant future. Therefore, the shortsighted DM can only directly assign probabilities to uncertainty that resolves in the first stage. This constraint has two consequences. First, the DM does not initially hold beliefs over the final states, but only over the signals he might learn in the first stage. Second, he understands only under which states that signal could not have been obtained, but is unable to independently form a prior over the remaining states. In particular, he is oblivious to any correlation in the resolution of uncertainty across periods, and hence cannot tell apart two signals with the same support, even if they correlate differently with the states in that support (see, for instance, the defendant example in Section 2.3). Since

14

the DM has no perception of correlation, it is without loss to treat those two signals as the same and to identify each signal with the event consisting of the states that are not ruled out by it.

As an example, consider a departmental recruiting committee that tries to fill one job opening from a given set of candidates, each of which belongs to a different group. The committee members cannot assess the probability of each candidate to be a good match for the department (the true state), but they do have a good idea of the likelihood that any particular subset of candidates will clear the bar set by their respective groups within the department (the signal). The committee is not able to compare or interpret the bar set by each group, and so perceives only a distribution over possible collections of approved candidates (an event). How can the committee base their decision on this incomplete information?

Being otherwise rational, the shortsighted DM (e.g., the committee) would like to infer the distribution over states, as well as the underlying information structure, from beliefs over first stage events. Since the DM does not perceive any correlation between events and states (beyond his understanding of which states have been ruled out), it seems natural that he will consider only information structures that likewise feature no correlation, that is, he will try to find a generalized partition. We call this protocol for inference *shortsightedly rational*, and ask whether following the protocol is always possible. The next result answers this question in the affirmative: it establishes that the DM can always find a pair of prior and generalized partition that jointly generate the given distribution over events.[10]

Theorem 3. *Let π be a probability measure over $2^S \backslash \emptyset$. There exists $\mu \in \Delta(S)$ and a generalized partition ρ on $2^{\sigma(\mu)}$ such that $\rho(I)\mu(I) = \pi(I)$ for all $I \in 2^{\sigma(\mu)}$.*

A proof is in Appendix D. Observe that such μ and ρ exist if and only if for any $s \in S$ either $\mu(s) = 0$ or $\sum_{\{I \in 2^{\sigma(\mu)} | s \in I\}} \rho(I) = \sum_{\{I \in 2^{\sigma(\mu)} | s \in I\}} \frac{\pi(I)}{\mu(I)} = 1$ holds. We establish that a solution to this non-linear system of equations exists via a fixed point theorem. In light of Theorem 2, it is clear that the support of μ must be such that the support of π constitutes a uniform cover of it.

The pair (μ, ρ) in the theorem may not be unique: Suppose, for example, that for all $I \in \text{supp}(\pi)$ it is the case that $s \in I$ if and only if $s' \in I$. Then the relative weight μ assigns to s versus s' can not be identified. However, as the following result shows, uniqueness is guaranteed if the DM can conceive of learning the state already in the first stage (i.e., $\pi(\{s\}) > 0$ for all $s \in S$).

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[10]Once more, Theorem 3 and Theorem 4 below are not restricted to the analysis of a boundedly rational DM, but can be more broadly applied whenever the DM receives a partial description of the true information structure, in the form of the probability distribution over events it induces.

Theorem 4. *Let π be a probability measure over $2^S \backslash \emptyset$ with $\pi(\{s\}) > 0$ for all $s \in S$. If $\mu \in \Delta(S)$ and ρ is a generalized partition on $2^{\sigma(\mu)}$ such that $\rho(I)\mu(I) = \pi(I)$ for all $I \in 2^{\sigma(\mu)}$, then for any other (μ', ρ') with this feature, $\mu' = \mu$ and $\rho' = \rho$.*

A proof is in Appendix E. To recapitulate, Theorem 3 implies that a shortsighted DM who follows the protocol described above will display cognitive inertia for *any* information structure he faces. Suppose the analyst happens to know the true information structure and that the condition in Theorem 4 is met. If the analyst is also willing to assume that the DM is shortsightedly rational, then the theorem implies that he can determine the information structure inferred by the DM, without relying on any choice data.

5. Related Literature

Our model builds on a general subjective-learning representation of preferences over menus of acts, which identifies signals with posterior distributions over states (Definition 5 in Section 2.3). Such representation was first derived in DLST. They further derived a partitional learning representation, in which signals correspond to events that partition the state space. Partitional learning is also studied in De Oliveira et al. (2016), who adopt the same menu choice approach but, unlike us, allow the DM to choose his information structure, and in Lu (2015), who uses the random choice approach to elicit information. A dynamic version of partitional learning appears in Dillenberger, Krishna, and Sadowski (2016).

In terms of generality, the generalized partition representation lies in between the two extreme cases studied in DLST. For a Bayesian DM, a salient feature of partitional learning is that the relative weights of states in the learned event do not shift. In this paper we show that this non-shifting-weights property corresponds to inertial behavior, which links ex ante and (anticipated) ex post rankings of acts. We then identify the *least restrictive* model with this property. Sections 2.3 and 3 demonstrate the merit of the generality permitted by our model (over partitional learning). In Appendix A we show how one can strengthen our main axiom to give a different characterization of partitional learning then the one provided in the aforementioned papers.

Starting with Sims (1998, 2003), numerous models of rational inattention were suggested to explain and match the inertia found in macroeconomic data, such as sluggish adjustment of prices in reaction to demand shocks (see, among many others, Mackowiak and Wiederholt 2009 and 2015; Steiner, Stewart, and Matějka 2016; and the empirical findings in Andersen et al. 2015). These models are based on the assumption that economic agents have limited attention which they allocate optimally, leading them to update their beliefs only infrequently. The inertia addressed by these models is, then, a delay in the reaction to new information the

more so the lower the stakes. In contrast, inertial behavior in our paper captures immediate but potentially incomplete adjustment that is independent of the stakes. Another related model, though one without any information arrival, is proposed in Vega-Redondo (1993). In his model, risk averse agents have precise perceptions of the consequences associated with the action they are currently performing, but only imprecise perceptions of all other actions, leading to status quo bias (which he refers to as inertial behavior).

Our notion of inertial behavior resembles the following familiar requirement, which is often referred to as dynamic consistency in the context of objective information: the ex ante comparison of alternatives that differ only on some event $I \subseteq S$ is the same as the ex post comparison of those alternatives, contingent upon learning that event I has occurred.[11] Breaking this requirement into two parts, violations of the axiom might emerge from either (i) an inconsistency between ex ante preferences and anticipated ex post preferences, or (ii) an inconsistency between anticipated ex post preferences and actual ex post preferences. Siniscalchi (2011), for example, explicitly focusses on (ii).[12] Importantly, in Siniscalchi's paper the information structure is objective – modeled as a fixed filtration, or partition if restricted to two stages – and signals are observable, so that it is possible to elicit anticipated ex post preferences by directly asking the DM whether he prefers to receive act f or g contingent on learning event I, while otherwise receiving the same act h in either case. In contrast, in our paper information is subjective, and hence inertial behavior rules out (i), but must be silent on (ii).

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[11]See, for example, Axiom 2 in Ghirardato (2002).

[12]Siniscalchi refers to anticipated ex post preferences as *conjectural* ex post preferences.

Appendices

A. Comparison with Partitional Learning

We now show how Axiom 1 can be modified to capture the idea of deterministic signals and thereby to characterize partitional learning. The axiomatization here is different than the one provided in DLST.

Axiom 2 (Partition). *For any two acts $f, g \in \mathcal{F}_+$ and irreducible event $I \subseteq S$,*

$$\{fI0\} \succeq \{gI0\} \Rightarrow \{fI0, gI0\} \sim \{fI0\}.$$

In the words of Kreps (1979), Axiom 2 can be viewed as a requirement of strategic rationality given an event. Recall that if I is irreducible then the DM can foresee learning an event J that (weakly) contains I. If signals are deterministic and I is irreducible, then there is no $I' \subset I$ the DM also foresees learning. It is thus obvious why for a Bayesian DM partitional learning satisfies this property; $fI0$ and $gI0$ agree outside I, and if $fI0$ does better on I, it will do better on any superset of I. Conversely, if information in the subjective-learning representation of Definition 5 is not partitional, then there are two posteriors $\pi, \pi' \in \text{supp}(p)$, such that $\text{supp}(\pi) = I$, $\text{supp}(\pi') = I'$, $I \neq I'$, and $I \cap I' \neq \phi$. Let $f = c$ and $g = c + \varepsilon$. Without loss of generality, suppose $I \cap I' \neq I$. If $\varepsilon > 0$ and small enough, then $\{fI0\} \succeq \{g(I \cap I')0\}$ while, since $g(I \cap I')0$ does better than $fI0$ conditional on I', $\{fI0, g(I \cap I')0\} \succ \{fI0\}$.

Since larger sets are better (preference for flexibility), $\{fI0\} \cup \{gI'0 \,|\, I' \subset I\} \succeq \{fI0\}$. Further, by payoff domination, $\{fI0, gI0\} \succeq \{fI0\} \cup \{gI'0 \,|\, I' \subset I\}$. Therefore, Axiom 2 implies Axiom 1, while, as is obvious from our results, the converse is false.

B. Proof of Theorem 1

Necessity: Suppose $\succeq$ admits a generalized partition representation (μ, ρ), that is,

$$V(F) = \sum_{I \in 2^{\text{supp}(\mu)}} \max_{f \in F} \left[\sum_{s \in I} f(s) \mu(s) \right] \rho(I)$$

represents $\succeq$.

Claim 1. *Event $I \subseteq S$ is irreducible if and only if $\rho(J) > 0$ for some $J \supseteq I$.*

Proof. This follows immediately from the following two observations:

1. For any J with $J \cap I \neq I$,

$$\max_{f \in \{cI'0 | I' \subset I\}} \left[\sum_{s \in J} f(s) \mu(s) \right] = c \sum_{s \in J \cap I} \mu(s) = \sum_{s \in J} cI0(s) \mu(s);$$

2. For any $J \supseteq I$, $\max_{f \in \{cI'0 | I' \subset I\}} \left[\sum_{s \in J} f(s) \mu(s) \right] < c \sum_{s \in I} \mu(s) = \sum_{s \in J} cI0(s) \mu(s)$.

∎

Consider an irreducible event I and two acts $f, g \in \mathcal{F}_+$. According to the generalized partition representation,

$$\{fI0\} \succeq \{gI0\} \Leftrightarrow \sum_{s \in I} \left[f(s) \mu(s) \sum_{I \subseteq S} \rho_s(I) \right] \geq \sum_{s \in I} \left[g(s) \mu(s) \sum_{I \subseteq S} \rho_s(I) \right]$$

Since ρ is a generalized partition, $\sum_{I \subseteq S} \rho_s(I) = 1$ for all $s \in \text{supp}(\mu)$, and hence $\{fI0\} \succeq \{gI0\} \Leftrightarrow \sum_{s \in I} f(s) \mu(s) \geq \sum_{s \in I} g(s) \mu(s)$. Therefore, $\{fI0\} \succeq \{gI0\}$ implies that for any event $J \supseteq I$ with $J \in \text{supp}(\rho)$ we have

$$\max_{f' \in \{fI0, gI0\}} \left[\sum_{s \in J} f'(s) \mu(s) \right] = \max \left[\sum_{s \in I} f(s) \mu(s), \sum_{s \in I} g(s) \mu(s) \right] =$$
$$= \sum_{s \in I} f(s) \mu(s) = \max_{f' \in \{fI0\} \cup \{gI'0 | I' \subset I\}} \left[\sum_{s \in J} f'(s) \mu(s) \right]$$

and also implies that for any event $J \in \text{supp}(\rho)$ with $J \cap I \neq I$ we have

$$\max_{f' \in \{fI0, gI0\}} \left[\sum_{s \in J} f'(s) \mu(s) \right] = \max_{f' \in \{fI0\} \cup \{gI'0 | I' \subset I\}} \left[\sum_{s \in J} f'(s) \mu(s) \right].$$

Therefore, $\{fI0\} \succeq \{gI0\} \Rightarrow \{fI0, gI0\} \sim \{fI0\} \cup \{gI'0 | I' \subset I\}$.

Sufficiency: Suppose that $\succeq$ admits a subjective learning representation and satisfies Axiom 1. For each $s \in S$, let $\mu(s) = \int_{\text{supp}(p)} \pi(s) \, dp(\pi)$. Since the measure p over $\Delta(S)$ in a subjective-learning representation is unique, so is μ. It is easy to see that μ is a probability measure and that for each $g, f \in \mathcal{F}$, $\{f\} \succ \{g\} \Leftrightarrow \sum_{s \in S} f(s) \mu(s) > \sum_{s \in S} g(s) \mu(s)$.

Let:
$$\Pi_0 := \left\{ \pi \,\middle|\, \exists s, s' \in \text{supp}(\pi) \cap \text{supp}(\mu) \text{ with } \frac{\pi(s)}{\pi(s')} \neq \frac{\mu(s)}{\mu(s')} \right\}$$

and for $\varepsilon > 0$

$$\Pi_\varepsilon := \left\{ \pi \,\middle|\, \exists s, s' \in \text{supp}(\pi) \cap \text{supp}(\mu) \text{ with } \frac{\pi(s)}{\pi(s')} \geq (1 + \varepsilon) \frac{\mu(s)}{\mu(s')} \right\}.$$

Finally, for any $s, s' \in S$ let

$$\Pi_{s,s',\varepsilon} := \left\{ \pi \,\middle|\, s, s' \in \mathrm{supp}\,(\pi) \cap \mathrm{supp}\,(\mu) \ \text{and}\ \frac{\pi\,(s)}{\pi\,(s')} \geq (1+\varepsilon)\,\frac{\mu\,(s)}{\mu\,(s')} \right\}.$$

Claim 2. *If* $p\,(\Pi_0) > 0$, *then there exist* $s, s' \in \mathrm{supp}(\mu)$ *and* $\varepsilon > 0$, *such that* $p\,(\Pi_{s,s',\varepsilon}) > 0$.

Proof. First note that there exists $\varepsilon > 0$ such that $p\,(\Pi_\varepsilon) > 0$. If it was not the case, then there would be $(\varepsilon_n) \downarrow 0$ such that

$$p\left(\left\{ \pi \,\middle|\, \exists s, s' \in \mathrm{supp}\,(\pi) \cap \mathrm{supp}\,(\mu) \ \text{with}\ \frac{\pi\,(s)}{\pi\,(s')} \notin \left((1-\varepsilon_n)\,\frac{\mu\,(s)}{\mu\,(s')}, (1+\varepsilon_n)\,\frac{\mu\,(s)}{\mu\,(s')} \right) \right\}\right) = 0$$

for all n. But for any s, s' we would have $\left((1-\varepsilon_n)\,\frac{\mu(s)}{\mu(s')}, (1+\varepsilon_n)\,\frac{\mu(s)}{\mu(s')} \right) \to \frac{\mu(s)}{\mu(s')}$ and therefore $p\,(\Pi_0) = 0$, which is a contradiction.

Second, since S is finite (and thereby also $\mathrm{supp}(\mu)$, and $\mathrm{supp}(\mu) \times \mathrm{supp}(\mu)$) we can write $p\,(\Pi_\varepsilon) = \sum_{s,s' \in \mathrm{supp}(\mu) \times \mathrm{supp}(\mu)} p\,(\Pi_{s,s',\varepsilon})$. This immediately implies that at least for one pair s, s' we must have $p\,(\Pi_{s,s',\varepsilon}) > 0$. $\blacksquare$

Now fix s, s' such that $p\,(\Pi_{s,s',\varepsilon}) > 0$. Let $f := \frac{1}{2}$, and for $\gamma, \beta > 0$ such that $\frac{\beta}{\gamma} \in \left(\frac{\mu(s)}{\mu(s')}, (1+\varepsilon)\,\frac{\mu(s)}{\mu(s')} \right)$, define an act g by

$$g\,(\widehat{s}) := \begin{cases} \frac{1}{2} + \gamma & \widehat{s} = s \\ \frac{1}{2} - \beta & \widehat{s} = s' \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

Note that for any I with $s, s' \in I$

$$\pi \in \Pi_{s,s',\varepsilon} \Rightarrow \frac{1}{2} < \sum_{\widehat{s} \in I} g\,(\widehat{s})\,\pi\,(\widehat{s}) ; \ \text{and} \tag{3i}$$

$$\frac{1}{2} > \sum_{\widehat{s} \in I} g\,(\widehat{s})\,\mu\,(\widehat{s}) . \tag{3ii}$$

Let $\mu_{s,s',\varepsilon}\,(s) := \int_{\Pi_{s,s',\varepsilon}} \pi\,(s)\,dp\,(\pi)$. We now argue that there exists an irreducible $I \subseteq \mathrm{supp}(\mu_{s,s',\varepsilon})$ such that

(a) $f \not\succeq_I g$;

(b) $\{fI0\} \succeq \{gI0\}$.

First, $s, s' \in \mathrm{supp}(\pi)$ for every $\pi \in \Pi_{s,s',\varepsilon}$. Because S is finite, $\Pi_{s,s',\varepsilon}$ can be partitioned into finitely many sets of posteriors, where π and π' are in the same cell of the partition

20

if and only if they have the same support. At least one of these cells must have positive weight under p. Choose I to be the support of the posteriors in such a cell. By (3i), we have $\{fI0, gI0\} \succ \{fI0\} \cup \{gI'0 \mid I' \subset I\}$, or $f \not\succeq_I g$. This establishes part (a). Part (b) is immediately implied by (3ii).

The combination of (a) and (b) constitutes a violation of Axiom 1. Therefore $p(\Pi_0) = 0$, that is, for all but a measure zero of posteriors, $\pi \in \mathrm{supp}(p)$ and $I \subset S$ imply that

$$\pi(s) = \begin{cases} \frac{\mu(s)}{\mu(I)} & s \in I \\ 0 & \text{otherwise} \end{cases}$$

Note that this result implies that if $\pi, \pi' \in \mathrm{supp}(p)$ are such that $\pi \neq \pi'$ then $\mathrm{supp}(\pi) \neq \mathrm{supp}(\pi)$. Therefore, we can index each element $\pi \in \mathrm{supp}(p)$ by its support $\mathrm{supp}(\pi) \in 2^{\mathrm{supp}(\mu)}$ and denote a typical element by $\pi(\cdot \mid I)$, where $\pi(s \mid I) = 0$ if $s \notin I \in 2^{\mathrm{supp}(\mu)}$. There is then a unique measure $\widehat{p}$ such that for all $F \in \mathcal{K}(\mathcal{F})$

$$V(F) = \sum_{I \in 2^{\mathrm{supp}(\mu)}} \max_{f \in F} \left[\sum_{s \in S} f(s) \pi(s \mid I) \right] \widehat{p}(I), \tag{3}$$

and $\mu(s) = \sum_{I \mid s \in I} \pi(s \mid I) \widehat{p}(I)$.

We have already established that $\pi(s \mid I) = \frac{\mu(s)}{\mu(I)}$ for all $s \in I \in \mathrm{supp}(\widehat{p})$. Define $\rho(I) := \frac{\widehat{p}(I)}{\mu(I)}$ and substitute $\mu(s) \rho(I)$ for $\pi(s \mid I) \widehat{p}(I)$ in (3). Bayes' law implies that

$$\rho_s(I) := \begin{cases} \rho(I) & \text{if } s \in I \\ 0 & \text{if } s \notin I \end{cases}$$

is indeed a probability measure for all s.

C. Proof of Theorem 2

(if) Let Ψ be a uniform cover of S'. Let $k \geq 1$ be the smallest number of times that S' is covered by Ψ. Set $\rho(I) = \frac{\beta(I)}{k}$ for all $I \in \Psi$.
(only if) Suppose that $\rho : 2^{S'} \to [0,1]$ is a generalized partition, with $\mathrm{supp}(\rho) = \Psi$. In addition to $\rho(I) = 0$ for $I \notin \Psi$, the conditions that ρ should satisfy can be written as $\mathbf{A}\rho_\Psi = 1$, where $\mathbf{A}$ is a $|S'| \times |\Psi|$ matrix with entries $a_{i,j} = \begin{cases} 1 & s \in I \\ 0 & s \notin I \end{cases}$, ρ_Ψ is a $|\Psi|$-dimensional vector with entries $(\rho(I))_{I \in \Psi}$, and $\mathbf{1}$ is a $|S'|$-dimensional vector of ones.

Suppose first that $\rho(I) \in \mathbb{Q} \cap (0,1]$ for all $I \in \Psi$. Rewrite the vector ρ_Ψ by expressing all entries using the smallest common denominator, $\xi \in \mathbb{N}_+$. Then Ψ is a generalized partition of size ξ. To see this, let $\beta(I) := \xi \rho(I)$ for all $I \in \Psi$. Then $\sum_{I \in \Psi \mid s \in I} \beta(I) = \sum_{I \in \Psi \mid s \in I} \xi \rho(I) = \xi$

for all $s \in S'$.

It is thus left to show that if $\rho_\Psi \in (0,1]^{|\Psi|}$ solves $\mathbf{A}\rho_\Psi = \mathbf{1}$, then there is also $\rho'_\Psi \in [\mathbb{Q} \cap (0,1]]^{|\Psi|}$ such that $\mathbf{A}\rho'_\Psi = \mathbf{1}$.

Let $\widehat{P}$ be the set of solutions for the system $\mathbf{A}\rho_\Psi = \mathbf{1}$. Then, there exists $X \in \mathbb{R}^k$ (with $k \leq |\Psi|$) and an affine function $f : X \to \mathbb{R}^{|\Psi|}$ such that $\widehat{\rho_\Psi} \in \widehat{P}$ implies $\widehat{\rho_\Psi} = f(x)$ for some $x \in X$. We first make the following two observations:

(i) there exists f as above, such that $x \in \mathbb{Q}^k$ implies $f(x) \in \mathbb{Q}^{|\Psi|}$;

(ii) there exists an open set $\widetilde{X} \subseteq \mathbb{R}^k$ such that $f(x) \in \widehat{P}$ for all $x \in \widetilde{X}$

To show (i), apply the Gauss elimination procedure to get f and X as above. Using the assumption that $\mathbf{A}$ has only rational entries, the Gauss elimination procedure (which involves a sequence of elementary operations on $\mathbf{A}$) guarantees that $x \in \mathbb{Q}^k$ implies $f(x) \in \mathbb{Q}^{|\Psi|}$.

To show (ii), suppose first that $\rho^* \in \widehat{P} \cap (0,1)^{|\Psi|}$ and $\rho^*_\Psi \notin \mathbb{Q}^{|\Psi|}$. By construction, $\rho^*_\Psi = f(x^*)$, for some $x^* \in X$. Since $\rho^*_\Psi \in (0,1)^{|\Psi|}$ and f is affine, there exists an open ball $B_\varepsilon(x^*) \subset \mathbb{R}^k$ such that $f(x) \in \widehat{P} \cap (0,1)^{|\Psi|}$ for all $x \in B_\varepsilon(x^*)$, and in particular for $x' \in B_\varepsilon(x^*) \cap \mathbb{Q}^k$ $(\neq \phi)$. Then $\rho'_\Psi = f(x') \in [\mathbb{Q} \cap (0,1]]^{|\Psi|}$. Lastly, suppose that $\rho^*_\Psi \in \widehat{P} \cap (0,1]^{|\Psi|}$ and that there are $0 \leq l \leq |\Psi|$ sets $I \in \Psi$, for which $\rho(I)$ is uniquely determined to be 1. Then set those l values to 1 and repeat the above procedure for the remaining system of $|\Psi| - l$ linear equations.

D. Proof of Theorem 3

First observe that such μ and ρ exist if, and only if, for any $s \in S$ either $\mu(s) = 0$ or $\sum_{\{I \in 2^{\sigma(\mu)} | s \in I\}} \rho(I) = \sum_{\{I \in 2^{\sigma(\mu)} | s \in I\}} \frac{\pi(I)}{\mu(I)} = 1$ holds. If $\mu(s) > 0$, then $\mu(I) \geq \pi(I)$ for any set $I \in 2^S$. Consequently, let

$$\mathcal{L} = \left\{ \mu \in \Delta(S) \,\middle|\, \mu(I) \geq \pi(I) \text{ for all } I \in 2^S \right\}$$

Claim 3. *The set $\mathcal{L}$ is nonempty, closed, and convex.*

Proof of Claim 3: To see that $\mathcal{L}$ is nonempty, consider the measure μ over S defined by $\mu(s_i) = \sum_{\{I | s_i \in I \text{ and } s_j \notin I \text{ for } j < i\}} \pi(I)$. Clearly $\mu(s_i) \in [0,1]$ for all i and $\sum_i \mu_i = \sum_{\{I \in 2^S \setminus \emptyset\}} \pi(I) = 1$. Furthermore, if we let $i_I = \min\{i | s_i \in I\}$, then $\pi(I) = \mu(s_{i_I})$ and thus $\mu(I) = \sum_{s \in I} \mu(s) \geq \pi(I)$, so $\mu \in \mathcal{L}$. For convexity, take $\mu, \mu' \in \mathcal{L}$. Since $\Delta(S)$ is a mixture space, $\alpha\mu + (1-\alpha)\mu' \in \Delta(S)$. And since $\mu(I) \geq \pi(I)$ and $\mu'(I) \geq \pi(I)$ for all $I \in 2^S$, $(\alpha\mu + (1-\alpha)\mu')(I) = \alpha\mu(I) + (1-\alpha)\mu'(I) \geq \pi(I)$ and hence $\alpha\mu + (1-\alpha)\mu' \in \mathcal{L}$. Lastly, $\mathcal{L}$ is closed since it is the intersection of closed half-spaces with the closed set $\Delta(S)$. $\square$

Define $f : \mathcal{L} \to \mathbb{R}^{|S|}$ by

$$[f\mu](s) = \begin{cases} 0 & \text{if } \mu(s) = 0 \\ \mu(s) \times \left[\sum_{\{I \in \text{supp}(\pi) | s \in I\}} \frac{\pi(I)}{\mu(I)}\right] & \text{otherwise} \end{cases}$$

Claim 4. $f(\mu) \in \mathcal{L}$ and f is continuous.

Proof of Claim 4: To see that $f(\mu) \in \mathcal{L}$, pick $\mu \in \mathcal{L}$. We first show that $f(\mu) \in \Delta(S)$. To see this, note that $[f\mu](s) \geq 0$ and that

$$\sum_{s \in S} [f\mu](s) = \sum_{s \in S} \mu(s) \times \left[\sum_{\{I \in \text{supp}(\pi) | s \in I\}} \frac{\pi(I)}{\mu(I)}\right]$$

$$= \sum_I \sum_{s \in I} \mu(s) \times \left[\frac{\pi(I)}{\mu(I)}\right]$$

$$= \sum_I \left[\frac{\pi(I)}{\mu(I)}\right] \sum_{s \in I} \mu(s)$$

$$= \sum_I \left[\frac{\pi(I)}{\mu(I)}\right] \mu(I) = \sum_I \pi(I) = 1.$$

Now pick $I' \in \text{supp}(\pi)$ and note that

$$\sum_{s \in I'} [f\mu](s) = \sum_{s \in I'} \mu(s) \times \left[\sum_{\{I \in \text{supp}(\pi) | s \in I\}} \frac{\pi(I)}{\mu(I)}\right] \geq \sum_{s \in I'} \mu(s) \times \frac{\pi(I')}{\mu(I')} = \pi(I')$$

hence $f(\mu) \in \mathcal{L}$.

For continuity, observe that f is obviously continuous on $\text{int}(\mathcal{L})$. Also notice that $\pi(\{s\}) > 0$ implies $\mu(s) > 0$ for all $\mu \in \mathcal{L}$. Let $S' = \{s \in S \,|\, \pi(\{s\}) = 0\} \subset S$. Let $\langle \mu_k \rangle \in \text{int}(\mathcal{L})$ be such that $\mu_k \to \overline{\mu}$, where $\overline{\mu}(s) = 0$ if, and only if $s \in J$, for some $J \subseteq S$. For any $s \in J$ we have

$$[f\mu_k](s) = \mu(s) \times \left[\sum_{\{I \in \text{supp}(\pi) | s \in I\}} \frac{\pi(I)}{\mu_k(I)}\right]$$

$$= \mu_k(s) \times \left[\sum_{\{I \in \text{supp}(\pi), \, I \subseteq S' | s \in I\}} \frac{\pi(I)}{\mu_k(I)} + \sum_{\{I \in \text{supp}(\pi), \, I \nsubseteq S' | s \in I\}} \frac{\pi(I)}{\mu_k(I)}\right]$$

$$= \mu_k(s) \times \left[0 + \sum_{\{I \in \text{supp}(\pi), \, I \nsubseteq S' | s \in I\}} \frac{\pi(I)}{\mu_k(I)}\right]$$

and

$$\lim_{\mu(s) \to 0} \mu(s) \times \sum_{\{I \in \text{supp}(\pi), \, I \nsubseteq S' | s \in I\}} \frac{\pi(I)}{\mu(I)} = 0.$$

Clearly $[f\mu_k](s) \to [f\overline{\mu}](s)$ for any $s \notin J$. Therefore, f is continuous. $\square$

From the two claims above, we conclude that f is a continuous function from a convex compact subset of $\mathbb{R}^{|S|}$ to itself. By Brouwer's fixed-point theorem, f has a fixed point, μ^*. But note that if μ^* is a fixed point of f and $\mu^*(s) > 0$, then $\sum_{\{I \in 2^S \setminus \emptyset | s \in I\}} \frac{\pi(I)}{\mu^*(I)} = 1$, and thus the pair (μ^*, ρ) with $\rho(I) = \frac{\pi(I)}{\mu^*(I)}$ satisfies all the requirements. ∎

E. Proof of Theorem 4

Suppose, contrary to the theorem, that $\mu \neq \mu'$ (otherwise $\rho = \rho'$ follows immediately.) Without loss, order states such that for some l and j with $k - j > l$ we have $\mu'(s_i) > \mu(s_i)$ if and only if $i \in \{1, ..., l\}$, and $\mu(s_i) > \mu'(s_i)$ if and only if $i \in \{k - j, ..., k\}$.

Let $\mu_0 = \mu$, $\overline{s}^0 = s_1$, and $\underline{s}^0 = s_k$. Define inductively $\rho_t(I) := \frac{\pi(I)}{\mu_t(I)}$, $\Delta\mu_t(s) = \mu_t(s) - \mu'(s)$,

$$
\mu_{t+1}(s) := \begin{cases}
\mu_t(s) & s \notin \{\overline{s}^t, \underline{s}^t\} \\
\mu_t(s) + \min\left(|\Delta\mu_t(\overline{s}^t)|, |\Delta\mu_t(\underline{s}^t)|\right) & s = \overline{s}^t \\
\mu_t(s) - \min\left(|\Delta\mu_t(\overline{s}^t)|, |\Delta\mu_t(\underline{s}^t)|\right) & s = \underline{s}^t
\end{cases}
$$

$\overline{s}^t := \arg\min\limits_{s:\mu_t(s)<\mu'(s)} \sum\limits_{I:s\in I} \rho_t(I)$, and $\underline{s}^t := \arg\max\limits_{s:\mu_t(s)>\mu'(s)} \sum\limits_{I:s\in I} \rho_t(I)$. Note that there is finite T with $\mu_T = \mu'$, at which point the process terminates. Hence, if we establish that ρ_t is never a generalized partition for $t > 0$, we are done.

Claim 5 (Induction beginning). ρ_1 is not a generalized partition. In particular

1. $\displaystyle\sum_{I:s_1\in I} \rho_1(I) < \min\left\{1, \min_{s\neq s_1}\left\{\sum_{I:s\in I} \rho_1(I)\right\}\right\}$

2. $\displaystyle\sum_{I:s_k\in I} \rho_1(I) > \max\left\{1, \max_{s\neq s_k}\left\{\sum_{I:s\in I} \rho_1(I)\right\}\right\}$.

Proof of Claim 5: $\mu_1(I) \geq \mu(I)$ if and only if $s_1 \in I$, with strict inequality for $s_2 \notin I$. Part 1 then follows immediately, and part 2 is analogous. □

Induction hypothesis. ρ_t is not a generalized partition. In particular

1. There is s^t with $\mu(s^t) > \mu_t(s^t)$ and $1 < \displaystyle\sum_{I:s^t\in I} \rho_t(I) \geq \max_{s\neq s^t}\left\{\sum_{I:s\in I} \rho_t(I)\right\}$.

2. There is s_t with $\mu(s_t) < \mu_t(s_t)$ and $1 > \displaystyle\sum_{I:s_t\in I} \rho_t(I) \leq \min_{s\neq s_t}\left\{\sum_{I:s\in I} \rho_t(I)\right\}$.

Claim 6 (Induction step). If μ_t and ρ_t satisfy the hypothesis, then so do μ_{t+1} and ρ_{t+1}.

24

Proof of Claim 6: Note that (i) $\mu_{t+1}(I) > \mu_t(I)$ if and only if $\overline{s}^t \in I$ and $\underline{s}^t \notin I$ (and in particular for $\{\overline{s}^t\}$) and (ii) $\mu_{t+1}(I) < \mu_t(I)$ if and only if $\underline{s}^t \in I$ and $\overline{s}^t \notin I$.

Suppose, contrary to the claim, that ρ_{t+1} is a generalized partition. Then it must be that $\sum_{I:s^t \in I} \rho_{t+1}(I) = 1$. But by (1) of the hypothesis, $1 < \sum_{I:s^t \in I} \rho_t(I) \geq \sum_{I:\overline{s}^t \in I} \rho_t(I)$ and then by (i) $\sum_{I:\overline{s}^t \in I} \rho_{t+1}(I) < 1$. Hence, ρ_{t+1} is not a generalized partition. In particular, due to (i) and the choice of $\overline{s}^t$, $\sum_{I:\overline{s}^t \in I} \rho_{t+1}(I) < \sum_{I:s \in I} \rho_{t+1}(I)$ for all s with $\mu_{t+1}(s) < \mu'(s)$, and hence (1) of the claim holds. (2) follows analogously. $\square$

This establishes the Theorem. $\blacksquare$

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