Does the Real Term Structure Forecast Consumption Growth?*

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ABSTRACT: Do people substitute consumption over time in response to a change in the expected real return? Do they react at both short and long horizons? This paper answers these questions by testing a general relationship that comes from the Euler equation: the *k*-period future consumption growth is driven by a *k*-period expected real return. Two methods are proposed: 1) for each maturity, the first method tests whether the expected real return can forecast consumption growth, and 2) in each period, the second method tests whether a term structure of expected real return can forecast consumption growth of different horizons. Using US quarterly data, both methods find the elasticity of intertemporal substitution (EIS) to be positive but not significantly different from zero, and robustness tests do not change this conclusion. My findings suggest that people do not substitute intertemporally over short or long horizons.

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1. INTRODUCTION

Do people substitute consumption over time in response to a change in the expected real return? Do they react at both short and long horizons? This paper answers these questions by testing a general implication that comes from the Euler equation: the k-period future consumption growth is driven by a k-period expected return. Log-linearizing the Euler equation, the k-period future consumption growth can be written as a function of an intercept (which comprises the constant second moments), a k-period expected return and an expectation error (Section 2).

To test that implication, two different empirical strategies are available. The first is the *time-series regression*: For each maturity[1], I test whether the expected real return can forecast consumption growth, and a different EIS is estimated for each maturity. The second is the *cross-section regression* which allows the EIS to change over time: For each period, I test whether the term structure of the expected real return can forecast the term structure of consumption growth (which is defined as the consumption growth from t to $t+m$, where $m=1,...,M$ and M is the longest horizon). Since we do not observe the real term structure for the US until very recently, I assume the Fisher equation holds and estimate the real term structure based on a model of inflation. Specifying inflation as a random walk trend plus a transitory cycle, both of which are subject to Markov switching shocks, I forecast inflation for different horizons and subtract the forecasts from the correspondent zero-coupon nominal yields to obtain the real term structure (Section 3).

Using US quarterly data from 1970Q1 to 2003Q4, both methods find the EIS to be positive but not significantly different from zero (Sections 4 and 5). The time-series regression suffers from small sample bias due to the persistence of the explanatory variable and overlapping observations of the dependent variable. Based on the results from a Monte Carlo experiment, the estimated EIS for all horizons are insignificantly different from zero. Splitting the sample at 1984Q1, the time at which the Great Moderation period begins, the estimated EIS for all horizons are slightly more significant. The GLS-corrected EIS estimate fluctuates around zero and is insignificantly different fromzero over most of the sample period. Both results imply that consumers are reluctant to substitute intertemporally over short or long horizons. Two robustness checks (Section

[1] In this paper the words *horizon* and *maturity* are used interchangeably.

6) use different measures of inflation forecasts and consumption and the main conclusion remains intact.

What can be gained from looking at consumption growth of longer horizons? According to Parker and Julliard (2005)[2], there are three arguments that consumption growth of longer horizons will more accurately measure intertemporal substitution. First, as pointed out by Wilcox (1992), sampling error, imputation procedures and definitional difficulties lead to serially correlated measurement error in consumption, and the error leads to delayed measure of actual consumption. Second, if utility is not separable, the marginal utility of consumption will be a function of leisure, habits or other variables in the utility function. Provided that these variables are stationary and correlated with the real returns, they will have a similar impact as that of a serially correlated measurement error. Third, consumption may be costly to adjust due to various constraints (e.g. it is costly to obtain the relevant information). For these three reasons, using consumption growth of longer horizons will provide a more accurate measure of how people substitute consumption over time.

The idea of linking the real term structure to consumption growths of different horizons originates from Harvey (1988)[3]. He tests whether the expected real term structure contains information on future consumption growth. He finds evidence for predictability during the 1970s and 1980s, and he shows that the real term structure has higher forecasting power than lagged consumption growth and lagged stock returns. My work relies on a similar theoretical framework as Harvey's, but the time-series regression equation is different (he considers the forecasting power of real yield *spread*), and he does not take into account the small sample bias that arises from using a relatively short quarterly sample. Harvey does not have the cross-section regression.

This paper also belongs to the large literature on the empirical performance of the consumption capital asset pricing models (C-CAPM)[4]. Besides testing the more restrictive random-walk hypothesis of Hall's (1978), most studies in the literature deal

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[2] They explain stock returns using consumption growths of different horizons, which is different from my purpose here.

[3] Another relevant paper is Chapman (1997) and he also considers consumption growth and real yields of different maturities. But his paper, with an aim of matching the predictions from a real business cycle model, differs from Harvey (1988) and the current paper by only looking at the cross correlations between consumption growth and real yields.

[4] See Merton (1971, 1973), Lucas (1978) and Breeden (1979).

with the relationship between real consumption growth and *ex post* real stock return or short-term real interest rate[5]. The results usually suggest that consumption growth cannot explain real returns. For example, to explain real stock returns with consumption growth requires a larger risk aversion parameter in the utility function (i.e. the equity premium puzzle due to Mehra and Prescott (1985)). The focus of this paper is different. I aim at using the real term structure to predict consumption growth, and unlike the real stock return the real term structure only has inflation risk[6].

2. THE CONSUMPTION EULER EQUATION – THE CRRA CASE

In the consumption capital asset pricing model (C-CAPM), the representative agent allocates resources to consumption and assets. The agent lives in an exchange economy and faces the following maximization problem at time *t*:

$$\max_{\{C_{t+\tau}\}_{\tau=0}^{\infty}} \sum_{\tau=0}^{\infty} \beta^{\tau} E_t\left[U\left(C_{t+\tau}\right)\right],$$

where β is the subjective discount rate and C_t is real consumption at time *t*. Since this paper only considers the expected real returns to default-free zero-coupon government yields, the only assets that the agent can buy are nominal bonds of different maturities with no risk of default. The agent uses the endowment and proceeds from the sale of bonds owned last period to finance consumption and bond acquisitions for the current period. The first-order condition (Euler equation) from the optimization problem is:

$$E_t\left[\beta \frac{U'\left(C_{t+1}\right)}{U'\left(C_t\right)}\left(1+r_t^{(1)}\right)\right]=1 \text{ or } \beta E_t\left[U'\left(C_{t+1}\right)\left(1+r_t^{(1)}\right)\right]=U'\left(C_t\right)$$

Since inflation from time *t* to time *t*+1 is unknown at time *t*, the one-period real return from time *t* to time *t*+1 $r_t^{(1)}$ is also unknown at time *t*. An optimizing consumer does not only equalize the marginal utilities of today and tomorrow but she also equalizes those of the more distant future. In general, the first-order condition for any horizon *m* holds:

[5] See Hall (1981), Hansen and Singleton (1983) and Mankiw, Rotemberg and Summers (1985). A recent example is Yogo (2004).
[6] More discussion on inflation risk is available in the Section 2. Data from the Treasury Inflation-Protected Securities (TIPS) market also contains liquidity risk, and the sample is too short for my purpose.

$$E_t\left[\beta^m \frac{U'(C_{t+m})}{U'(C_t)}\left(1+r_t^{(m)}\right)^m\right]=1 \text{ or } \beta^m E_t\left[U'(C_{t+m})\left(1+r_t^{(m)}\right)^m\right]=U'(C_t),$$

where $r_t^{(m)}$ is the unknown real return per period over the coming *m* periods. Following Hansen and Singleton (1983), the ratio of the marginal utility and the real return of any horizon *m* are assumed to be jointly log-normally distributed[7]:

$$(1)\qquad \begin{aligned}&\ln E_t\left[\beta^m \frac{U'(C_{t+m})}{U'(C_t)}\left(1+r_t^{(m)}\right)^m\right]=\\ &E_t\left[\ln\left(\beta^m \frac{U'(C_{t+m})}{U'(C_t)}\left(1+r_t^{(m)}\right)^m\right)\right]+\frac{1}{2}\operatorname{var}_t\left[\ln\left(\beta^m \frac{U'(C_{t+m})}{U'(C_t)}\left(1+r_t^{(m)}\right)^m\right)\right]=0\end{aligned}.$$

To simplify the exposition, let the utility function be the constant relative risk aversion (CRRA) type:

$$U(C_t)=\frac{C_t^{1-\gamma}-1}{1-\gamma}.$$

Plug the utility function into (1) and take the logarithm, I have the log-linearized form of the Euler equation (logged consumptions are in lowercase):

$$(2)\qquad -\gamma\left(E_t c_{t+m}-c_t\right)+m\ln\beta+E_t m\ln\left(1+r_t^{(m)}\right)+\frac{1}{2}\operatorname{var}_t\left[\ln\left(\beta^m\left(\frac{C_{t+m}}{C_t}\right)^{-\gamma}\left(1+r_t^{(m)}\right)^m\right)\right]=0.$$

For a small real return, I can use the approximation $\ln\left(1+r_t^{(m)}\right)\approx r_t^{(m)}$. Divide both sides of (2) by $m\gamma$. To obtain further insights from the Euler equation, the real return is decomposed into the nominal return and inflation: the term

$$\operatorname{var}_t\left[\ln\left(\beta^m\left(\frac{C_{t+m}}{C_t}\right)^{-\gamma}\left(1+r_t^{(m)}\right)^m\right)\right]$$

can be rewritten as

$$\operatorname{var}_t\left[\ln\left(\beta^m\left(\frac{C_{t+m}}{C_t}\right)^{-\gamma}\left(1+i_t^{(m)}\right)^m\left(1+\pi_t^{(m)}\right)^{-m}\right)\right]$$

[7] Harvey (1988) also makes use of the same assumption.

where $\pi_t^{(m)}$ is the inflation over the coming *m* periods in annual term, and $i_t^{(m)}$ is the nominal yield per period of maturity *m* that is known today. For small nominal return and inflation, the variance term is approximately equal to

$$\mathrm{var}_t\left[m\ln\beta-\gamma\left(c_{t+m}-c_t\right)+mi_t^{(m)}-m\pi_t^{(m)}\right].$$

Finding the variance for the variables unknown at time *t*, the above term can be rewritten as:

$$\gamma^2\,\mathrm{var}_t\left(c_{t+m}\right)+m^2\,\mathrm{var}_t\left(\pi_t^{(m)}\right)+2\gamma m\,\mathrm{cov}_t\left(c_{t+m},\pi_t^{(m)}\right)=v_t\,.$$

Finally we have:

(3) $$\frac{1}{m}\left(E_tc_{t+m}-c_t\right)=\frac{\ln\beta}{\gamma}+\frac{1}{\gamma}E_tr_t^{(m)}+\frac{1}{2\gamma m}v_t\,.$$

When $\pi_t^{(m)}$ is high (which lowers the real return), the future consumption is also high when $\mathrm{cov}\left(c_{t+m},\pi_t^{(m)}\right)>0$ and this leads to a large v_t and $E_tr_t^{(m)}$ has to decrease in (3) to keep the equality. In this case holding a nominal zero-coupon bond for *m* periods gives you lower real return when you are in "good state," and the term v_t can be interpreted as an inflation risk premium. In words, the consumption growth per period in the coming *m* periods, $\frac{1}{m}\left(E_tc_{t+m}-c_t\right)$, is a function of an intercept, the expected real return over the coming *m* periods $E_tr_t^{(m)}$, and a conditional variance term v_t. The effect of the expected real return on the expected consumption growth is determined by the elasticity of intertemporal substitution (EIS)$1/\gamma$ (the inverse of which is the coefficient of relative risk aversion for the CRRA case).

There are two approaches to test result (3). The conventional approach is to assume the conditional variance term v_t to be constant $v^{(m)}$ (i.e. allowing the second moments to vary among horizons)[8] and estimate the following equation using time series data on consumption and real returns, separately for each maturity *m*:

(4) $$\frac{1}{m}\left(c_{t+m}-c_t\right)=\alpha^{(m)}+\frac{1}{\gamma}E_tr_t^{(m)}+\varepsilon_{t+m}\ \text{ for }\ t=1,\ldots,T\ \text{ and }\ m=1,\ldots,M$$

[8] Since all second moments are constant (or constant in a sub-sample), I assume away precautionary saving.

where $\alpha^{(m)} = \frac{\ln \beta}{\gamma} + v^{(m)}$ and $\varepsilon_{t+m} = \frac{1}{m}(c_{t+m} - c_t) - \frac{1}{m} E_t(c_{t+m} - c_t)$ is the expectation error. Let's call (4) the "time-series regression", and there are *M* such regression to estimate. Harvey (1988) estimates a variant of (4) in two different ways: 1) based on an IMA(1,1) model for inflation, he obtains the expected real returns by subtracting the inflation forecasts from the nominal zero-coupon yields, and estimates a variant of (4) by least squares, and 2) using the *ex post* real returns, he estimates a variant of (4) with instrumental variables. Estimating (4), based on Harvey's first strategy, answers the question of whether expected real return today predicts future consumption growth, separately for each horizon *m*. Harvey's second strategy is outside the scope of this paper.

The second approach to test result (3), which is similar to the regression strategy of Nelson and Siegel's (1987) and Fama and MacBeth's (1973), is to treat the term structures of consumption growth and expected real return at each time *t* as cross-sectional data. Notice that in (3) consumption growth of any horizon *m* is controlled by the same set of parameters, and at each time *t* the following regression can be estimated:

(5) $$\frac{1}{m}(c_{t+m} - c_t) = \alpha_t^{(m)} + \frac{1}{\gamma_t} E_t r_t^{(m)} + \varepsilon_{t+m} \text{ for } t = 1,\dots,T \text{ and } m = 1,\dots,M$$

Let's call (5) the "cross-sectional regression", and there are *T* such regression to estimate. I estimate (5) repeatedly and allow both the intercept term α and the EIS $1/\gamma$ to vary over time[9]. In addition to checking whether expected real return today predicts future consumption growth separately for each horizon *m*, estimating (5) answers the question of whether the effect of expected return on consumption growth varies over time. To simplify notation, I drop the expectation sign for the expected real yield $E_t r_t^{(m)}$ for the rest of this paper.

As shown in the Appendix, similar reduced-form linear relationships can be derived from the ordinal certainty equivalence (OCE) and the Epstein-Zinn's (1987) recursive utility settings.

[9] In order to be consistent with the optimization problem above, we assume at each time *t* the agent does not expect the parameters to change in the future (the EIS is a random walk), and the shock to the EIS is uncorrelated with consumption. In that case the error term contains both expectation errors of consumption and the EIS.

3. THE REAL TERM STRUCTURE AND OTHER DATA

This section first describes the model for inflation from which forecasts are made and then discusses the construction of the real term structure and other data issues.

3.1 An Unobserved-Component Model for Inflation

Obtaining a real term structure requires a model for inflation to generate forecast for different horizons. Consider the following parsimonious trend-cycle model for inflation π_t in state-space form:

(6) $$\pi_t = \tau_t + \omega_t \ ,$$

(7) $$\tau_t = \tau_{t-1} + u_t \ ,$$

(8) $$\omega_t = \theta_1 \omega_{t-1} + \theta_2 \omega_{t-2} + v_t \ ,$$

and $\begin{pmatrix} u_t \\ v_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{u,S_t} & 0 \\ 0 & \sigma^2_{v,S_t} \end{pmatrix} \right)$.

Inflation is decomposed into a random walk trend τ_t and a second-order autoregressive cycle ω_t. The shocks to the trend and cycle are uncorrelated. The AR(2) cycle is constrained to be stationary. A similar example of such decomposition for inflation is Stock and Watson (2007). They specify inflation as a stochastic trend plus white noise, with the two shocks subject to stochastic volatility. The AR(2) specification is used to pick up the serial correlation in the transitory component (as in Kang, Kim and Morley (2006)). Over the sample period the inflation dynamics changed dramatically (e.g. volatility rose and fell in the 1970s and 1980s), and to capture the uncertainty people faced when forecasting inflation I allow two regimes for the state shocks. The transition probabilities are:

$$\Pr\left[S_t = 1 \mid S_{t-1} = 1\right] = p \text{ and } \Pr\left[S_t = 0 \mid S_{t-1} = 0\right] = q$$

There are eight parameters in this model $\left\{\theta_1, \theta_2, \sigma^2_{u,S_t=1}, \sigma^2_{u,S_t=0}, \sigma^2_{v,S_t=1}, \sigma^2_{v,S_t=0}, p, q\right\}$, and they are assumed to be in the representative agent's information set. However, at each time *t*, the agent has to infer the current regime using data only up to time *t* (i.e. the filtered probability), instead of using all data (i.e. the smoothed probability)[10]. As the

[10] See Kim and Nelson (1999) for details.

empirical results show, the model identifies a high-variance and a low-variance regime for inflation. The agent is uncertain about which regime he is currently in, and he can only use the available data and the known parameters to infer the probability of being in each regime. The agent forecasts inflation for different horizons based on the known parameters and the inferred regime.

I consider four quarterly inflation series: the GDP deflator, the PCE deflator, the CPI and the CPI without food and energy (the "core CPI"). The two CPI series are monthly, and I convert them to quarterly frequency by taking the monthly averages. All series are seasonally adjusted, and inflation is defined by 400 times the first difference of the logarithm of the price index. The core CPI is available from 1957Q1 to 2007Q2, and the other three series are available from 1947Q1 to 2007Q2. The state-space model (5)-(7) is estimated by maximum likelihood (for details see Kim and Nelson (1999)), and the results are in Table 1. Figures 1-4 show the filtered and smoothed trend and low-variance probability for the four inflation series.

3.2 The Real Term Structure and Other Data

Data for the nominal term structure are generated by the program provided by Robert Bliss for maturity of 1 quarter to 5 years, with an interval of 1-quarter (See Bliss (1997) for details on the data). They are Fama-Bliss unsmoothed yields for 1970:01 to 2003:12 (end-of-month observations)[11], and I convert the yields into quarterly data by taking the average of the monthly observations. The real term structure is obtained by subtracting the inflation forecasts from the nominal yields. Figure 5 plots the 1-quarter real yield and 5-year real yield for the four inflation series. The real term structure has some unsurprising features: the long real yield is usually higher and less variable than the short real yield, during the 1970s both yields are close to zero and occasionally negative (i.e. monetary policy is "accommodative"), and under the Volcker period both yields are high (i.e. monetary is "tight").

All data except the nominal yields used in this paper are from the Federal Reserve Bank at St. Louis's *FRED* database. Real consumption data on nondurable goods and

[11] I also use this shorter sample period to estimate the inflation model, and the inflation forecasts are very similar.

services (C_t) is converted into quarterly data by using the quarterly averages, and into per capita data by dividing it by quarterly average US population. Consumption growth for the coming *m* quarters is defined as $\frac{400}{m}\ln\left(C_{t+m}/C_t\right)$ (i.e. all growth rates are in annual terms). Table 2 shows the statistical properties of consumption growths and the real yields. The mean of the 1-quarter real yield ranges from 1.3 to 1.9 percent, and the 5-year yield ranges from 2.7 to 3.3 percent. The two yields are equally volatile with a standard deviation of above 2 percent. Consumption growths of 1 quarter and 5 years are about 2 percent for the full sample. The standard deviation of the 1-quarter consumption is about 2 percent, and the 5-year consumption growth, being a moving average of the 1-quarter one, only has a standard deviation of 0.5 percent. First, I perform the augmented Dickey-Fuller unit root test with modified AIC on 1-quarter and 5-year consumption growths and real yields under different inflation measures. Results for the real yields are mixed: I reject the null of a unit root for the CPI and Core CPI real yields (except the 1-quarter CPI real yield), and I cannot reject for PCE and GDP ones. One conclusion to draw from the unit root tests is that all the real yields are highly persistent. For consumption, I clearly reject a unit root for the 1-quarter consumption growth, but I cannot reject one for the 5-year consumption growth. The persistence in 5-year consumption growth comes from overlapping observations (see discussion below). From the autocorrelation estimates for the series in Table 2 we can see that the 5-year yields are more persistent than the 1-quarter yields, and 5-year consumption growth is likewise more persistent than the 1-quarter consumption growth.

Figure 6 plots the real yield and consumption growth for horizons 1-quarter and 5-year, where real yield of maturity *m* at quarter *t* is matched by consumption growth from quarter *t* to quarter *t*+*m*. The plot for 1-quarter is visually striking: real yield seems to track future consumption growth closely, especially from the 1980s onward; the 5-year real yield has little to do with the future 5-year consumption growth, with the latter being much less volatile[12].

[12] Of course, most of the discrepancy for the 5-year case can be accounted by the forecast horizon being longer than the sampling interval, which results in serially correlated error. See below.

4. RESULTS FROM TIME-SERIES REGRESSION

4.1 Econometric Problems and a Monte Carlo Experiment

The predictive regression (4) is subject to at least two econometric problems[13]. First, a large horizon *m* leads to over-rejection of the null $1/\gamma = 0$: we find long-horizon predictability when there is none. Second, the Newey-West covariance estimator to correct for moving average error term becomes inconsistent when the sample size is small. In the conclusion of their paper Nelson and Kim (1987) suggest that “[t]he investigator would seem to be obliged to develop the empirical distribution of the statistic under the null hypotheses using simulations methods before drawing inferences (p. 660)”. This section conducts a Monte Carlo experiment to answer the question of whether real yield predicts consumption growth in each horizon.

Result (4) is a special case of the generic system:

(9) $y_t = \alpha + \beta x_{t-1} + u_t$,

(10) $x_t = \mu + \phi x_{t-1} + v_t$,

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} \sim i.i.d.\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{pmatrix} \right).$$

The above system is studied by Stambaugh (1986) and Mankiw and Shapiro (1985). For our purpose we can think of y_t as consumption growth and x_t as the expected real yield. The OLS estimator in the predictive regression (9) is consistent, though it is not unbiased in finite sample as x_{t-1} is not orthogonal to the error term u_t. Stambaugh proves that

$$E\left(\hat{\beta} - \beta\right) = \frac{\sigma_{uv}}{\sigma_v^2} E\left(\hat{\phi} - \phi\right),$$

where, as Kendall (1954) has shown,

$$E\left(\hat{\phi} - \phi\right) = \frac{-\left(1 + 3\phi\right)}{n} + O\left(n^{-2}\right).$$

The more persistent is x_{t-1}, the stronger the correlation is between the two errors, or the smaller the sample size, the larger will be the small sample bias for the estimate

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[13] See Nelson and Kim (1993), Stambaugh (1986) and Mankiw and Shapiro (1985) for earlier references. A recent work is Valkanov (2003).

of β (i.e. the real yield may appear to be a predictor for consumption growth when it is not). For our persistent expected real yield (See Section 3) and small sample size, there will be small sample bias in the estimate for the EIS when the shocks to consumption growth and expected real yield are contemporaneously correlated.

For consumption growth of horizon more than one quarter, regression (4) suffers from the Hansen-Hodrick's (1980) overlapping observation problem. When the growth horizon is longer than the sample frequency, the expectation error in (4) will be a moving average process. A conventional remedy is to use the autocorrelation and heteroskedasticity consistent Newey-West (1987) standard error. Unfortunately, the Newey-West estimator gives correct inference only asymptotically, and for a small sample its performance can be problematic.

I deal with the two problems by conducting a simple Monte Carlo experiment to find out the empirical distribution for regression equation (4)[14]. According to (9), the data-generating process (DGP) for the Euler equation is written as:

$$r_t^{(m)} = \mu_m + \phi_m r_{t-1}^{(m)} + v_t,\ v_t \sim i.i.d.\left(0, \sigma_v^2\right)$$

$$\Delta c_t = \alpha + \frac{1}{\gamma} r_{t-1}^{(1)} + u_t,\ u_t \sim i.i.d.\left(0, \sigma_u^2\right)$$

where the two shocks are contemporaneously correlated with a correlation of ρ_{uv}. The null is $1/\gamma = 0$. First the 1-quarter consumption growth is regressed on a constant to obtain α, and the square of the standard error of regression is used as the parameter σ_u^2 in the DGP. Per annum consumption growths of longer horizons are constructed with the 1-quarter consumption growth. An AR(1) model is fitted for the *m*-period expected real yield to obtain the parameters μ_m, ϕ_m and σ_v^2. The parameter ρ_{uv} is obtained by the correlation between the residuals of the 1-quarter consumption growth regression and the 1-quarter expected real yield[15]. Notice that a different set of parameters is used for different inflation measures, different real yield maturities and different sub-samples. Each experiment has 5000 runs. I run OLS for (4) for each horizon and obtain the

[14] Instead of conducting the Monte Carlo experiment, I can follow Nelson and Kim (1993) and use the randomization method instead. Since I do not find much non-normal features (e.g. fat tails) in both the consumption data and real yield data, the Monte Carlo method is appropriate.

[15] The results are similar if a different correlation is calculated for each maturity of the expected real yield.

coefficient estimate for $1/\gamma$, the standard error, the *t*-statistic, and the *R*-squared. Then I compare the statistics with the counterparts from the Monte Carlo experiment to make inference[16].

4.2 Results

Tables 2 and 3 show the full-sample results for the four inflation measures. Looking at columns 2 to 5, it would seem that the real yields have predictability power for consumption growth, especially for longer horizons. But the critical values from the Monte Carlo experiment tell us that none of the estimates for $1/\gamma$ are significant, and the corrected *R*-squareds (by subtracting the Monte Carlo *R*-squared from the one of the data) for most of the regressions are close to zero. All we can conclude from these results is that a 1% increase in real yield today increases future consumption growth by less than 0.1%, and the amount of increase is larger for more distant consumption growth but the effect is imprecisely estimated for all horizons. Notice that the empirical *t*-ratios are skewed, reflecting the negative bias in the slope estimate (the reason of which is the small positive correlation between the two shocks in the DGP), but the results show that even after taking the bias into account the slope estimates are still not significantly larger than zero.

Next the sample is split into two: 1970Q1 to 1983Q4 and 1984Q1-2003Q4. The two sub-samples roughly correspond to the pre-Great Moderation and Great Moderation periods, and it is well-documented that between the two periods the second moments decrease for most macroeconomic time series (See Kim and Nelson (1999)). By splitting the sample, I allow a break in the second moments (which are contained in the intercept) between the two samples. Tables 4-5 show the results for the early sample, and Tables 6-7 show those for the later one. The results are more encouraging. The corrected *R*-squared is higher for most of the regressions, the estimates for $1/\gamma$ are in general higher, and some of them are significant at the 10% level. In the early sample, a 1% increase in real yield today significantly increases future consumption growth by about 0.1% for

[16] Equivalently, I can use the Newey-West standard errors in both the estimation with the data and the Monte Carlo experiment. The *t*-statistics in the data and the experiment will both be lower, but my conclusions will remain the same.

horizons above 4 years. In the later sample, a 1% increase in real yield today significantly increases future consumption growth by about 0.2% for horizons below 1 year. Since most of the slope coefficients are still imprecisely estimated, the results in the split samples only provide weak evidence for intertemporal substitution in consumption[17].

Results for the time-series regression (4) suggest that people do not substitute over time, or at most very little. In the next section I test the theory from the cross-section perspective using regression (5) and avoid some of the econometric problems in regression (4).

5. RESULTS FOR CROSS-SECTIONAL REGRESSION

5.1 A Generalized Least Squares (GLS) Procedure

Result (5) relaxes the identifying assumption that the intercept α and $1/\gamma$ are constant over time, and replaces it with the identifying assumption that the EIS $1/\gamma$ is the same for all horizons. Unlike the estimation procedure in Section 4, in which the Euler equation for each horizon is treated as a time-series regression, now (5) is estimated as cross sections. In vector form we have, for each quarter *t*:

$$(11) \qquad \mathbf{y} = \boldsymbol{\alpha} + \frac{1}{\gamma}\mathbf{r} + \boldsymbol{\varepsilon},$$

where $\mathbf{y} = \begin{pmatrix} 400(c_{t+1} - c_t) \\ 400(c_{t+2} - c_t)/2 \\ \vdots \\ 400(c_{t+M} - c_t)/M \end{pmatrix}$, $\boldsymbol{\alpha} = \begin{pmatrix} \alpha^{(1)} \\ \alpha^{(2)} \\ \vdots \\ \alpha^{(M)} \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} r_t^{(1)} \\ r_t^{(2)} \\ \vdots \\ r_t^{(M)} \end{pmatrix}$, and $\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_{t+1} \\ \varepsilon_{t+2} \\ \vdots \\ \varepsilon_{t+M} \end{pmatrix}$.

In words, at each quarter *t* the *M* different consumption growths are stacked as the dependent variable $\mathbf{y}$, and the right-hand side has a horizon-dependent constant $\boldsymbol{\alpha}$ and the vector of expected real yield of different maturities $\mathbf{r}$. This cross-sectional regression

[17] To achieve better identification, we can restrict the EIS to be the same among maturities and estimate (4) in as a panel with *fixed maturity effects*. For the CPI real yields (results from other inflation measures are similar), with cross-section GLS weights and cross-section White correction for the standard error (available in EViews), the point estimate of the EIS is 0.079 and the *t* statistic is 3.27. Despite the potential small sample bias in the panel estimation (which is beyond the scope of this paper), people do appear to be substituting quite significantly over time, under the identification assumption that EIS is constant both over time and across maturities.

is repeatedly estimated for all quarters, and the error term in the regression can be written as (the multiplication of 400 is omitted for simplicity):

$$\varepsilon_{t+1} = (c_{t+1} - c_t) - E_t(c_{t+1} - c_t)$$

and $\operatorname{var}(\varepsilon_{t+1}) = \sigma^2$

$$\varepsilon_{t+2} = \frac{1}{2}(c_{t+2} - c_t) - \frac{1}{2}E_t(c_{t+2} - c_t) = \frac{1}{2}(c_{t+2} - c_t + c_{t+1} - c_{t+1}) - \frac{1}{2}E_t(c_{t+2} - c_t + c_{t+1} - c_{t+1})$$

$$= \frac{1}{2}(c_{t+2} - c_{t+1}) - \frac{1}{2}E_t(c_{t+2} - c_{t+1}) + \frac{1}{2}(c_{t+1} - c_t) - \frac{1}{2}E_t(c_{t+1} - c_t)$$

and $\operatorname{var}(\varepsilon_{t+2}) = \frac{1}{2}\sigma^2$.

In general we have:

$$\varepsilon_{t+M} = \frac{1}{M}(c_{t+M} - c_t) - \frac{1}{M}E_t(c_{t+M} - c_t)$$

$$= \frac{1}{M}(c_{t+M} - c_t + c_{t+M-1} - c_{t+M-1}\cdots) - \frac{1}{M}E_t(c_{t+M} - c_t + c_{t+M-1} - c_{t+M-1}\cdots)$$

$$= \frac{1}{M}(c_{t+M} - c_{t+M-1}) - \frac{1}{M}E_t(c_{t+M} - c_{t+M-1}) + \cdots + \frac{1}{M}(c_{t+1} - c_t) - \frac{1}{M}E_t(c_{t+1} - c_t)$$

and $\operatorname{var}(\varepsilon_{t+M}) = \frac{1}{M}\sigma^2$.

The above derivation depends on the assumption that consumption is a random walk (i.e. the expected real return does not explain consumption growth). To check if the estimates are sensitive to this GLS assumption, I have also run the regression without the GLS correction and the results are very similar. The distribution of the error term can thus be written as $\boldsymbol{\varepsilon} \sim i.i.d\left(\mathbf{0}, \sigma^2\boldsymbol{\Omega}\right)$,

where $\boldsymbol{\Omega} = \begin{bmatrix} 1 & 1/4 & \cdots & 1/M^2 \\ 1/4 & 1/2 & \cdots & 1/(M-1)^2 \\ \vdots & \vdots & \ddots & 1/(M-2)^2 \\ 1/M^2 & 1/(M-1)^2 & 1/(M-2)^2 & 1/M \end{bmatrix}$ and σ^2 is the constant variance of the 1-quarter expectation error which is assumed to be the same for any forecasting horizon. A GLS procedure can be applied to (11). Since (11) is not a time-series regression (i.e. how result (5) behaves over time does not matter), the overlapping observations problem mentioned in Section 4 goes away. In Section 2 it is shown that the intercept term $\boldsymbol{\alpha}$ is a function of the discount rate and the second moments, and the intercept term can be different for different maturities. To estimate (11) first I assume the same intercept applies to all horizons ($\alpha^{(1)} = \alpha^{(2)} = \cdots = \alpha^{(M)}$), and second I use data demeaned by maturity to estimate (11) to allow for different intercept terms for different horizons[18].

5.2 Results

To save space Figures 7 and 8 only show the estimates for $1/\gamma$ in (11) using the CPI inflation (results from the other three measures are similar). The GLS estimator is $\begin{bmatrix} \hat{\alpha} \\ \widehat{1/\gamma} \end{bmatrix} = \left(\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{y}$ where $\mathbf{X} = \begin{bmatrix} \boldsymbol{\iota} & \mathbf{r} \end{bmatrix}$ and the covariance matrix is $\widehat{\text{var}}\begin{pmatrix} \hat{\alpha} \\ \widehat{1/\gamma} \end{pmatrix} = \left(\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X}\right)^{-1}\hat{\sigma}^2$. When assuming the same intercept for all horizons, the estimates are in general close to zero, and they are occasionally significantly different from zero (using the 95% confidence interval). The estimates for the early sample are mostly of the wrong sign, and those after the 1980s are mostly positive. The results using the demeaned data are similar, though the estimates are even closer to zero[19]. The null of a constant EIS over the sample period for both regressions is clearly rejected, and for most of the period the EIS fluctuates around zero, and occasionally it has a large positive or negative value[20].

A more vigorous way to interpret the cross-sectional results is to take the theory as formulated in Section 2 seriously and postulate the EIS as a constant parameter: the estimates in Figures 7 and 8 should fluctuate (due to sampling error) around the true constant value. A *t* test of the form as in Fama and MacBeth (1973) is carried out to test if null that the constant EIS is zero ($1/\gamma = 0$). The test statistics is calculated as

[18] Using the different means for the two sub-samples in Section 4 does not change the results much.
[19] A cross-section of 20 observations clearly belongs to the family of small samples. Using the data produced by the Monte Carlo experiment in Section 4, I estimate the cross-section equation (5) under the null of no effect. The critical values for the slope are slightly larger, and some of the cross-section slope estimates become insignificantly different from zero.
[20] There is a strong positive correlation between the NBER recessions and the EIS estimates, i.e. the EIS is higher during a recession. I have no explanation for this result.

$\frac{\overline{1/\gamma}}{s\left(\widehat{1/\gamma}\right)/\sqrt{n}}$ where $s\left(\widehat{1/\gamma}\right)$ is the standard deviation of the EIS estimates and *n* is the number of cross-section regressions. For the regressions with the original data the test statistics is -4.44 and that with the demeaned data is -2.39. The *t* test seems to suggest that, under the assumption of a constant EIS, we cannot reject the null hypothesis that the EIS is negative, or the utility function is non-concave, consistent with the findings in Mankiw, Rotemberg and Summers (1985). To account for potential small sample bias in this *t* test, the same Monte Carlo experiment (with the null $1/\gamma = 0$) in the previous section is carried out to obtain the empirical critical values. Based on 5000 Monte Carlo runs, the 95% empirical critical values for the *t* test are -3.94 and 3.79, and we conclude once again that there is little evidence that the EIS is significantly different from zero.

6. ROBUSTNESS CHECKS

Both the time-series regression (4) and the cross-section regression (5) provide weak evidence for intertemporal substitution by the consumers[21]. The result is possibly due to inappropriate measures on inflation forecasts or consumption, and this section provides two robustness checks using different measures of the two variables.

6.1 Measurement Error In The Real Yields?

The model for inflation proposed in Section 3 is potentially mis-specified and it might provide wrong inflation forecasts[22]. If the model for forecasting inflation used by the investors is different from the trend-cycle model proposed here, the generated real

[21] In the New Keynesian model, output is a function of expected output and *ex ante* real return (the Euler equation for output). Most studies find that *ex ante* real return affects output very little and the effect is usually insignificant (e.g. Fuhrer and Rudebusch (2004) and Cho and Moreno (2006)). Since the output Euler equation is derived from the same optimization problem in Section 2 by equating output and consumption, its poor performance is consistent with my findings with the consumption Euler equation. Also, the weak response of future consumption growth to real yield seems to be contradictory to the well-known finding that the nominal yield spread is a good predictor of future GDP growth. But as Estrella (2004) has shown, in a simple three-equation New Keynesian model, there is some monetary policy rule that makes a small EIS consistent with the predictability.

[22] An alternative argument will be that the inflation data investors use is different from the current inflation data which is revised and updated. Estimating the Euler equation with “vintage data” is left for future research.

yields will also be different from the real yields the investors have in mind. The "wrong" real yields cannot tell us whether people substitute consumption over time.

I have experimented with several other models for inflation that assume a unit root (e.g. IMA(1,1) as used by Harvey (1988)), and found that the forecasts are similar to those from the trend-cycle model. The reason is simple: once a unit root is assumed, the data suggests an unimportant transitory component in inflation.

One alternative model for inflation that gives different forecasts will be a model that does not assume a unit root in inflation (e.g. a VAR model as in Ang, Bekaert and Wei (2007)). Using such a model will be controversial when making long-horizon forecasts. For example, during the high inflation period during the late 1970s, a stationary model for inflation will imply that the 5-year forecast will be close to the sample mean. If we believe that during the high inflation period people predict the high inflation will persist (i.e. people do not believe the Fed can bring down inflation), such forecasts will be implausible. Of course, there are other more ad hoc models that take into account time-varying behavior of inflation (e.g. split sample or rolling window), but the trend-cycle model in Section 3 is chosen to keep the model parsimonious.

To check whether the results in Section 4 and 5 are robust, I estimate an AR(1) model with a constant term for CPI inflation recursively: The initial sample is 1947Q2-1970Q1, and the coefficient estimates are used to forecast inflation in 1 quarter to 5 years. The next set of forecasts is based on the updated sample 1947Q2-1970Q2 and the updated set of parameters. The "stationary" inflation forecasts are subtracted from the nominal yields to obtain expected real yields. Results for (4) and (5) based on this set of expected real yields are similar to those based on the trend-cycle model: the EIS estimates for (4) for all horizons are slightly higher in magnitude but are still insignificant, and the EIS estimates for (5) fluctuate similarly around zero though they have more extreme values.

6.2 Measurement Error in Consumption Data?

Another potential explanation for the lack of intertemporal substitution is that the consumption data is of low quality. If there is a classical measurement error (e.g. white noise) in consumption growth, the standard error for the slope estimate will increase, and it may explain the insignificant results in the time-series regression (4).

Besides the measurement error as a white noise, Hahm (1998) also argues that "… for both homeowners and renters, it would be prohibitively costly to intertemporally substitute their housing consumption in response to frequently changing real interest rates". Using US quarterly data from 1953Q1 to 1994Q4, he excludes housing expenditure from service expenditure and finds that the elasticity of intertemporal substitution is about 0.3 and significant. However, I have re-estimated regressions (4) and (5) using 1) nondurable and service expenditure minus housing and 2) only nondurable expenditure, and find similar results as those for nondurable and service expenditure.

7. CONCLUDING REMARKS

The expected real yield is a poor predictor of future consumption growth for any horizon. Based on a consumer optimization problem, the consumption growth will realize in *k* periods should increase with the current *k*-period expected real yield, and the size of the increase is determined by the EIS. Empirical results show that such relationship does not exist, or at most it is weak one. Hall's (1988) hypothesis that the EIS is low seems to hold even for longer horizons.

There are two lessons to learn. If we take the consumer optimization problem seriously and believe that the implications from the problem are *insensitive* to the choice of the utility function, then the findings are bad news for monetary policy. According to the interest rate transmission mechanism, the central bank affects the real economy in the following manner: the central bank changes the short real rate with the Taylor rule and the long real rate according to the expectation theory, the real rates affect real consumption/output through the Euler equation, and the change in the real activity affects inflation through the Phillips curve. In such a framework, a small and imprecisely estimated EIS means that the mechanism breaks down at the Euler equation and monetary policy is ineffective and unreliable.

The second lesson is that if we believe that people do substitute over time, then the findings imply that the optimization problem or the functional form usually adopted in the literature is inadequate for producing intertemporal substitution. A more complicated functional form (e.g. hyperbolic discounting) or a different setup of the problem (e.g. a non-separable leisure choice) might help; otherwise the forward-looking assumption should be abandoned (see Startz (2007)).

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Table 1 – Maximum Likelihood Estimates for the Inflation Model

Parameter	CPI Inflation	Core CPI Inflation	GDP Inflation	PCE Inflation
θ_1	0.385	0.169	0.044	0.002
	(0.345)	(0.405)	(0.349)	(0.390)
θ_2	0.118	0.168	0.000	0.000
	(0.239)	(0.161)	(0.022)	(0.022)
$\boldsymbol{p}$	0.970	0.951	0.979	0.971
	(0.015)	(0.023)	(0.013)	(0.016)
$\boldsymbol{q}$	0.921	0.823	0.927	0.876
	(0.042)	(0.077)	(0.045)	(0.071)
$\sigma^2_{u,S_t=1}$	1.153	2.440	3.237	5.154
	(4.277)	(1.378)	(1.558)	(2.364)
$\sigma^2_{u,S_t=0}$	0.086	0.051	0.105	0.140
	(0.097)	(0.048)	(0.048)	(0.061)
$\sigma^2_{v,S_t=1}$	9.597	3.408	3.333	2.485
	(5.658)	(1.473)	(1.353)	(1.462)
$\sigma^2_{v,S_t=0}$	1.158	0.217	0.450	0.590
	(0.295)	(0.055)	(0.082)	(0.115)
Likelihood	-460.265	-263.15	-389.213	-399.306
Observations	1947Q2-2007Q2	1957Q2-2007Q2	1947Q2-2007Q2	1947Q2-2007Q2
	241	201	241	241

Note: I estimate a trend-cycle model for inflation, with Markov switching in the state shock variances. Standard errors in parenthesis are obtained by delta method.

Table 2 – Unit-Root Tests and Statistical Properties of Consumption Growth and Real Yields

	Mean	SD	Unit Root p-value	Autocorrelation				
				1st Lag	2nd Lag	3rd Lag	4th Lag	5th Lag
1-Quarter Con Growth	2.074	1.946	0.000	0.327	0.163	0.314	-0.021	-0.132
5-Year Con Growth	2.057	0.501	0.069	0.961	0.903	0.836	0.742	0.646
1-Quarter Yield (CPI)	1.342	2.340	0.349	0.840	0.731	0.732	0.639	0.544
5-Year Yield (CPI)	2.749	2.642	0.084	0.914	0.836	0.794	0.699	0.605
1-Quarter Yield (CPI-C)	1.329	2.103	0.053	0.807	0.661	0.580	0.470	0.397
5-Year Yield (CPI-C)	2.739	2.218	0.058	0.887	0.787	0.705	0.593	0.523
1-Quarter Yield (PCE)	1.843	2.236	0.627	0.870	0.753	0.710	0.615	0.520
5-Year Yield (PCE)	3.352	2.401	0.478	0.930	0.846	0.786	0.691	0.593
1-Quarter Yield (GDP)	1.933	2.308	0.524	0.871	0.768	0.736	0.676	0.596
5-Year Yield (GDP)	3.341	2.389	0.597	0.933	0.863	0.818	0.771	0.701

Note: I use the augmented Dickey-Fuller unit root test with modified AIC. The last observation for the 5-year consumption growth is on 2002Q1, which is the consumption growth from 2002Q1 to 2007Q1. All unit root tests are tests on the level of the series and include an intercept.

Table 2 – Estimate for $1/\gamma$ in Equation (4): Full Sample (1970Q1 – 2003Q4) for GDP and PCE Inflations

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	-0.088	0.064	-1.379	0.014	-2.138	-1.840	1.515	1.817	0.008
2	-0.064	0.052	-1.230	0.011	-2.871	-2.428	2.099	2.527	0.014
3	-0.033	0.047	-0.710	0.004	-3.480	-2.993	2.508	3.056	0.021
4	-0.017	0.044	-0.390	0.001	-4.003	-3.411	2.855	3.537	0.027
5	0.004	0.041	0.093	0.000	-4.574	-3.821	3.068	3.731	0.032
6	0.019	0.038	0.487	0.002	-5.040	-4.181	3.290	3.959	0.037
7	0.024	0.036	0.683	0.003	-5.360	-4.499	3.600	4.500	0.043
8	0.033	0.034	0.969	0.007	-5.552	-4.692	3.788	4.554	0.048
9	0.042	0.032	1.284	0.012	-5.999	-5.025	4.032	4.936	0.052
10	0.050	0.031	1.630	0.019	-6.361	-5.306	4.239	5.173	0.059
11	0.058	0.029	1.991	0.029	-6.565	-5.576	4.270	5.263	0.061
12	0.061	0.027	2.255	0.037	-6.880	-5.753	4.461	5.477	0.069
13	0.062	0.026	2.441	0.043	-7.028	-5.959	4.561	5.579	0.071
14	0.063	0.024	2.613	0.048	-7.306	-6.107	4.624	5.693	0.077
15	0.063	0.023	2.738	0.053	-7.568	-6.273	4.804	5.891	0.081
16	0.059	0.022	2.663	0.051	-7.799	-6.473	5.033	6.174	0.083
17	0.055	0.021	2.603	0.049	-8.060	-6.834	5.074	6.339	0.088
18	0.051	0.020	2.528	0.047	-8.270	-7.013	5.185	6.402	0.089
19	0.048	0.020	2.450	0.044	-8.706	-7.237	5.221	6.389	0.095
20	0.044	0.019	2.340	0.041	-8.489	-7.161	5.334	6.738	0.100

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	-0.036	0.066	-0.548	0.002	-2.103	-1.814	1.511	1.835	0.008
2	-0.014	0.054	-0.264	0.001	-2.850	-2.423	2.127	2.526	0.014
3	0.020	0.048	0.418	0.001	-3.602	-2.990	2.496	3.053	0.021
4	0.033	0.045	0.733	0.004	-4.163	-3.501	2.866	3.488	0.026
5	0.049	0.042	1.161	0.010	-4.517	-3.848	3.056	3.741	0.032
6	0.063	0.038	1.654	0.020	-4.992	-4.280	3.341	4.072	0.038
7	0.068	0.036	1.909	0.026	-5.337	-4.596	3.502	4.305	0.043
8	0.073	0.034	2.150	0.033	-5.491	-4.616	3.752	4.608	0.047
9	0.079	0.032	2.477	0.044	-5.960	-5.072	3.935	4.874	0.053
10	0.084	0.030	2.790	0.055	-6.191	-5.183	4.015	4.807	0.056
11	0.088	0.028	3.110	0.067	-6.630	-5.504	4.298	5.253	0.062
12	0.090	0.027	3.405	0.080	-6.971	-5.874	4.297	5.379	0.068
13	0.089	0.025	3.569	0.087	-7.037	-6.096	4.407	5.409	0.071
14	0.086	0.024	3.657	0.091	-7.287	-6.167	4.640	5.767	0.076
15	0.083	0.022	3.698	0.093	-7.627	-6.446	4.500	5.610	0.079
16	0.076	0.021	3.570	0.088	-7.647	-6.607	4.701	5.755	0.083
17	0.071	0.021	3.424	0.082	-7.875	-6.754	4.945	6.108	0.088
18	0.065	0.020	3.279	0.076	-8.181	-6.935	4.787	6.112	0.090
19	0.060	0.019	3.153	0.072	-8.295	-7.024	5.017	6.345	0.095
20	0.056	0.018	3.023	0.067	-8.710	-7.255	5.268	6.274	0.099

Note: Columns 2-5 show the OLS estimates for equation (4) of different horizons. Columns 6-10 show the critical values and R-squared obtained from the Monte Carlo experiment (for 5000 runs) as described in Section 3.

Table 3 – Estimate for $1/\gamma$ in Equation (4): Full Sample (1970Q1 – 2003Q4) for CPI and Core CPI Inflations

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	0.072	0.063	1.142	0.010	-2.118	-1.843	1.574	1.893	0.008
2	0.071	0.052	1.370	0.014	-2.882	-2.483	2.076	2.484	0.014
3	0.103	0.046	2.243	0.036	-3.511	-2.935	2.393	2.908	0.019
4	0.104	0.043	2.424	0.042	-4.077	-3.422	2.755	3.400	0.026
5	0.106	0.039	2.701	0.052	-4.384	-3.741	2.920	3.595	0.031
6	0.111	0.035	3.146	0.069	-4.781	-4.077	3.322	4.135	0.037
7	0.111	0.033	3.350	0.077	-5.153	-4.372	3.387	4.111	0.040
8	0.109	0.031	3.496	0.084	-5.347	-4.603	3.671	4.458	0.045
9	0.110	0.029	3.754	0.095	-5.875	-4.984	3.723	4.629	0.051
10	0.108	0.027	3.921	0.103	-6.242	-5.213	4.015	4.842	0.055
11	0.104	0.026	4.017	0.107	-6.340	-5.443	4.144	5.022	0.059
12	0.099	0.024	4.108	0.112	-6.699	-5.764	4.272	5.219	0.063
13	0.093	0.023	4.084	0.111	-6.650	-5.717	4.449	5.556	0.067
14	0.086	0.021	4.002	0.107	-6.919	-5.931	4.510	5.439	0.070
15	0.080	0.020	3.947	0.105	-7.346	-6.205	4.503	5.653	0.076
16	0.073	0.019	3.737	0.096	-7.266	-6.259	4.473	5.680	0.076
17	0.065	0.019	3.491	0.085	-7.655	-6.478	4.852	5.990	0.082
18	0.058	0.018	3.240	0.075	-7.651	-6.494	4.733	5.884	0.082
19	0.052	0.017	3.015	0.066	-7.988	-6.872	4.769	6.010	0.088
20	0.048	0.017	2.865	0.060	-8.237	-6.921	4.982	6.247	0.091

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	0.012	0.070	0.177	0.000	-2.062	-1.745	1.525	1.837	0.007
2	-0.001	0.058	-0.015	0.000	-2.934	-2.510	2.080	2.561	0.014
3	0.022	0.052	0.429	0.001	-3.409	-2.947	2.447	2.988	0.020
4	0.030	0.049	0.610	0.003	-3.800	-3.277	2.819	3.425	0.025
5	0.046	0.046	1.018	0.008	-4.154	-3.509	2.877	3.615	0.029
6	0.058	0.042	1.387	0.014	-4.672	-3.999	3.245	4.029	0.035
7	0.060	0.039	1.531	0.017	-4.926	-4.079	3.430	4.158	0.037
8	0.065	0.037	1.736	0.022	-5.348	-4.467	3.648	4.445	0.044
9	0.072	0.035	2.035	0.030	-5.469	-4.631	3.863	4.633	0.047
10	0.079	0.033	2.367	0.040	-5.628	-4.739	3.930	4.732	0.049
11	0.085	0.031	2.726	0.053	-5.862	-5.034	4.042	4.988	0.054
12	0.087	0.029	2.990	0.063	-6.120	-5.150	4.127	5.045	0.057
13	0.086	0.027	3.132	0.068	-6.256	-5.383	4.095	4.976	0.059
14	0.083	0.026	3.225	0.072	-6.639	-5.643	4.504	5.426	0.066
15	0.082	0.024	3.377	0.079	-6.799	-5.620	4.444	5.322	0.068
16	0.079	0.023	3.407	0.081	-6.969	-5.837	4.356	5.403	0.069
17	0.076	0.022	3.400	0.081	-7.056	-5.864	4.482	5.487	0.071
18	0.070	0.021	3.292	0.077	-7.284	-6.122	4.635	5.756	0.076
19	0.065	0.020	3.175	0.072	-7.165	-6.104	4.667	5.922	0.078
20	0.062	0.020	3.157	0.072	-7.320	-6.131	4.638	5.630	0.079

Note: Columns 2-5 show the OLS estimates for equation (4) of different horizons. Columns 6-10 show the critical values and R-squared obtained from the Monte Carlo experiment (for 5000 runs) as described in Section 3.

Table 4 – Estimate for $1/\gamma$ in Equation (4): Early Sample (1970Q1 – 1983Q4) for GDP and PCE Inflations

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	-0.236	0.100	-2.368	0.094	-2.119	-1.806	1.601	1.970	0.019
2	-0.190	0.084	-2.265	0.087	-2.992	-2.442	1.989	2.395	0.033
3	-0.141	0.077	-1.831	0.058	-3.481	-2.873	2.326	2.889	0.044
4	-0.107	0.074	-1.446	0.037	-3.871	-3.339	2.561	3.166	0.056
5	-0.058	0.071	-0.824	0.012	-4.253	-3.610	2.869	3.528	0.066
6	-0.016	0.066	-0.237	0.001	-4.743	-4.026	3.019	3.714	0.077
7	0.013	0.062	0.206	0.001	-5.067	-4.213	3.120	3.938	0.087
8	0.038	0.058	0.653	0.008	-5.407	-4.498	3.440	4.202	0.098
9	0.060	0.055	1.091	0.022	-5.579	-4.659	3.538	4.331	0.104
10	0.081	0.051	1.567	0.044	-5.695	-4.815	3.549	4.411	0.113
11	0.095	0.048	1.987	0.068	-6.177	-5.229	3.646	4.599	0.122
12	0.103	0.044	2.363	0.094	-6.402	-5.369	3.755	4.655	0.131
13	0.107	0.040	2.656	0.116	-6.608	-5.624	3.707	4.725	0.137
14	0.111	0.037	2.989	0.142	-6.573	-5.540	3.790	4.776	0.142
15	0.112	0.035	3.236	0.162	-7.055	-5.863	3.847	4.884	0.151
16	0.105	0.032	3.245	0.163	-6.985	-6.028	3.867	4.807	0.157
17	0.104	0.031	3.391	0.176	-7.121	-5.927	3.918	4.904	0.158
18	0.105	0.028	3.679	0.200	-7.299	-6.129	4.155	5.106	0.170
19	0.106	0.027	4.010	0.229	-7.297	-5.992	3.918	4.957	0.168
20	0.108	0.024	4.415	0.265	-7.495	-6.096	3.840	4.949	0.170

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	-0.156	0.103	-1.512	0.041	-2.226	-1.881	1.426	1.754	0.019
2	-0.110	0.086	-1.275	0.029	-3.059	-2.605	1.861	2.325	0.035
3	-0.058	0.078	-0.741	0.010	-3.654	-3.176	2.330	2.886	0.050
4	-0.030	0.074	-0.412	0.003	-4.164	-3.628	2.512	3.061	0.062
5	0.009	0.070	0.135	0.000	-4.922	-4.094	2.729	3.499	0.077
6	0.049	0.064	0.776	0.011	-5.211	-4.323	2.895	3.775	0.087
7	0.071	0.059	1.209	0.026	-5.619	-4.734	3.087	3.882	0.100
8	0.086	0.055	1.575	0.044	-6.041	-5.130	3.326	4.171	0.112
9	0.101	0.051	1.964	0.067	-6.021	-5.205	3.468	4.521	0.118
10	0.113	0.048	2.368	0.094	-6.899	-5.711	3.656	4.686	0.136
11	0.119	0.044	2.728	0.121	-7.350	-6.111	3.593	4.572	0.144
12	0.124	0.040	3.102	0.151	-7.571	-6.500	3.789	4.828	0.155
13	0.122	0.037	3.327	0.170	-7.968	-6.664	4.067	5.154	0.171
14	0.120	0.034	3.514	0.186	-8.417	-6.931	4.079	5.112	0.178
15	0.115	0.032	3.614	0.195	-8.390	-7.074	3.836	4.902	0.182
16	0.105	0.030	3.519	0.187	-8.732	-7.374	4.217	5.348	0.193
17	0.100	0.028	3.531	0.188	-8.647	-7.384	3.959	5.094	0.196
18	0.098	0.027	3.701	0.202	-8.855	-7.300	4.104	5.271	0.206
19	0.097	0.025	3.909	0.221	-9.107	-7.593	4.120	5.095	0.209
20	0.097	0.023	4.184	0.245	-9.382	-8.032	3.958	5.233	0.220

Note: Columns 2-5 show the OLS estimates for equation (4) of different horizons. Columns 6-10 show the critical values and R-squared obtained from the Monte Carlo experiment (for 5000 runs) as described in Section 3.

Table 5 – Estimate for $1/\gamma$ in Equation (4): Early Sample (1970Q1 – 1983Q4) for CPI and Core CPI Inflations

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	0.015	0.100	0.147	0.000	-2.201	-1.867	1.412	1.755	0.019
2	0.023	0.085	0.272	0.001	-3.082	-2.577	1.872	2.214	0.034
3	0.070	0.075	0.933	0.016	-3.754	-3.183	2.222	2.744	0.049
4	0.080	0.070	1.139	0.023	-4.201	-3.596	2.431	3.168	0.062
5	0.103	0.064	1.594	0.045	-4.685	-3.973	2.745	3.451	0.073
6	0.128	0.057	2.231	0.084	-5.231	-4.446	2.905	3.717	0.089
7	0.137	0.052	2.611	0.112	-5.498	-4.750	3.009	3.845	0.099
8	0.142	0.049	2.924	0.137	-6.156	-5.083	3.137	4.010	0.109
9	0.147	0.045	3.278	0.166	-6.300	-5.353	3.445	4.318	0.120
10	0.147	0.041	3.578	0.192	-6.772	-5.700	3.381	4.251	0.130
11	0.143	0.038	3.761	0.208	-7.233	-6.127	3.526	4.457	0.146
12	0.137	0.035	3.956	0.225	-7.450	-6.226	3.735	4.653	0.157
13	0.128	0.032	3.980	0.227	-7.539	-6.422	3.729	4.727	0.161
14	0.119	0.030	3.962	0.225	-8.174	-6.767	3.649	4.705	0.173
15	0.112	0.028	3.983	0.227	-8.339	-6.924	3.911	4.939	0.182
16	0.102	0.027	3.830	0.214	-8.732	-7.454	3.862	5.032	0.192
17	0.095	0.025	3.763	0.208	-8.472	-7.233	4.038	4.998	0.196
18	0.090	0.024	3.838	0.214	-8.928	-7.403	4.141	5.239	0.203
19	0.087	0.022	3.938	0.223	-8.811	-7.331	3.967	4.955	0.205
20	0.086	0.021	4.170	0.244	-9.134	-7.522	3.907	4.927	0.213

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	-0.067	0.107	-0.628	0.007	-2.272	-1.900	1.511	1.859	0.020
2	-0.087	0.090	-0.963	0.017	-2.860	-2.465	1.916	2.300	0.033
3	-0.052	0.083	-0.630	0.007	-3.565	-2.955	2.285	2.802	0.046
4	-0.032	0.079	-0.408	0.003	-4.053	-3.361	2.559	3.165	0.057
5	0.008	0.073	0.110	0.000	-4.366	-3.818	2.707	3.446	0.066
6	0.045	0.067	0.673	0.008	-4.621	-3.873	3.026	3.685	0.075
7	0.065	0.063	1.037	0.020	-5.000	-4.230	3.046	3.787	0.085
8	0.080	0.059	1.368	0.033	-5.185	-4.486	3.424	4.366	0.098
9	0.096	0.055	1.759	0.054	-5.558	-4.701	3.460	4.313	0.105
10	0.110	0.050	2.185	0.081	-6.062	-4.956	3.553	4.442	0.114
11	0.117	0.046	2.532	0.106	-6.112	-5.149	3.555	4.529	0.120
12	0.120	0.042	2.835	0.130	-6.225	-5.339	3.781	4.746	0.132
13	0.117	0.039	2.991	0.142	-6.450	-5.421	3.700	4.658	0.136
14	0.113	0.036	3.125	0.153	-6.619	-5.591	3.795	4.825	0.143
15	0.112	0.033	3.348	0.172	-7.199	-5.899	3.936	4.905	0.151
16	0.108	0.031	3.455	0.181	-7.028	-5.979	3.887	5.034	0.156
17	0.106	0.029	3.605	0.194	-7.343	-6.003	3.954	5.129	0.161
18	0.102	0.027	3.751	0.207	-7.005	-5.859	3.805	4.986	0.159
19	0.100	0.026	3.902	0.220	-7.140	-5.978	3.812	4.707	0.163
20	0.101	0.024	4.289	0.254	-7.253	-6.036	3.941	5.013	0.175

Note: Columns 2-5 show the OLS estimates for equation (4) of different horizons. Columns 6-10 show the critical values and R-squared obtained from the Monte Carlo experiment (for 5000 runs) as described in Section 3.

Table 6 – Estimate for $1/\gamma$ in Equation (4): Late Sample (1984Q1 – 2003Q4) for GDP and PCE Inflations

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	0.154	0.091	1.692	0.035	-2.217	-1.924	1.377	1.714	0.014
2	0.149	0.069	2.153	0.056	-3.148	-2.762	2.001	2.459	0.027
3	0.150	0.059	2.525	0.076	-3.871	-3.337	2.258	2.803	0.037
4	0.137	0.055	2.479	0.073	-4.450	-3.802	2.618	3.264	0.049
5	0.126	0.051	2.465	0.072	-4.953	-4.207	3.055	3.648	0.059
6	0.110	0.047	2.323	0.065	-5.572	-4.749	3.251	4.023	0.071
7	0.094	0.046	2.049	0.051	-5.946	-4.946	3.612	4.440	0.080
8	0.085	0.044	1.941	0.046	-6.147	-5.212	3.799	4.714	0.087
9	0.083	0.043	1.941	0.046	-6.673	-5.534	3.935	4.820	0.096
10	0.083	0.042	1.998	0.049	-7.158	-6.023	4.174	5.141	0.107
11	0.086	0.040	2.143	0.056	-7.538	-6.337	4.018	5.161	0.114
12	0.084	0.039	2.169	0.057	-7.441	-6.291	4.485	5.574	0.121
13	0.083	0.038	2.198	0.058	-7.494	-6.271	4.615	5.769	0.120
14	0.081	0.036	2.241	0.060	-7.770	-6.628	4.564	6.000	0.131
15	0.087	0.036	2.452	0.072	-8.180	-6.795	4.757	6.128	0.137
16	0.092	0.035	2.619	0.083	-8.196	-6.904	5.042	6.219	0.143
17	0.099	0.035	2.835	0.097	-8.753	-7.315	4.881	5.996	0.150
18	0.102	0.035	2.906	0.102	-8.657	-7.181	5.067	6.356	0.151
19	0.101	0.035	2.896	0.103	-8.781	-7.265	5.187	6.361	0.159
20	0.093	0.035	2.696	0.092	-8.754	-7.181	5.205	6.764	0.157

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	0.197	0.095	2.065	0.052	-2.133	-1.833	1.457	1.783	0.013
2	0.184	0.073	2.526	0.076	-3.060	-2.620	2.013	2.455	0.025
3	0.185	0.062	2.988	0.103	-3.645	-3.119	2.377	3.078	0.036
4	0.173	0.058	2.999	0.103	-4.375	-3.659	2.856	3.494	0.048
5	0.156	0.053	2.934	0.099	-4.930	-4.055	3.047	3.779	0.057
6	0.140	0.050	2.829	0.093	-5.146	-4.463	3.372	4.096	0.067
7	0.126	0.048	2.646	0.082	-5.735	-4.894	3.661	4.479	0.078
8	0.118	0.046	2.574	0.078	-5.785	-5.038	3.752	4.728	0.083
9	0.119	0.044	2.687	0.085	-6.009	-5.080	3.949	4.846	0.088
10	0.121	0.043	2.815	0.092	-6.771	-5.531	4.142	5.256	0.099
11	0.124	0.042	2.984	0.102	-6.896	-5.839	4.308	5.318	0.107
12	0.123	0.040	3.075	0.108	-6.938	-5.732	4.592	5.774	0.109
13	0.123	0.039	3.173	0.114	-7.412	-6.123	4.371	5.463	0.115
14	0.121	0.037	3.279	0.121	-7.156	-6.048	4.663	5.736	0.117
15	0.129	0.036	3.575	0.142	-7.488	-6.342	4.564	5.720	0.123
16	0.137	0.035	3.859	0.164	-7.405	-6.107	4.747	5.973	0.125
17	0.144	0.035	4.134	0.186	-7.822	-6.504	4.731	5.878	0.134
18	0.147	0.035	4.238	0.195	-7.930	-6.636	4.905	6.153	0.140
19	0.146	0.034	4.280	0.201	-7.702	-6.476	4.848	6.241	0.139
20	0.141	0.034	4.187	0.196	-7.777	-6.555	4.623	5.781	0.139

Note: Columns 2-5 show the OLS estimates for equation (4) of different horizons. Columns 6-10 show the critical values and R-squared obtained from the Monte Carlo experiment (for 5000 runs) as described in Section 3.

Table 7 – Estimate for $1/\gamma$ in Equation (4): Late Sample (1984Q1 – 2003Q4) for CPI and Core CPI Inflations

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	0.232	0.093	2.492	0.074	-2.172	-1.850	1.453	1.780	0.013
2	0.211	0.071	2.963	0.101	-2.884	-2.460	2.130	2.621	0.024
3	0.211	0.060	3.511	0.136	-3.523	-3.001	2.474	2.996	0.034
4	0.196	0.056	3.505	0.136	-4.083	-3.418	2.822	3.430	0.044
5	0.173	0.052	3.332	0.125	-4.490	-3.769	3.071	3.690	0.053
6	0.153	0.048	3.185	0.115	-5.101	-4.288	3.394	4.158	0.063
7	0.141	0.046	3.063	0.107	-5.173	-4.406	3.658	4.575	0.070
8	0.131	0.044	2.959	0.101	-5.396	-4.498	3.821	4.841	0.075
9	0.131	0.043	3.076	0.108	-5.716	-4.796	3.830	4.767	0.081
10	0.130	0.041	3.154	0.113	-5.782	-4.954	4.215	5.234	0.090
11	0.127	0.040	3.197	0.116	-6.322	-5.261	4.164	5.172	0.095
12	0.123	0.038	3.198	0.116	-6.320	-5.312	4.280	5.276	0.097
13	0.120	0.037	3.238	0.118	-6.323	-5.438	4.412	5.326	0.100
14	0.117	0.035	3.316	0.124	-6.318	-5.341	4.144	5.232	0.103
15	0.120	0.034	3.504	0.138	-6.583	-5.567	4.389	5.304	0.108
16	0.122	0.034	3.654	0.149	-6.656	-5.555	4.563	5.444	0.111
17	0.122	0.033	3.734	0.157	-6.898	-5.638	4.659	5.694	0.117
18	0.119	0.033	3.651	0.153	-6.817	-5.729	4.392	5.665	0.116
19	0.113	0.032	3.532	0.146	-7.066	-5.867	4.379	5.364	0.120
20	0.107	0.032	3.376	0.137	-7.171	-5.927	4.418	5.548	0.123

					Monte Carlo Critical Values				
Hor	**Est**	**S.D.**	**t-stat**	**R-sq**	**-2.5%**	**5.0%**	**95.0%**	**97.5%**	**R-sq**
1	0.203	0.104	1.946	0.046	-2.210	-1.885	1.434	1.750	0.013
2	0.196	0.079	2.461	0.072	-3.133	-2.655	1.967	2.406	0.026
3	0.185	0.068	2.718	0.087	-3.909	-3.282	2.293	2.886	0.038
4	0.167	0.064	2.610	0.080	-4.321	-3.618	2.753	3.357	0.046
5	0.148	0.059	2.516	0.075	-4.743	-4.037	2.972	3.692	0.056
6	0.124	0.055	2.243	0.061	-5.294	-4.470	3.306	4.051	0.067
7	0.102	0.053	1.911	0.045	-5.527	-4.847	3.475	4.328	0.075
8	0.093	0.052	1.793	0.040	-5.928	-5.014	3.708	4.617	0.083
9	0.090	0.050	1.785	0.039	-6.404	-5.330	3.723	4.668	0.088
10	0.091	0.049	1.855	0.042	-6.649	-5.549	3.919	4.964	0.096
11	0.100	0.048	2.098	0.053	-6.729	-5.632	4.128	5.096	0.103
12	0.103	0.046	2.222	0.060	-6.867	-5.634	4.096	5.082	0.102
13	0.105	0.045	2.361	0.067	-6.767	-5.821	4.229	5.412	0.109
14	0.108	0.043	2.525	0.076	-7.197	-6.013	4.335	5.389	0.114
15	0.115	0.042	2.759	0.090	-7.061	-5.911	4.470	5.537	0.117
16	0.120	0.041	2.955	0.103	-7.341	-6.152	4.581	5.698	0.122
17	0.126	0.040	3.179	0.119	-7.347	-6.247	4.394	5.496	0.126
18	0.126	0.039	3.247	0.125	-7.355	-6.169	4.436	5.361	0.125
19	0.126	0.038	3.300	0.130	-7.621	-6.402	4.372	5.379	0.132
20	0.121	0.038	3.199	0.124	-7.790	-6.475	4.493	5.582	0.135

Note: Columns 2-5 show the OLS estimates for equation (4) of different horizons. Columns 6-10 show the critical values and R-squared obtained from the Monte Carlo experiment (for 5000 runs) as described in Section 3.

Figure 1A: CPI Inflation, Filtered Trend and Smoothed Trend

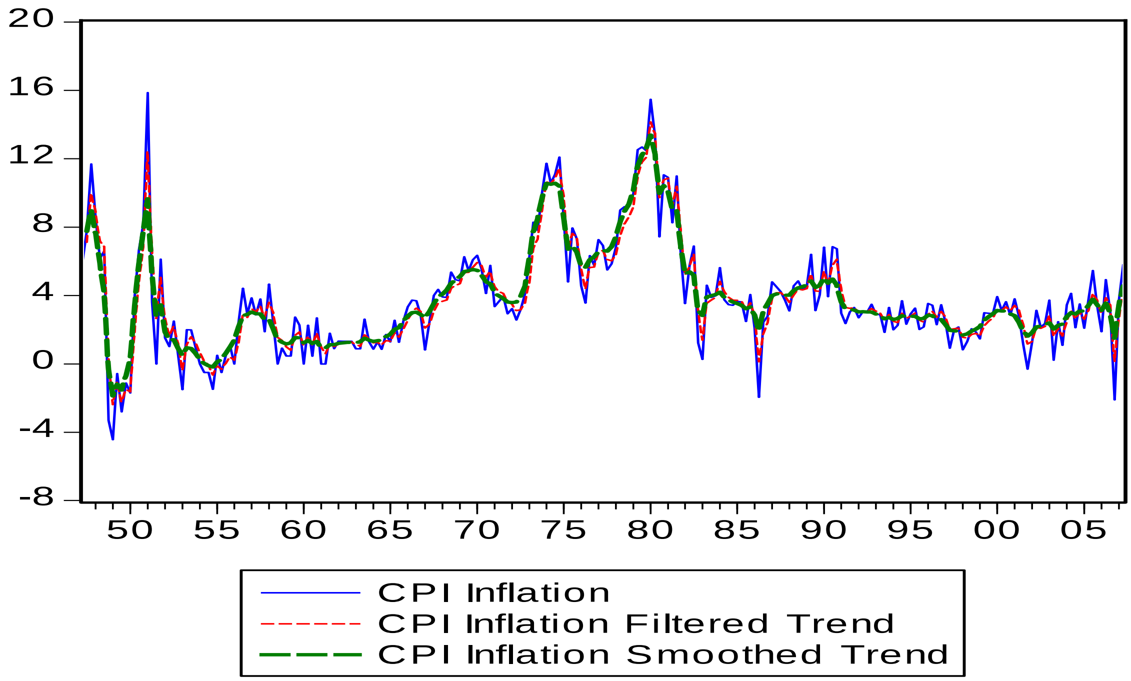
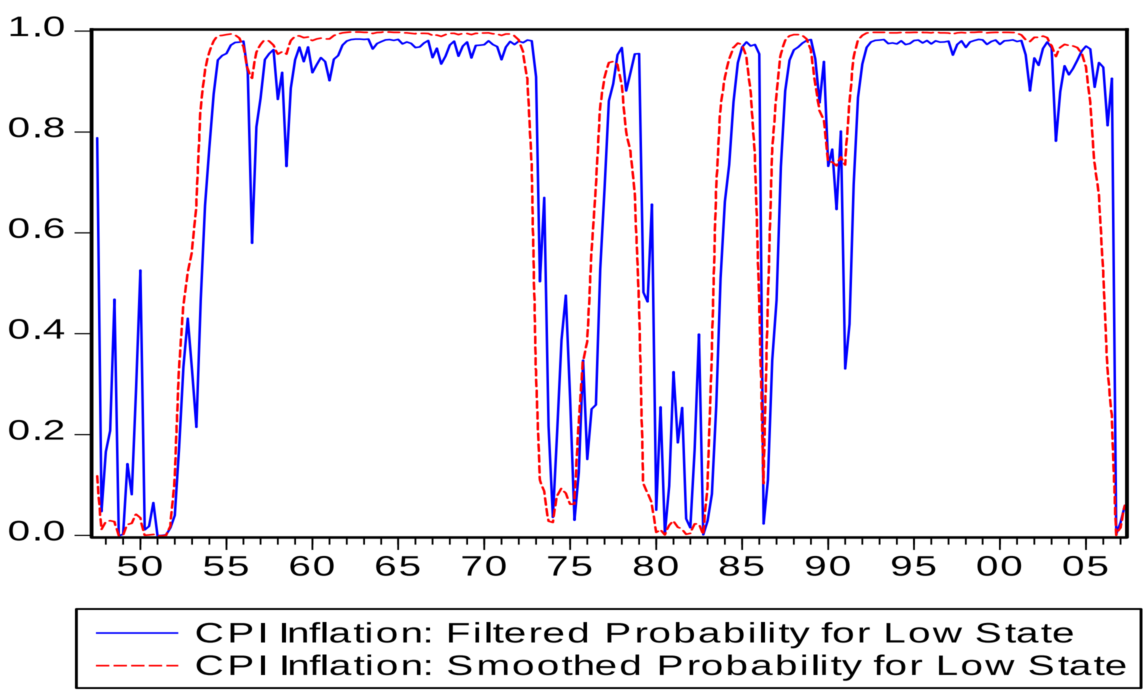


Figure 1B: CPI Inflation Filtered and Smoothed Probabilities for Low State



Note: A trend-cycle model as discussed in Section 3 is estimated for each inflation measure.

Figure 2A: Core CPI Inflation, Filtered Trend and Smoothed Trend

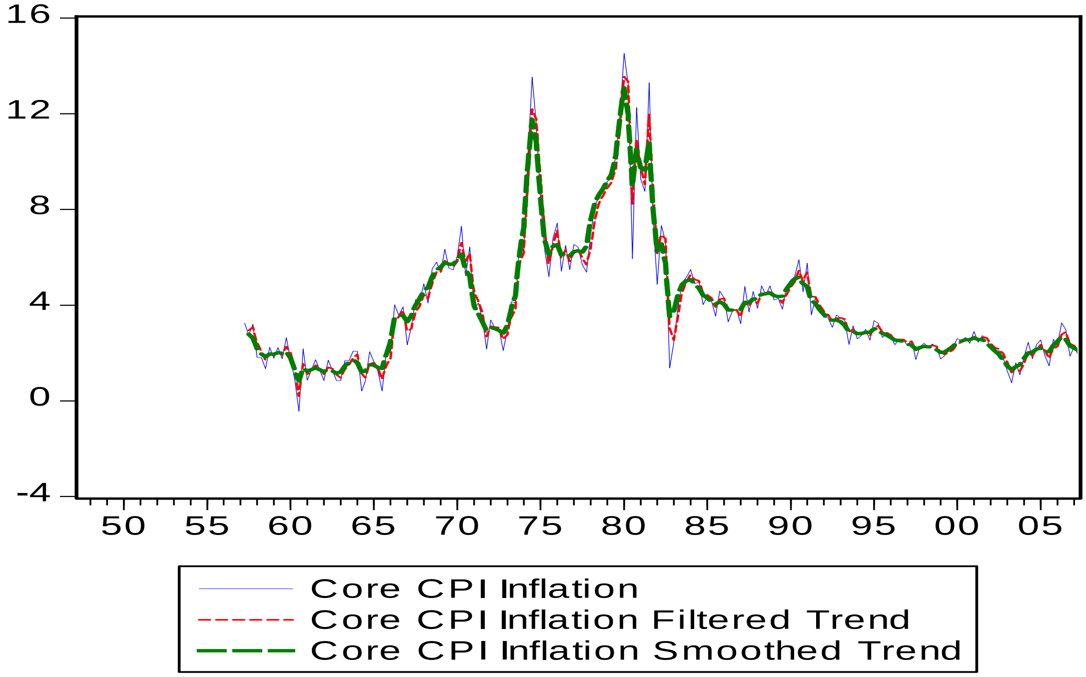
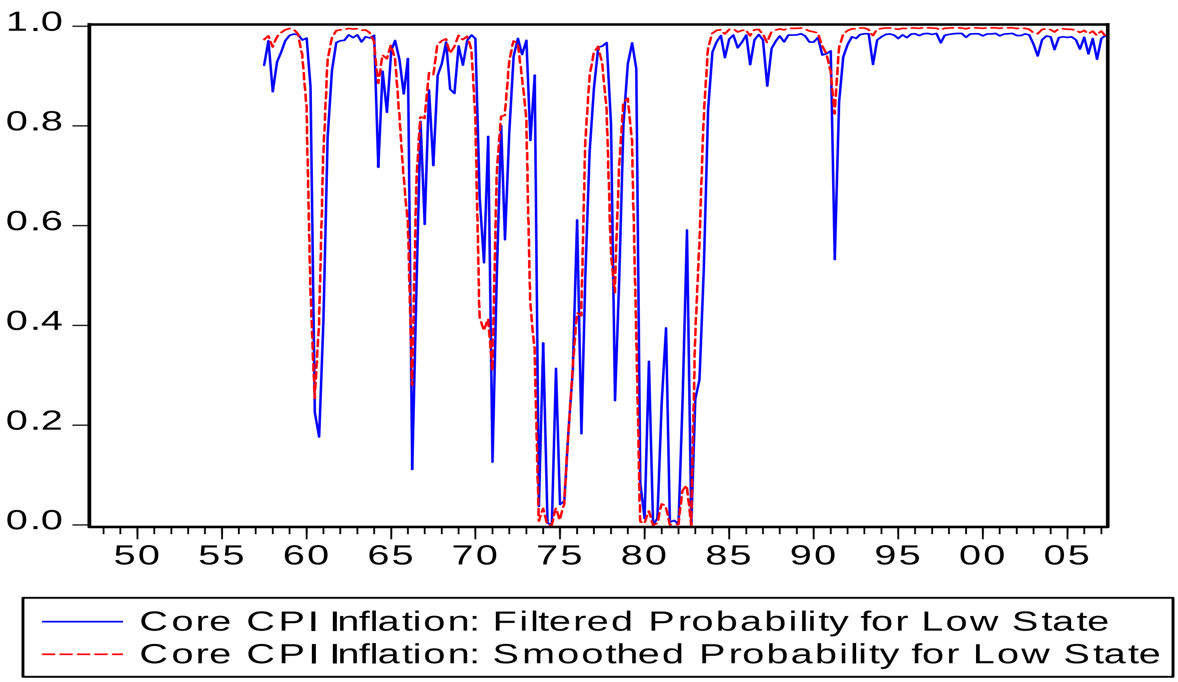


Figure 2B: Core CPI Inflation Filtered and Smoothed Probabilities for Low State



Note: A trend-cycle model as discussed in Section 3 is estimated for each inflation measure.

Figure 3A: GDP Inflation, Filtered Trend and Smoothed Trend

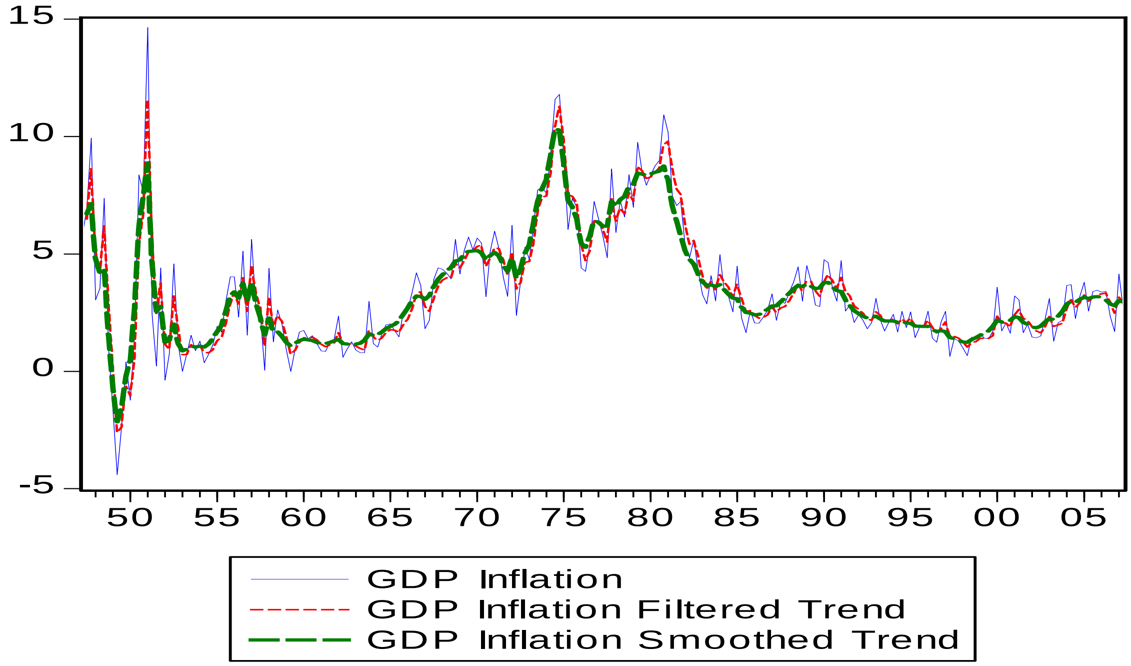
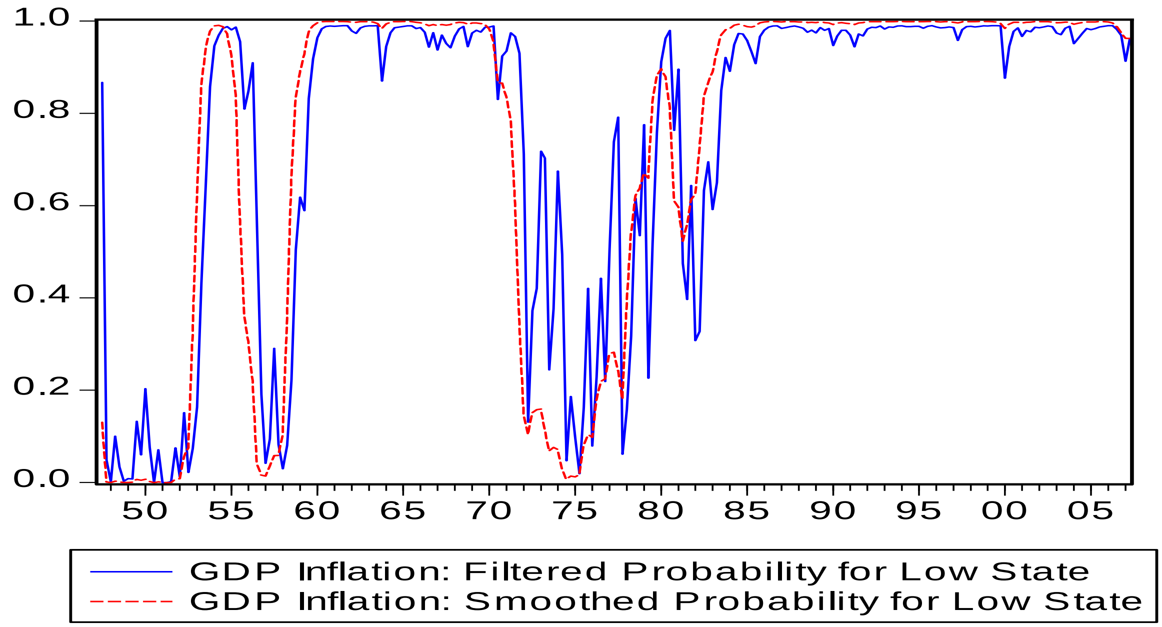


Figure 3B: GDP Inflation Filtered and Smoothed Probabilities for Low State



Note: A trend-cycle model as discussed in Section 3 is estimated for each inflation measure.

Figure 4A: PCE Inflation, Filtered Trend and Smoothed Trend

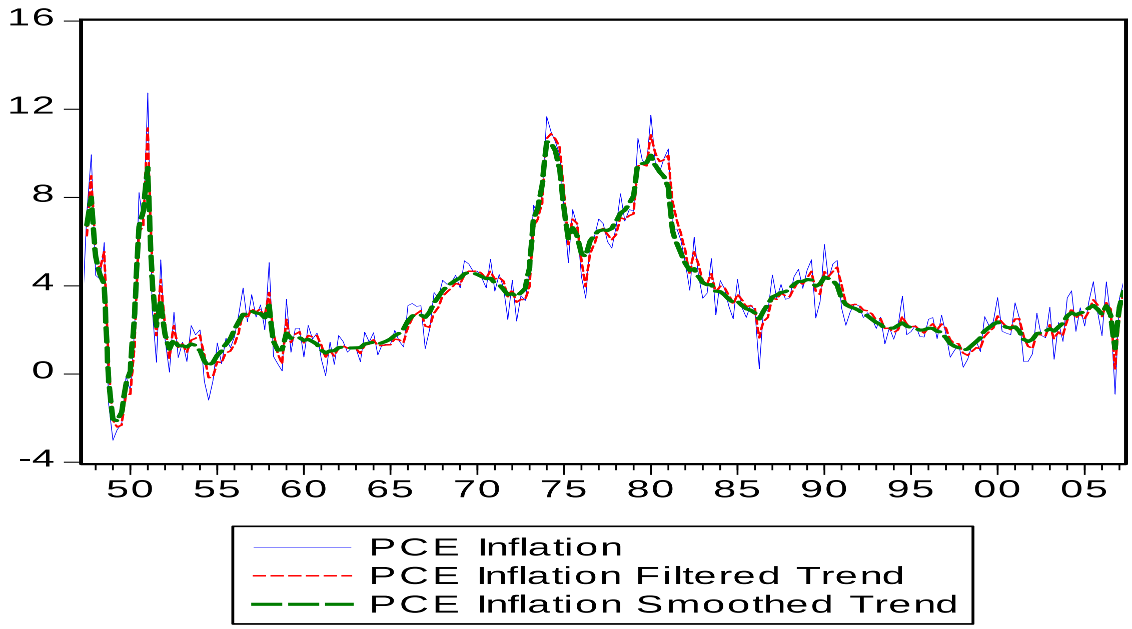
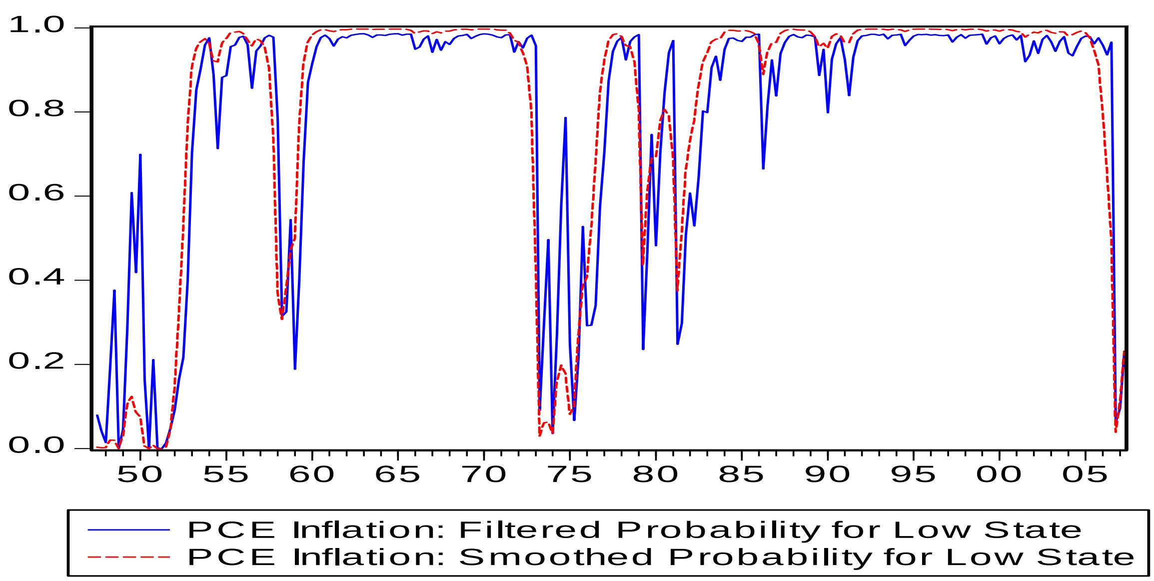


Figure 4B: PCE Inflation Filtered and Smoothed Probabilities for Low State



Note: A trend-cycle model as discussed in Section 3 is estimated for each inflation measure.

Figure 5A: The Real Term Structure Implied by CPI Inflation

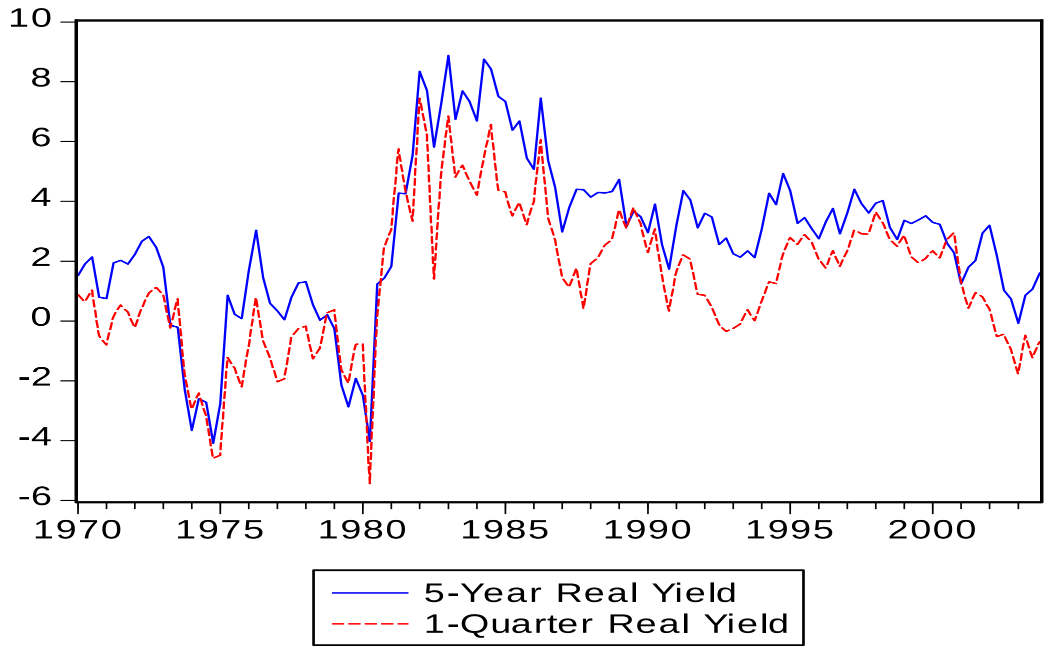
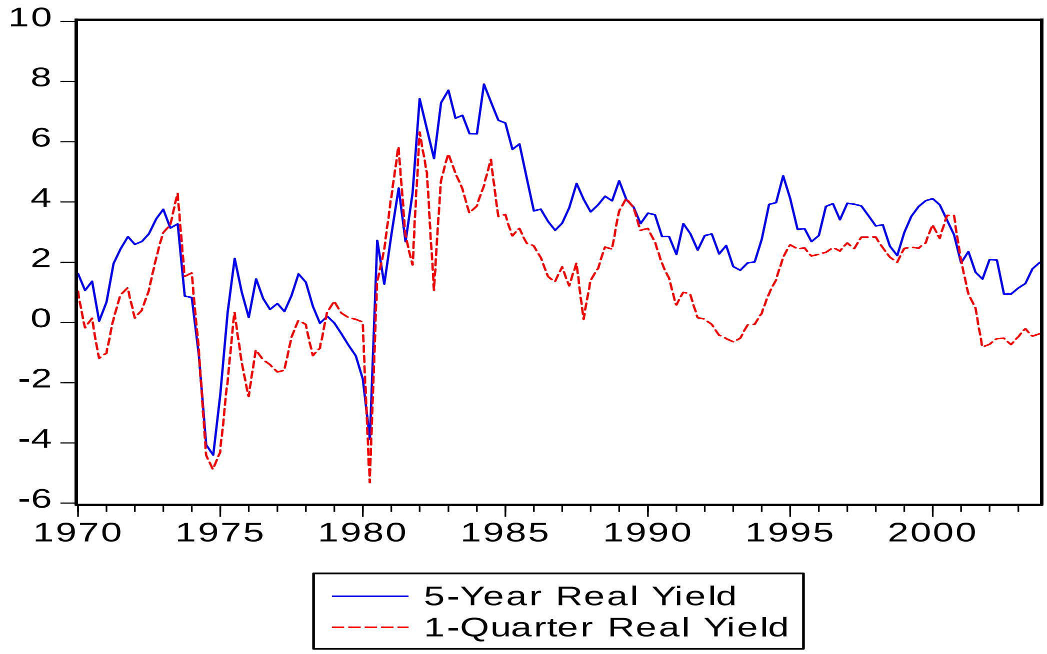


Figure 5B: The Real Term Structure Implied by Core CPI Inflation



Note: The real yield for each maturity *m* is obtained by subtracting the inflation forecast of horizon *m* from the *m*-period nominal yield. See Section 3 for details.

Figure 5C: The Real Term Structure Implied by GDP Inflation

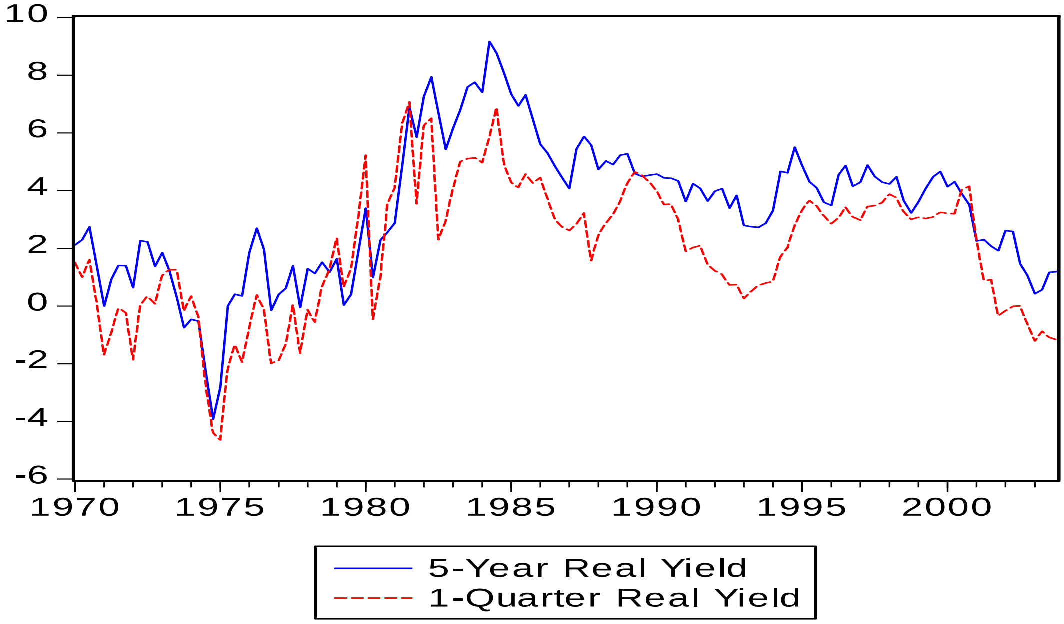
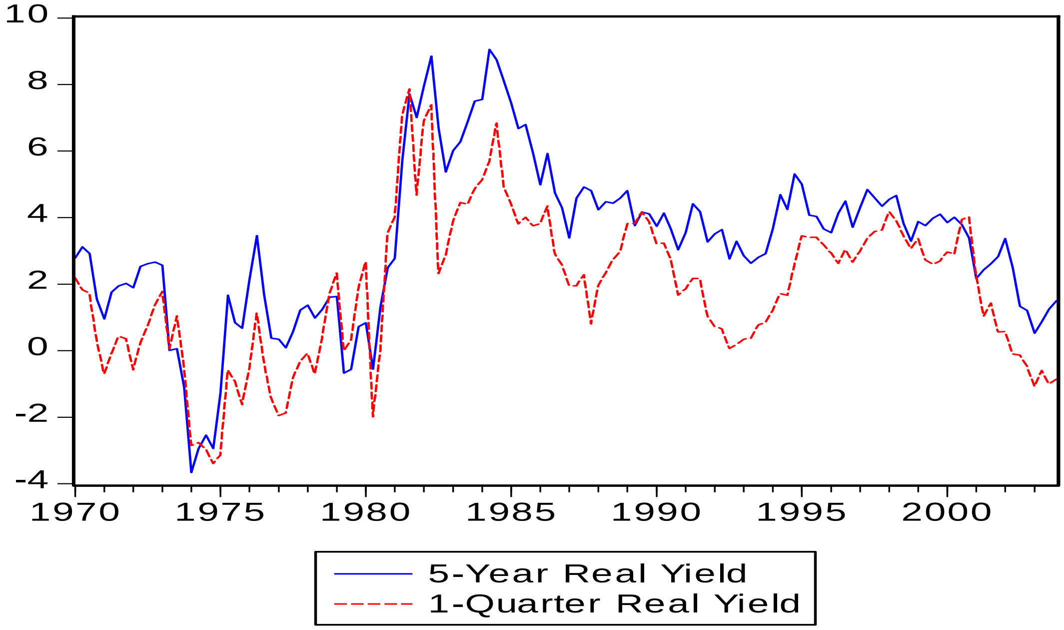
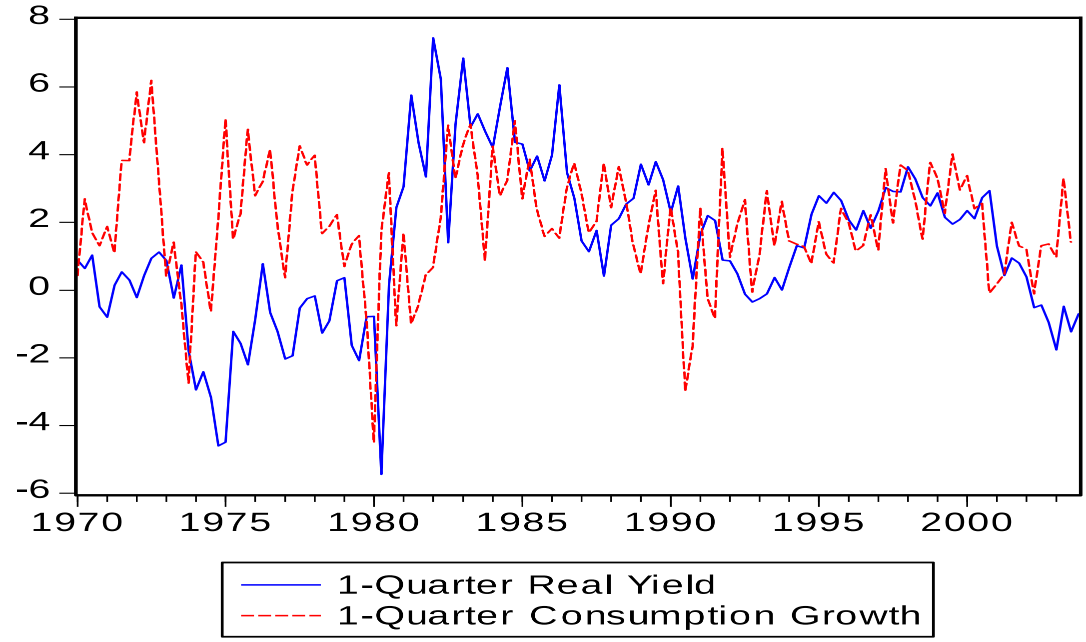


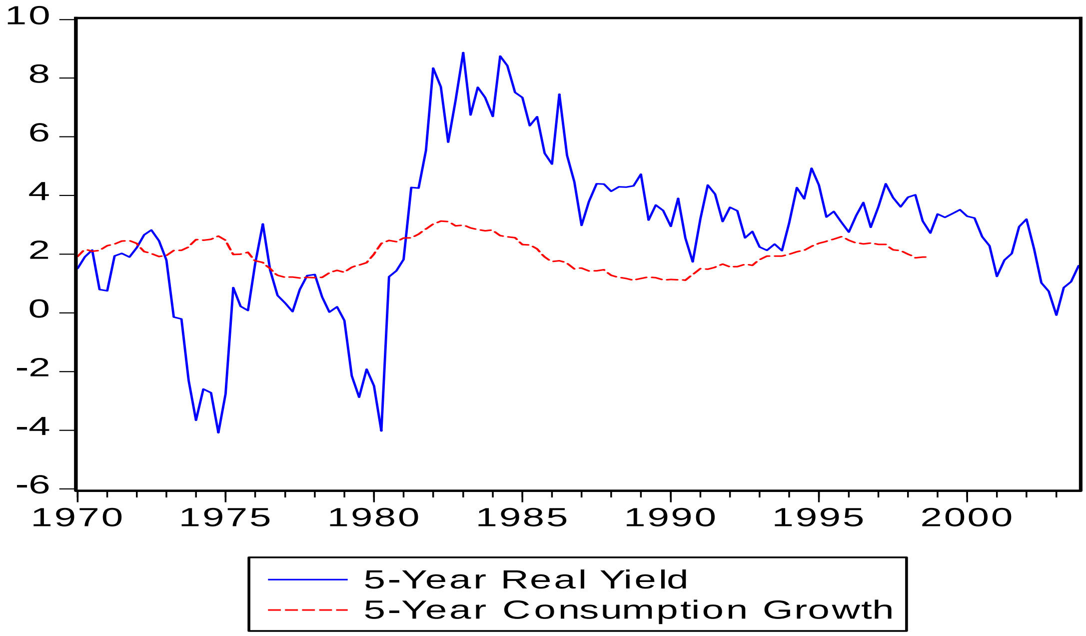
Figure 5D: The Real Term Structure Implied by PCE Inflation



Note: The real yield for each maturity *m* is obtained by subtracting the inflation forecast of horizon *m* from the *m*-period nominal yield. See Section 3 for details.

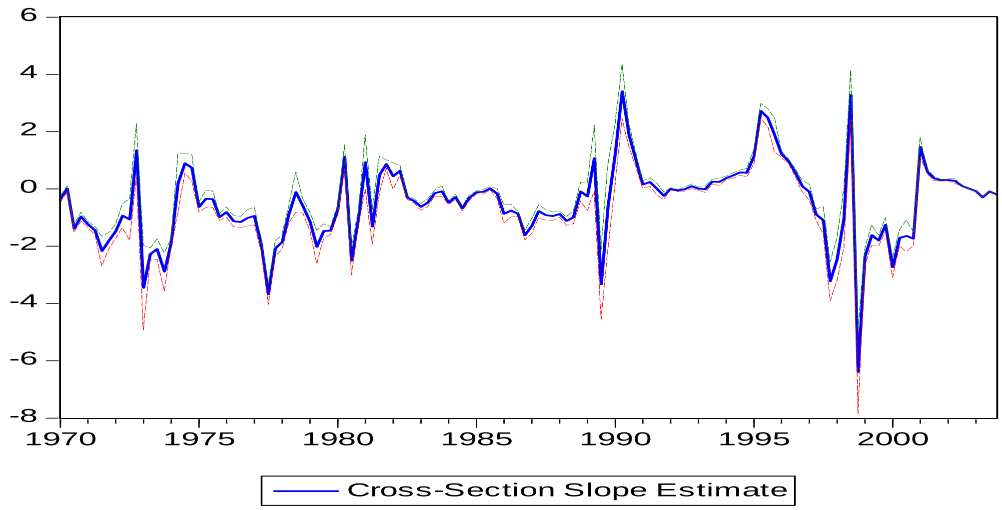
Figure 6: Consumption Growth and Real Yield (CPI) for 3-Month and 5-Year Horizons





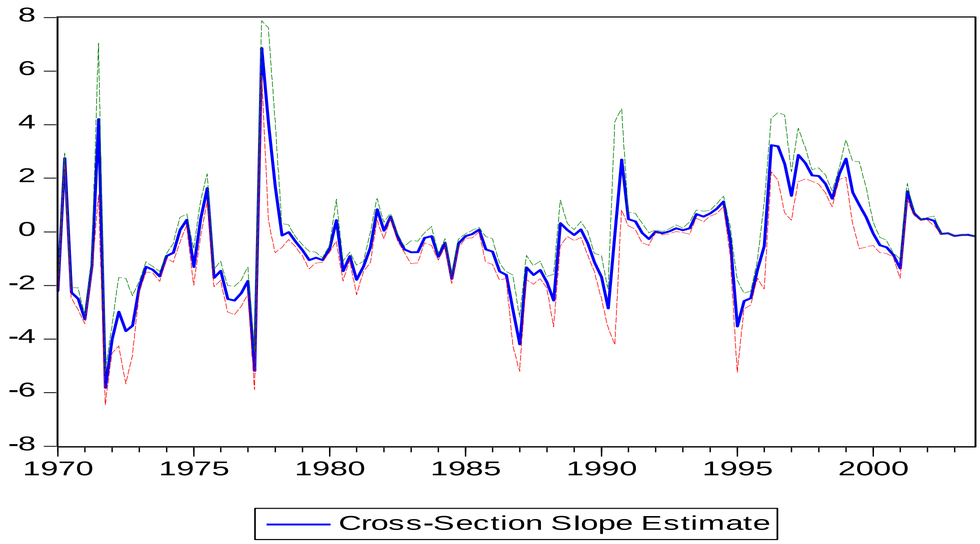
Note: Real yield of maturity m at quarter *t* is matched by consumption growth from quarter t to quarter *t+m*.

Figure 7: Cross-section Estimate for (5) with CPI Inflation



Note: The dashed lines are the 95% confidence intervals. See Section 5 for details of the cross-section regression.

Figure 8: Cross-section Estimate for (5) with Demeaned Data and CPI Inflation



Note: The dashed lines are the 95% confidence intervals. See Section 5 for details of the cross-section regression.

APPENDIX TO SECTION 2

A.1 The Ordinal Certainty Equivalence (OCE) Case

The OCE model proposed by Seldon (1978) avoids the inverse relationship between the EIS and risk aversion parameter. Now the problem becomes:

$$\max_{\{C^*_{t+\tau}\}_{\tau=0}^{\infty}} \sum_{\tau=0}^{\infty} \beta^{\tau} E_t \left[U\left(C^*_{t+\tau}\right)\right]$$

where

(6) $$V\left(C^*_{t+\tau}\right) = E_t\left[V\left(C_{t+\tau}\right)\right]$$

Now the argument that enters the utility function directly is the "certain equivalent" consumption $C^*_{t+\tau}$. The "certain equivalent" is defined by (6). The curvature of the function U controls the level of intertemporal substitution and the curvature of the function V controls the level of risk aversion (See Attanasio and Weber (1989) for details). Define the inverse function $G = V^{-1}$, I write down the Euler equation for this problem as:

$$\beta^m U'\left(G'\left(E_t\left[V'(C_{t+m})\left(1+r_t^{(m)}\right)^m\right]\right)\right) = U'\left(G'\left(V'(C_t)\right)\right)$$

Following the convention, I use the CRRA functional forms for both functions: $V(C) = \dfrac{C^{1-\theta}-1}{1-\theta}$ and $U\left(C^*\right) = \dfrac{C^{*1-\gamma}-1}{1-\gamma}$. The above Euler equation can be rewritten as (using the facts that $G' = V'^{-1}$ and $C_t = C_t^*$):

$$\beta^m C^{*-\gamma}_{t+m} C^{*\theta}_{t+m} E_t\left[C^{-\theta}_{t+m}\left(1+r_t^{(m)}\right)^m\right] = C_t^{-\gamma}$$

Writing in only observable consumption:

$$\beta^m E_t\left[\left(\frac{C_{t+m}}{C_t}\right)^{-\theta}\left(1+r_t^{(m)}\right)^m\right] E_t\left[\left(\frac{C_{t+m}}{C_t}\right)^{1-\theta}\right]^{(\theta-\gamma)/(1-\theta)} = 1$$

Of course, if $\theta = \gamma$ I get back the same Euler equation as in the CRRA case in Section 2. Again, when both the terms in the expectation are log-normally distributed, I have the similar linear approximation:

$$m\ln\beta-\theta\left(E_t c_{t+m}-c_t\right)+mE_t r_t^{(m)}+\frac{1}{2}\mathrm{var}_t\left(\ln\left(\left(\frac{C_{t+m}}{C_t}\right)^{-\theta}\left(1+r_t^{(m)}\right)^m\right)\right)$$

$$+\frac{\theta-\gamma}{1-\theta}\left[\left(1-\theta\right)\left(E_t c_{t+m}-c_t\right)+\frac{1}{2}\mathrm{var}_t\left(\ln\left(\left(\frac{C_{t+m}}{C_t}\right)^{1-\theta}\right)\right)\right]=0$$

->

$$m\ln\beta-\gamma\left(E_t c_{t+m}-c_t\right)+mE_t r_t^{(m)}+\frac{1}{2}\mathrm{var}_t\left(-\theta\left(c_{t+m}-c_t\right)+mr_t^{(m)}\right)$$

$$+\frac{\theta-\gamma}{1-\theta}\left[\frac{1}{2}\mathrm{var}_t\left(\left(1-\theta\right)\left(c_{t+m}-c_t\right)\right)\right]=0$$

->

$$\gamma\left(E_t c_{t+m}-c_t\right)$$

$$=m\ln\beta+mE_t r_t^{(m)}+\frac{m^2}{2}\mathrm{var}_t\left(r_t^{(m)}\right)-\theta m\,\mathrm{cov}\left(c_{t+m},r_t^{(m)}\right)+\frac{1}{2}\left(\theta-\gamma+\theta\gamma\right)\mathrm{var}_t\left(c_{t+m}\right)$$

->

$$\frac{1}{m}\left(E_t c_{t+m}-c_t\right)$$

$$=\frac{1}{\gamma}\ln\beta+\frac{1}{\gamma}E_t r_t^{(m)}+\frac{m}{\gamma 2}\mathrm{var}_t\left(r_t^{(m)}\right)-\frac{\theta}{\gamma}\mathrm{cov}\left(c_{t+m},r_t^{(m)}\right)+\frac{1}{2\gamma m}\left(\theta-\gamma+\theta\gamma\right)\mathrm{var}_t\left(c_{t+m}\right)$$

-> $$\frac{1}{m}\left(c_{t+m}-c_t\right)=\alpha^{(m)}+\frac{1}{\gamma}E_t r_t^{(m)}+\varepsilon_{t+m}$$

Now $\alpha^{(m)}$ is a function of γ, θ and the second moments, and the coefficient on the ex ante real return is again the elasticity of intertemporal substitution. The only difference between the OCE model and the CRRA model is the different interpretation of the intercept term.

A.2 The Epstein-Zin (1989) Recursive Utility Case

Notice that the above results are not sensitive to the choice of a utility function. For example, if we have the Epstein-Zin (1989) utility function (an extension of the Kreps and Porteus (1978) model):

$$U_t=\left[C_t^{1-\gamma}+\beta E_t\left(U_{t+1}^{1-\theta}\right)^{(1-\gamma)/(1-\theta)}\right]^{1/(1-\gamma)}$$

Now the parameter γ is the reciprocal of the EIS, and the parameter θ is the coefficient of relative risk aversion. Epstein and Zin (1989) show that two Euler equations can be derived:

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\frac{(1-\theta)\gamma}{(\gamma-1)}}\left(1+\bar{r}\right)^{\frac{\gamma-\theta}{1-\gamma}}\left(1+r_t^{(1)}\right)\right]=\beta^{\frac{(1-\theta)}{(\gamma-1)}}$$

and $$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\frac{(1-\theta)\gamma}{(\gamma-1)}}\left(1+\bar{r}\right)^{\frac{1-\theta}{1-\gamma}}\right]=\beta^{\frac{(1-\theta)}{(1-\gamma)}}$$

The term $\bar{r}$ stands for the unobservable return to aggregate wealth (which includes human capital). The first Euler equation can be rewritten in the multi-period version as:

$$E_t\left[\left(\frac{C_{t+m}}{C_t}\right)^{\frac{(1-\theta)\gamma}{(\gamma-1)}}\left(1+\bar{r}\right)^{\frac{m(\gamma-\theta)}{1-\gamma}}\left(1+r_t^{(m)}\right)^m\right]=\beta^{\frac{m(1-\theta)}{(\gamma-1)}}$$

Assuming consumption growth, return to aggregate wealth, and real return are jointly lognormally distributed, Attanasio and Weber (1989) show that the Euler equation can be written as:

$$\frac{1}{m}\left(c_{t+m}-c_t\right)=\alpha^{(m)}+\frac{1}{\gamma}E_t r_t^{(m)}+\varepsilon_{t+m}$$

where

$$\alpha^{(m)}=\frac{\ln\beta}{\gamma}+\frac{1}{m\gamma}\frac{1}{2}\left[\left(\frac{\gamma-\theta}{1-\gamma}\right)\mathrm{var}_t\left(1+\bar{r}\right)-\mathrm{var}_t\left(r_t^{(m)}\right)-\gamma^2\left(\frac{1-\theta}{1-\gamma}\right)^2\mathrm{var}_t\left(c_{t+m}\right)\right.$$
$$\left.-\gamma\left(\frac{1-\theta}{1-\gamma}\right)\mathrm{cov}\left(r^{(m)},c_{t+m}\right)+\frac{\gamma-\theta}{1-\gamma}\mathrm{cov}_t\left(r_t^{(m)},1+\bar{r}\right)\right]$$

and $$\varepsilon_{t+m}=\frac{1}{m}\left(c_{t+m}-E_t c_{t+m}\right)$$

Using the Epstein-Zin utility function only leads to a different interpretation of the coefficients: 1) the intercept term is a more complicated function of the discount rate and conditional variance and covariance terms, and 2) the effect of the expected real return on consumption growth is controlled by the EIS $1/\gamma$, and its reciprocal does not reflect the degree of risk aversion. The linear relationship between consumption growth and real yield still holds, and results (4) and (5) are still valid.