Because of delays to input payments, production linkages coincide with trade credit linkages. This paper investigates how these interconnected production and financial linkages of firms propagate shocks upstream and downstream in an input-output model. Theoretically, I obtain that financial frictions in trade and financial linkages of firms are necessary features for generating strong upstream propagation of financial shocks. Meanwhile, four types of firm level heterogeneities play important roles in firm’s sensitivity to shocks: 1) the positions in the production network, 2) the levels of liquidity constraints, 3) the degrees of mismatching maturities, in terms of accounts-payables and accounts-receivables, and 4) the forms of production functions. Empirically, I observe that upstream propagation of financial shocks is stronger than downstream propagation in the U.S. data, which conforms to my theoretical prediction. Finally, I use the model to compare credit policies that target liquidity to different sectors. I show that after an aggregate liquidity shock credit policies have a stronger impact by supplying liquidity to downstream sectors.

1 Introduction

An economy is an entangled network of specialized productions with a substantial amount of inter-firm trade within and across sectors. In a trade, the product of one firm can be purchased and consumed by another firm as inputs. Also, in many cases there is a waiting period between the point when a cost is incurred and when cash flow materializes. Therefore, the production network also leads to a trade credit network. An idiosyncratic shock could spread out through the production and financial linkages of firms, which may lead to aggregate fluctuations. In order to explore this phenomenon, the following questions must
be answered. *How does the interaction of the production network and trade credit network affect the propagation of shocks? What forms of firm heterogeneity are the most powerful driver of the difference in sector sensitivity to shocks? How can credit policy target liquidities of different sectors?*

Following the collapse of Lehman Brothers Holdings Inc., there was a significant amount of output contraction. One widely accepted explanation is that the Wall Street meltdown affects the economic outcomes on Main Street through the credit channel. As an illustration, Figure 1 plots the credit spread index and the excess bond premium index of non-financial firms constructed by Gilchrist and Zakrajsek (2010). An increase of the credit spread reflects the raise of borrowing cost, while an increase of the excess bond premium reflects a reduction in the risk appetite of the financial sector and a contraction in the supply of credit. In Figure 1 both series experienced a huge spike by the end of 2008, which means that the economy had a severe credit contraction. There has been extensive research on why a credit crunch inflicts such strong negative impact on aggregate real economic activities (e.g., Cur-
However, sectoral output falls very differently after the shock (as illustrated in Figure 2). The mechanism underlying this phenomenon is not well understood. The nature of the financial frictions in conjunction with the input-output structure in the economy can have important implications about which sector are most affected by certain types of shocks. Meanwhile, a branch of the network research emphasizes the strong economic impact of the idiosyncratic supply or demand shocks through the input-output transmission (e.g., Gabaix (2011), Carvalho (2008), Acemoglu et al. (2012, 2015), Baqee (2015)). However, the propagation of idiosyncratic financial shocks hasn’t been thoroughly examined. Furthermore, during extreme economic conditions, the central bank target lending to help specific sectors or institutions. The implications of credit policies by targeting liquidity to specific sectors cannot be well studied in a representative firm model, while it can be investigated through a network economy with financial frictions.

As an effort towards filling this gap, this paper built an input-output model with financial linkages of firms to study the propagation and the impact of financial shocks, as well as the implications of credit policy.

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1 For example, the Fed granted the Continental Illinois National Bank and Trust Company access to the discount window from May 1984 through February 1985 despite the company’s effective insolvency, and indirectly channeled credit to commercial paper market to relieve the financial stress of the Penn Central Transportation Co. in 1970 (Price 2012). Likewise, the Fed lending to private entities and other central banks reached $1.5 trillion by the end of 2008 after the Lehman bankruptcy in September 2008.
In this paper, I assume that firms are inter-connected through a production and a trade credit network (as in Long and Plosser (1983), Kiyotaki and Moore (1997)). To capture the financial friction in production, I assume that firms have a working capital requirement on inputs, and they satisfy this requirement through bank lending. To obtain endogenous credit spread and borrowing rate, I introduce financial intermediaries into the model. Banks lend funds obtained from households to firms, given an incentive constraint (as in Gertler and Karadi (2011)). In my model, firms are financially interlocked through trade. On the one hand, suppliers can use payments from customers as working capital. On the other hand, the balance sheets of trading parties are interlocked through accounts-receivable and accounts-payable. In particular, this paper emphasizes the role of financial linkages of firms in the propagation of financial shocks.

I compare three types of network economies with different degrees of financial friction in trade, namely (1) a deferred-payment (DP) economy, (2) a cash-in-advance (CIA) economy, and (3) a trade-credit (TC) economy. First, producers in the DP economy have a working capital requirement on labor and capital inputs. While labor and capital costs have to be borrowed and paid before production, intermediate inputs are paid after production has been completed. As a result, there is no financial friction in trade. Second, producers in the CIA-network economy have a working capital requirement on all production inputs. There is financial friction in trade and financial linkages of firms. Finally, producers in the TC-network economy have a working capital requirement on all production inputs, but they could defer a proportion of their intermediate input payments using trade credit.

Although we know that production processes are sequential, the direction of shock propagation remains to be explored. Specifically, do they propagate upstream or downstream? As illustrated in Figure 3, I obtain in my model that in all three types of economies, supply-side shocks create powerful downstream propagation, because they lead to significant changes of the prices of intermediate inputs. In comparison, demand-side shocks generate strong upstream propagation, as affected industries adjust their production levels and thus input demands. *Remarkably, financial shocks propagate differently in the three types of models.*
the DP-network model, financial shocks mainly generate downstream propagation. Because of delays to intermediate input payments, intermediate input demand is less sensitive to interest rate shocks than labor and capital demands. A firm substitutes labor or capital with intermediate inputs after a positive interest rate shock. This substitution effect mitigates the shock impact on intermediate input demand and alleviates the upstream propagation power of financial shocks. In the CIA-network model, the same type of shock mainly generates upstream propagation. Products are sold to downstream firms who make early payments and to households who make late payments. Because producers value early payments, there is a price discrimination based on the payment schedule. When the interest rate is high, downstream firms receive a bigger price discount. Thus, the downstream propagation of financial shocks is weak. Finally, a TC-network model is a hybrid of the other two models. In my simulation, upstream propagation is stronger than downstream when there is a moderate amount of trade credit.
Empirically, the U.S. data in which I investigated (see Section 4 for details) suggests that *upstream propagation of financial shocks is stronger than downstream propagation*. I use changes of sectoral bond yields as an instrument for idiosyncratic financial shocks. I obtain that a sector is more sensitive to financial shocks transmitted from its customers than to shocks transmitted from its suppliers. Moreover, I use changes of the credit spread index and the excess bond premium index as instruments for aggregate financial shocks. My results show that upstream sectors are more sensitive to aggregate financial shocks than downstream sectors. Therefore, the data supports the type of models with financial friction in trade and financial linkages of firms, such as the CIA- and TC-network models.

To achieve quantitative predictions, I calibrate the model using the U.S. data at the 2-digit NAICS level. The simulation result indicates that the share of intermediate inputs plays the most important role in the sectoral output sensitivity to aggregate supply shocks. Trade credit and capital share play important roles in the sector sensitivity to monetary policy shocks. Capital share and financial leverage play important roles in the sectoral output sensitivity to aggregate financial shocks. Additionally, I find that *credit policy has a stronger impact by targeting liquidity to downstream sectors, given that financial shocks mainly propagate upstream*.

**Literature review.** This project fits into three strands of literature: (1) the literature on production networks, (2) the literature on mismatching maturities in production networks, and (3) the literature on financial frictions. First, Long and Plosser (1983) start sectoral co-movements studies in the network model. Later, there is a rich literature on the aggregate volatility generated from idiosyncratic shocks (e.g., Gabaix (2011), Carvalho (2008), Acemoglu et al. (2012,2015), Baqee (2015)). Notably, Acemoglu et al. (2015) study the propagation of supply and demand shocks through input-output and geographic networks. My predictions on the propagation of those types of shocks are consistent with theirs. Nonetheless, I emphasize the transmission of financial shocks through the input-output and trade credit networks. Second, several theories have been put forth to explain why suppliers provide trade credit to customers (e.g. Petersen and Rajan (1997), Burkart and Ellioson (2004),
Cunat (2007)). My paper follows Kiyotaki and Moore (1997) in emphasizing the role of trade credit in the propagation of shocks. To my knowledge, this is the first paper which studies production and trade-credit networks in a general equilibrium framework. Finally, financial friction has been extensively studied in the literature after the 2008 financial crisis (Gertler and Karadi (2011), Curdia and Woodford (2010)). Bigio and La’O (2014), Su (2014) and this paper contribute to the network-based approaches to the amplification of financial shocks. My paper is different from Bigio and La’O (2014) by accommodating financial linkages of firms and micro-founded financial sector. Su (2014) presents a network model with financial friction in the capital input. His model is closer to the DP-network model in this paper. Nonetheless, without financial friction in trade, this type of network model can be allocationally equivalent to a horizontal one.

The structure of the paper is as follows: Section 2 presents a simple model to illustrate the propagation of shocks in a chain economy. Section 3 discusses my empirical findings on the propagation of financial shocks. Afterwards, Section 4 extends the model to a general one. Section 5 calibrates the general model and forms quantitative predictions. Section 6 discusses credit policy implications. Finally, Section 7 concludes.

2 Simple network models

In this section, I consider a set of simple three-firm models in order to study the propagation of shocks under various conditions of production network and financial frictions. My main focus is on three types of network models that differ in their payment schedules with regard to intermediate input purchases.

Before proceeding with this discussion, it will be useful to consider a horizontal economy as the benchmark. In this simple economy, three firms produce three types of specialized intermediate products. The flow of goods in this economy is illustrated in Figure 4 panel a. Under these circumstances, there are no connections among the firms, and labor is the sole
input. Intermediate sectors thus have the following production functions,

\[ m_1 = z_1 l_1^{\beta_1}, \quad m_2 = z_2 l_2^{\beta_2}, \quad m_3 = z_3 l_3^{\beta_3}, \]

where \( z_i \) denotes technology. The final output is

\[ Y^h = \Xi m_1^{\zeta_i^h} m_2^{\zeta_i^h} m_3^{\zeta_i^h}, \]  \( (1) \)

where \( \Xi \) is a constant, \( \zeta_i^h \) denotes the final share of product \( i \). Also, there is a representative household that solves the following problem,

\[ \max U(C) - V(L) = \log(C) - L, \]  \( (2) \)

\[ \text{s.t.} \quad PC + wL. \]

Households make choices of consumption \( C \) and labor supply \( L \) subject to their budget constraint. The aggregate price level is \( P \) and the wage rate is \( w \).

A *competitive equilibrium* in this horizontal economy is a group of endogenous variables \( \{m_i, l_i, L, Y, C, p_i, w\} \) that are functions of \( \{\beta_i, \zeta_i^h, z_i, \Xi\} \) for \( i \in \{1, 2, 3\} \) and a given aggregate price level \( P \), such that (1) each firm maximizes its own profits, (2) each household maximizes its utility (Equation \( 2 \)), and (3) the goods market and the labor market clear.

Three considerations are important in the discussion of the propagation of shocks. First, in this economy, the equilibrium sectoral outputs are,

\[ m_i^h = z_i (\beta_i \zeta_i^h)^{\beta_i}, \quad i \in \{1, 2, 3\}. \]

Secondly, I denote the influence factor as,

\[ v_i \equiv \frac{p_i m_i}{PY}, \]  \( (3) \)
which factor provides an intuitive measure of the significance of sectoral output on GDP. Clearly, in this simple economy, $v_i^h = \zeta_i^h$. Finally, the proportions of sectoral output used in the intermediate goods production are

$$\rho_i \equiv 1 - \frac{\zeta_i}{v_i} ,$$

where $1 - \rho_i$ is the proportion of sectoral output used in the final goods production. In the horizontal economy $\rho_i^h = 0$.

In what follows, network models are discussed within the context of a similar framework, while the above three variables play important roles in each economy. The sensitivity of sectoral output $m_i$ to shocks is the focal point. $v_i$ controls the importance of sector $i$ in aggregate output. $\rho_i$ is related to the intermediate input demand and governs the power of upstream propagation.

Figure 4: Horizontal VS Network Economy

2.1 The simple network models

Unlike in a horizontal economy, firms in a simple network are interconnected through trade. Consider a production network with three firms that are vertically linked. The input-output structure is illustrated in Figure 4 panel b. For the sake of simplicity, Firm 1 is the most upstream firm, while Firm 3 is the most downstream firm. All three firms supply input
that figures in the production of the final goods. The production functions of these firms are:

\[
m_1 = z_1 l_1, \\
m_2 = z_2^{\beta_2} m_{21}^{1-\beta_2}, \\
m_3 = z_3^{\beta_3} m_{32}^{1-\beta_3}. 
\]

The final output of the economy can thus be represented as,

\[
Y = \zeta_1 - \zeta_1 \zeta_2 - \zeta_3 - \zeta_1 y_1 - \zeta_2 y_2 - \zeta_3 y_3.
\]

Households under these conditions solve the same problem in like manner as they do in the horizontal economy (Equation 2). The goods market clearing conditions are: \( m_1 = m_{21} + y_1, \) \( m_2 = m_{32} + y_2, \) \( m_3 = y_3, \) and \( Y = C. \)

2.1.1 The standard network (SN)

In the absence of additional assumptions regarding financial friction, the standard network economy is the one presented above.

A competitive equilibrium in the context of a standard network economy describes a group of endogenous variables \( \{m_i, l_i, m_{21}, m_{32}, y_i, L, Y, C, p_i, w_i\} \) as functions of \( \{\beta_i, \zeta_i, z_i, \} \) for \( i \in \{1, 2, 3\} \) and a given aggregate price level \( P, \) such that, again, (1) each firm maximizes profits, (2) each household maximizes its utility, (3) the goods market clear.

Firms labor and intermediate input demand are,

\[
w_l = \beta_i p_i m_i \quad i \in \{1, 2, 3\},
\]

\[
p_{i-1} m_{i,i-1} = (1 - \beta_i)p_i m_i, \quad i \in \{2, 3\}.
\]

\(^2\text{With Cobb-Douglas production function, } \beta_1 = 1.\)
In equilibrium, the influence factors are,

\[ v_{1}^{sn} = \zeta_3 (1 - \beta_2)(1 - \beta_3) + \zeta_2 (1 - \beta_2) + \zeta_1, \quad v_{2}^{sn} = \zeta_3 (1 - \beta_3) + \zeta_2, \quad v_{3}^{sn} = \zeta_3. \]

The proportions of sectoral output used in the intermediate goods production are \( \rho_{i}^{sn} = 1 - \frac{\zeta_i}{v_{i}^{sn}} \). The sectoral outputs are,

\[ m_{1}^{sn} = \Upsilon_{1} z_1 v_{1}^{sn}, \]
\[ m_{2}^{sn} = \Upsilon_{2} z_2 z_1 (1 - \beta_2) v_{2}^{sn}, \]
\[ m_{3}^{sn} = \Upsilon_{3} z_3 z_2 z_1 (1 - \beta_2)(1 - \beta_3) v_{3}^{sn}, \]

where \( \Upsilon_{1} = 1, \quad \Upsilon_{2} = \beta_2 (1 - \beta_2)^{1 - \beta_2}, \quad \Upsilon_{3} = \beta_3 (1 - \beta_3)(1 - \beta_3) (1 - \beta_2)(1 - \beta_3). \)

The aggregate output is,

\[ Y^{sn} = \Theta z_{1}^{v_{1}^{sn}} z_{2}^{v_{2}^{sn}} z_{3}^{v_{3}^{sn}}, \tag{7} \]

where \( \Theta = \Upsilon_{1}^{\zeta_1} \Upsilon_{2}^{\zeta_2} \Upsilon_{3}^{\zeta_3} \).

Thus, compared with a horizontal economy, idiosyncratic shocks in a standard network spill over from one firm to another along the production chain in a network economy. Output sensitivity to shocks differs according to the network locations of these firms (as discussed in Section 2.2).

### 2.1.2 The deferred-payment network (DP)

I shall now consider three network models with financial frictions. In a deferred-payment (DP) network, a working capital requirement on labor exists in the absence of any financial friction in inter-firm trade. Firms in this case need to borrow in order to pay for their labor cost at the beginning of each production period with an exogenous interest rate \( R_i \).

The intermediate input cost is paid at the end of the production period. The competitive equilibrium in this economy is the same as that in the standard network economy, except that endogenous variables also depend on interest rates \( R_i \). The intermediate input demand
function is the same as Equation 6. The labor demand function is,

$$\beta_i p_i m_i = R_i w_i, \quad i \in \{1, 2, 3\}. \tag{8}$$

The influence factors and the share of sectoral output used in the intermediate goods production are the same as the ones used in a standard network economy,

$$v_i^{dp} = v_i^{sn}, \quad \rho_i^{dp} = \rho_i^{sn} \quad i \in \{1, 2, 3\}.$$ 

Notably, financial shocks (such as a shock to $R_i$) do not affect these factors. The sectoral outputs are,

$$m_1^{dp} = f(z_1, R_1) \tag{9}$$
$$m_2^{dp} = f(z_1, z_2, R_1, R_2) \tag{10}$$
$$m_3^{dp} = f(z_1, z_2, z_3, R_1, R_2, R_3) \tag{11}$$

Please refer to Appendix A.1 for the functional form of each $m_i$.

**Proposition 1.** A horizontal economy with working capital requirements on production inputs can be equivalent to a DP-network economy with respect to allocations. In either economy, the unique equilibrium allocation is given by,

$$Y = Y^{sn} R_1^{-v_1^{sn}} R_2^{-\beta_2 v_2^{sn}} R_3^{-\beta_3 v_3^{sn}},$$

if $\Xi = \frac{Y^{\zeta_1} Y^{\zeta_2} Y^{\zeta_3} (1-\beta_2)(1-\beta_2) v_3^{sn}}{(v_1^{sn})^\beta_1 (v_2^{sn})^{\beta_2} (v_3^{sn})^{\beta_3 v_3^{sn}}}$ and $\zeta_i^{h} = v_i^{sn}$. And, the labor allocation is, $l_i = \frac{\beta_i v_i^{sn}}{R_i}$.

Without financial friction in inter-firm trade, a DP-network economy is similar to a horizontal economy with the same type of working capital requirements.

### 2.1.3 The cash-in-advance network (CIA)

In a cash-in-advance (CIA) network economy, there is a working capital requirement on both labor and intermediate inputs. Both types of inputs have to be paid in advance of production.
Early payments received by suppliers are used to fund other input costs. However, products consumed by the final output producer \( y_i \) are paid at the end of the production period. Let \( p_{i+1,i} \) denote the intermediate input price paid by the firm \( i+1 \). Because early payments could be used as working capital by suppliers, the marginal benefit of selling one unit of the product \( i \) to a downstream firm \( i+1 \) is \( [p_{i+1,i} + p_{i+1,i}(R_i - 1)] \), where the second term comes from the interest saving. Thus, firm 2’s problem is,

\[
\begin{align*}
\max_{l_2,m_{21},m_{32},y_2} & \quad R_2 p_{32} m_{32} + p_2 y_2 - R_2 w l_2 - R_2 p_{21} m_{21} \\
\text{s.t.} & \quad z_2 l_2^{\beta_2} m_{21}^{1-\beta_2} \geq m_{32} + y_2.
\end{align*}
\]

Consequently, \( p_{32} = \frac{p_2}{R_2} \) at equilibrium. The labor demand of each firm follows Equation 8. Intermediate input demand can be expressed as,

\[
(1 - \beta_i) p_i m_i = R_i p_{i,i-1} m_{i,i-1}, \quad i \in \{2, 3\},
\]

The same definition used for the DP-network economy applies to the competitive equilibrium. The influence factors thus become,

\[
\begin{align*}
v_{1}^{cia} &= \zeta_1 + \frac{1 - \beta_2}{R_2} \left[ \zeta_2 + (1 - \beta_3) \zeta_3 \frac{R_2}{R_3} \right] \\
v_{2}^{cia} &= \zeta_2 + \frac{(1 - \beta_3) \zeta_3}{R_3} \\
v_{3}^{cia} &= \zeta_3,
\end{align*}
\]

where \( v_i = \frac{p_{i+1,i} m_{i+1,i} + p_i y_i}{p_y} \). Financial shocks to downstream firms impact the influence factor of upstream firms. Consequently, financial friction reduces the relative significance of upstream firms for total aggregate output. Moreover, \( \rho_i^{cia} = 1 - \frac{v_i}{v_{cia}} \). Financial friction increases the share of upstream output used in the final goods production \( (1 - \rho_i) \). The
sectoral outputs are,

\[ m_{cia}^1 = f(z_1, R_1, R_2, R_3) \] (12)
\[ m_{cia}^2 = f(z_1, z_2, R_2, R_3) \] (13)
\[ m_{cia}^3 = f(z_1, z_2, z_3, R_3) \] (14)

Functional forms are presented in Appendix A.1. The aggregate output is,

\[ Y_{cia} = Y_{sn}R_1^{-\xi_1}R_2^{-\xi_2}R_3^{-\xi_3} \] (15)

Compared with the DP-network model, financial friction in the CIA model generates a strong negative impact on the aggregate output. Financial shocks are amplified through the network structure and the friction in trade. Thus, a horizontal economy could not be allocationally equivalent to a network economy with financial friction in trade.

2.1.4 The trade-credit network (TC)

Trade credit is a short-term loan that a supplier provides to a customer upon purchase of its product, and that thus constitutes a form of deferred payments. In a TC-network economy, firms pay a proportion \((1 - \theta_i)\) of their intermediate input purchases at the beginning of the production period and pay the balance at the end. Therefore, each firm \(i\) has accounts payable that equal \(\theta_i\) of its total intermediate input costs, and has accounts receivable that equal \(\theta_j\) of their sales to firm \(j\). When \(\theta_i = 0\), the CIA-network model is operative; when \(\theta_i = 1\), a DP-network model is operative. Similar to the case with the CIA-network model, early payments received by suppliers can be used to pay their input costs. Thus, firms are interconnected through both the production network and the trade credit network. The financial linkages of these three firms are presented in Table 1. Obviously, upstream firms provide additional credit to downstream firms. Also, upstream firms have larger working capital than downstream firms.
Table 1: Financial linkages of firms in TC-network

<table>
<thead>
<tr>
<th></th>
<th>Account Payables</th>
<th>Account Receivables</th>
<th>Net working capital from trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>firm 1</td>
<td>0</td>
<td>$\theta_2 p_1 m_{21}$</td>
<td>$(1 - \theta_2)p_1 m_{21}$</td>
</tr>
<tr>
<td>firm 2</td>
<td>$\theta_2 p_1 m_{21}$</td>
<td>$\theta_3 p_2 m_{32}$</td>
<td>$-(1 - \theta_2)p_1 m_{21} + (1 - \theta_3)p_2 m_{32}$</td>
</tr>
<tr>
<td>firm 3</td>
<td>$\theta_3 p_2 m_{32}$</td>
<td>0</td>
<td>$-(1 - \theta_3)p_2 m_{32}$</td>
</tr>
</tbody>
</table>

Thus, firm 2 solves the following problem,

$$
\max_{l_2,m_{32},m_{23},y_2} \left[ (1 - \theta_3)R_2 + \theta_3 p_{32} m_{32} + p_2 y_2 - w_l R_2 - ((1 - \theta_2)R_2 + \theta_2) p_{21} m_{21} \right]
$$

s.t. $z_2 l_2^\beta_2 m_{21}^{1-\beta_2} \geq m_{32} + y_2$

The first term represents the benefit of selling products to firm 3. The marginal benefit of selling one unit of $m_{32}$ is $[p_{32} + (R_2 - 1)(1 - \theta_3)p_{32}]$, where the second term represents the benefit on interest cost saving. Through first order conditions, I find that

$$p_{32} = p_2 / [\theta_3 + (1 - \theta_3)R_2].$$

Close reality, price and trade credit are bundled together in the contract. The price is low when $\theta_3$ is small.

The definition of the competitive equilibrium in the TC-network economy is the same as the one in the standard network economy, except that endogenous variables also depend on interest rates $R_i$ and the trade credit ratio $\theta_i$. Labor demand follows Equation 8. The demand for intermediate input is,

$$[(1 - \theta_2)R_2 + \theta_2]p_{21} m_{21} = (1 - \beta_2)p_2 m_2$$

(17)
At equilibrium,

\[ v_{1c}^t = \zeta_1 + \frac{1 - \beta_2}{\theta_2 + (1 - \theta_2)R_2} \left[ \zeta_2 + (1 - \beta_3)\zeta_3 \frac{[\theta_3 + (1 - \theta_3)R_2]}{[\theta_3 + (1 - \theta_3)R_3]} \right] \]
\[ v_{2c}^t = \zeta_2 + \frac{(1 - \beta_3)\zeta_3}{[\theta_3 + (1 - \theta_3)R_3]} \]
\[ v_{3c}^t = \zeta_3 \]

The equilibrium level of production is,

\[ m_{1c}^t = f(z_1, R_1, R_2, R_3) \] (18)
\[ m_{1c}^t = f(z_1, z_2, R_1, R_2, R_3) \] (19)
\[ m_{1c}^t = f(z_1, z_2, z_3, R_1, R_2, R_3) \] (20)
\[ (21) \]

The functional forms are presented in Appendix A.1. The aggregate output is,

\[ Y^{tc} = Y^{sn} R_1^{-v_{1c}^n} R_2^{-\beta_2 v_2^n} R_3^{-\beta_3 v_3^n} \left( \frac{[\theta_2 + (1 - \theta_2)R_1]}{[\theta_2 + (1 - \theta_2)R_2]} \right)^{v_{1c}^n - \zeta_1} \left( \frac{[\theta_3 + (1 - \theta_3)R_2]}{[\theta_3 + (1 - \theta_3)R_3]} \right)^{v_{2c}^n - \zeta_2} \] (22)

Financial linkages among firms play important roles in the sensitivity of these firms to shocks. On the one hand, the working capital of upstream firms is more sensitive to economic fluctuations, which is consistent with the finding by Kalemli-Ozcan et al. (2012). In both the CIA and TC-network models, a shock to a downstream firm would impact the early payments to its upstream firms, which would in turn cause fluctuations in the working capital of these upstream firms. Working capital increases relatively more for upstream firms during the boom and declines relatively more for upstream firms during the recessions. On the other hand, trade credit increases the correlation of firms. Thus Raddatz (2008) shows empirically that trade credit increases sectoral output correlations. The TC-network model demonstrates that idiosyncratic interest rate shock spills over to all firms in the economy (Equations 18-20), while the spillover is in one direction in other types of network mod-


2.2 The propagation of shocks

Having established the three types of network models, I will now discuss how supply, demand and financial shocks propagate through the economy. Specifically, given that production processes are sequential, the question needs to be asked whether shocks propagate upstream or downstream. Propagation is *upstream* if a firm-specific shock spills over to the firm’s suppliers. Propagation is *downstream* if a firm-specific shock affects the firm’s customers. In the three types of economies, a shock to firm 2 could impact its supplier, firm 1, which would be an example of upstream propagation, or could impact its customer, firm 3, which would then be an example of downstream propagation.

2.2.1 Supply shocks

I begin the analysis of supply shocks by analyzing their transmission. In particular, I want to consider technology shocks in the three types of network economies. Clearly, the technologies employed by upstream firms have significant impact on the outputs of their downstream firms in all three types of economies (Equation 9-11, 12-14, 18-20).

Proposition 2. In all three types of network models with financial frictions (DP, CIA, TC), supply shocks generate strong downstream propagation. Moreover, downstream firms with larger shares of intermediate inputs \((1 - \beta_i)\) are more affected by this downstream propagation. The elasticity of sectoral output with respect to the technology of firm 2 \((z_2)\) is,

\[
\frac{\partial m_1}{\partial z_2} \frac{z_2}{m_1} = 0, \quad \frac{\partial m_2}{\partial z_2} \frac{z_2}{m_2} = 1, \quad \frac{\partial m_3}{\partial z_2} \frac{z_2}{m_3} = (1 - \beta_3),
\]

Technology shocks mainly generate downstream propagation because they lead to significant changes in intermediate input prices. Moreover, the share of intermediate inputs in the production function \(((1 - \beta_i))\) controls the power of this downstream propagation. Intuitively, the larger the intermediate input usage of a firm, the more sensitive the firm is to shocks
that flow down the production chain.

2.2.2 Demand shocks

To explain the propagation of demand shocks in a production chain, I modify the non-production side of the model slightly, as follows: 1) the aggregate consumption of households is measured as a composite of the three specialized products: \( C = c_1 \xi_1_c_2 \xi_2 c_3 \xi_3 \); 2) a government that collects lump-sum taxes to finance its expenditures \( \{g_1, g_2, g_3\} \) is posited. The budget constraints of households thus become \( \sum_i p_i c_i + T = wL \). The budget constraint of the government becomes \( \sum_i p_i g_i = T \). The market clearing conditions become

\[
\begin{align*}
    m_1 &= m_{21} + c_1 + g_1, \\
    m_2 &= m_{32} + c_2 + g_2, \\
    m_3 &= c_3 + g_3.
\end{align*}
\]

From this starting point, the impact of demand shocks such as government expenditure shocks \( (g_i \text{ shocks}) \) to the economy can be analyzed. Solving for the equilibrium of this economy generates the following result.

**Proposition 3.** In all three types of network models (DP, CIA, TC), when government expenditure is taken into account, demand shocks generate strong upstream propagation. Moreover, firms that have a larger proportion of output purchased by the impacted firms \( (\rho_i) \) are naturally more sensitive to the demand shock. The sensitivity of firm outputs to the government expenditure shock to product 2 \( (g_2) \) is,

\[
\begin{align*}
    \frac{\partial m_1}{\partial g_2} &= f(\rho_1), \\
    \frac{\partial m_2}{\partial g_2} &= 1, \\
    \frac{\partial m_3}{\partial g_2} &= 0,
\end{align*}
\]

with \( \frac{\partial f(\rho_1)}{\partial \rho_i} > 0 \).

As discussed in Appendix A.2, consumption and price levels of the economy remain constant. The affected firms adjust their production levels, and thus their input demands, after the government expenditure shock. As a consequence, demand-side shocks mainly create upstream propagation. Moreover, the share of sectoral output used as intermediate inputs \( \rho_i \) governs the strength of this upstream propagation. Intuitively, the larger this share is,
the more sensitive the firm is to shocks that flow up the production chain.

2.2.3 Financial shocks

Although the propagation of supply and demand shocks are the same in the three types of economies, the propagation effects of financial shocks differ significantly.

**Proposition 4.** (1) Financial shocks generate strong downstream propagation in a DP-network economy. The downstream propagation power is controlled by the share of intermediate input usage in downstream production \((1 - \beta_i)\). The elasticity of sectoral output with respect to the interest rate of firm 2 \((R_2)\) is,

\[
\frac{\partial m_{dp}^{1}}{\partial R_2} \frac{R_2}{m_{dp}^{1}} = 0, \quad \frac{\partial m_{dp}^{2}}{\partial R_2} \frac{R_2}{m_{dp}^{2}} = -\beta_2, \quad \frac{\partial m_{dp}^{3}}{\partial R_2} \frac{R_2}{m_{dp}^{3}} = -\beta_2(1 - \beta_3).
\]

(2) Financial shocks generate strong upstream propagation in a CIA-network economy. The extent of upstream propagation is controlled by a function of the customer’s intermediate inputs share \((1 - \beta_{i+1})\). The elasticity of sectoral output with respect to the interest rate of firm 2 \((R_2)\) is,

\[
\frac{\partial m_{cia}^{1}}{\partial R_2} \frac{R_2}{m_{cia}^{1}} = -(1 - \beta_2)f_{cia}^{1}(R_1, R_2, R_3), \quad \frac{\partial m_{cia}^{2}}{\partial R_2} \frac{R_2}{m_{cia}^{2}} = -f_{cia}^{2}(R_2, R_3), \quad \frac{\partial m_{cia}^{3}}{\partial R_2} \frac{R_2}{m_{cia}^{3}} = 0,
\]

with \(f_{cia}^{1} > 0\) (refer to Appendix A.2).

(3) Financial shocks generate strong upstream and downstream propagation in a TC-network economy. The elasticity of sectoral output with respect to the interest rate of firm 2 \((R_2)\) is,

\[
\frac{\partial m_{tc}^{1}}{\partial R_2} \frac{R_2}{m_{tc}^{1}} = -(1 - \beta_2)f_{tc}^{1}(R_1, R_2, R_3),
\]

\[
\frac{\partial m_{tc}^{2}}{\partial R_2} \frac{R_2}{m_{tc}^{2}} = -f_{tc}^{1}(R_1, R_2, R_3),
\]

\[
\frac{\partial m_{tc}^{3}}{\partial R_2} \frac{R_2}{m_{tc}^{3}} = -(1 - \beta_3)f_{tc}^{3}(R_1, R_2, R_3),
\]

with \(f_{tc}^{1} > 0\) (refer to Appendix A.2). If firm 2 receives more trade credit from firm 1 \((\theta_2)\),
then both upstream and downstream propagation effects are weaker. Also, firm 2 is less sensitive to the interest rate $R_2$ shock, i.e.

$$\partial f_{tc}^1 / \partial \theta_2 < 0, \quad \partial f_{tc}^2 / \partial \theta_2 < 0, \quad \partial f_{tc}^3 / \partial \theta_2 < 0.$$ 

If firm 2 provides more trade credit to firm 3 ($\theta_3$), then both upstream and downstream propagation effects are stronger. Also, firm 2 is more sensitive to the interest rate $R_2$ shock, i.e.

$$\partial f_{tc}^1 / \partial \theta_3 > 0, \quad \partial f_{tc}^2 / \partial \theta_3 > 0, \quad \partial f_{tc}^3 / \partial \theta_3 > 0.$$ 

Financial shocks generate downstream propagation when there is no financial friction in inter-firm trade (in the DP-network). Product price increases with the interest cost, which increase in turn generates downstream propagation. Meanwhile, two effects are affecting intermediate input demand, which is in turn related to the upstream propagation. On the one hand, the income effect decreases intermediate input demand, since production falls on account of the financial distress. On the other hand, the substitution effect increases the intermediate input demand, since intermediate input price is less affected by the interest rate than by labor input. Consequently, the upstream propagation effect is weaker than the downstream effect. Particularly, under the Cobb-Douglas production rubric, there is no upstream propagation because the substitution and the income effect cancel out each other.

Financial shocks nevertheless do propagate upstream in a CIA-network economy. Owing to the working capital requirement, an increase in the borrowing cost reduces intermediate input demand. In this case, a firm will not substitute labor with intermediate inputs because both are affected by the interest rate shock. Nonetheless, the distressed firm gives a larger discount to a downstream firm that pays in advance (Equation 16). Consequently, the downstream propagation effect is weaker. Particularly, again under the Cobb-Douglas set up, the downstream effect disappears.

TC-network is a mixture of early and late payments of intermediate input purchases. A
negative financial shock increases the interest rate and the production price, which increases in turn create supply impact and downstream propagation. In addition, an increase in the input cost reduces intermediate input demand and creates upstream propagation. The relative strength of upstream versus downstream effects depends on parameter values. With a moderate amount of trade credit, the upstream propagation effect is stronger than the downstream effect. Moreover, trade credit represents external loans that firms receive from their suppliers, and this form of credit relaxes the financial constraints on the firm. On the one hand, accounts-payable alleviates the financial constraint of the firm and weakens the transmission of shocks. On the other hand, accounts-receivable amplifies the financial stress of the firm and strengthens the transmission of shocks.

2.3 What do we learn from the simple model?

This section focuses on three types of network models with financial constraint. Remarkably, financial constraint generates a strongly negative impact on aggregate output. Moreover, depending on the payment schedule of the intermediate inputs, the propagation of financial shocks could be either upstream or downstream. Effects propagate downstream in the DP-network, upstream in the CIA-network and in both directions in the TC-network. In particular, I want to call attention to the fact that financial frictions on trade and financial linkages of firms are necessary preconditions for generating strong upstream propagation effects of financial shocks. On the contrary, the propagation effects of supply and demand shocks are consistent in all three economies, whereas supply shocks propagate downstream and demand shocks propagate upstream.

While my prediction of the propagation of supply and demand shocks is consistent with the empirical findings in the existing literature, the question remains regarding which propagation pattern of financial shocks is actually revealed in the data, so I will take up this question in the next section.
3 Empirical Findings

In what follows, I present my primary empirical findings on the propagation of financial shocks and on the measure of industry linkages and of downstreamness of subsectors.

3.1 Upstream versus downstream

A theoretical model could offer a precise prediction for the form of the propagation of shocks. Nonetheless, the direction taken by shocks would remain unclear in the data, because firm $i$ could be both a supplier and a customer of firm $j$. The concepts of upstreamness and downstreamness therefore need to be defined within the context of a more general network.

To begin with, $\omega_{ij}$ denotes the amount of $j$ used as an input in producing $1$ worth of $i$ output,

$$\omega_{ij} = \frac{p_{ij}m_{ij}}{p_im_i}$$

Thus, the direct requirements matrix $\Omega$ denotes the matrix of $\omega_{ij}$’s,

$$\Omega = \begin{pmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1n} \\ \omega_{21} & \omega_{22} & \cdots & \omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{n1} & \omega_{n2} & \cdots & \omega_{nn} \end{pmatrix} \quad (23)$$

I define

$$\mathcal{H} \equiv (I - \Omega)^{-1} \quad (24)$$

as the Leontief inverse of the economy, with entry denoted by $h_{ij}$. The Leontief inverse matrix equals the geometric summations of $\Omega$ and reflects both the direct and indirect use of $j$ as input of firm $i$. Therefore, $h_{ij}$ presents the total use (or demand) of $j$ for the production of $i$. It is closely related to the downstream impact from $j$ to $i$.

Meanwhile, $\hat{\Omega}$ is the matrix with entries $\hat{\omega}_{ij} = \frac{p_{ij}m_{ij}}{p_jm_j}$, that denotes sales from $j$ to $i$ nor-
malized by sales of industry \( j \). The Leontief inverse of the matrix \( \hat{\mathbf{H}} \equiv (I - \Omega)^{-1} \) has entries \( \hat{h}_{ij} \), which variable captures the total supply from \( j \) to \( i \). It is closely related to the upstream impact from \( i \) to \( j \).

Consequently, \( h_i = \sum_j h_{ij} \) captures the total usage of intermediate inputs for the production of \( i \). The larger the value of \( h_i \), the further downstream the firm is in the network. \( \hat{h}_i = \sum_j h_{ji} \) captures the total supply of intermediate input \( i \). The larger is \( \hat{h}_i \), the further upstream the firm is in the network.

### 3.2 Data sources

The industry-level data for manufacturing is obtained from the Industrial Production index by the Federal Reserve Board of the United States and the NBER-CES Manufacturing Industry Database (Becker, Grey and Marvakov, 2013). The former data reports the level of production for the period 1986-2015 on a monthly basis. Here I utilize the data at the 4-digit NAICS level. The latter database represents the level of real value added and productivity (defined as the real value added over total employment) from 1956 to 2009 at yearly intervals and the 6-digit NAICS level.

To measure the level of industry downstreamness and the linkages between industries, I use the Input-Output Table created by the Bureau of Economic Analysis. This table reports the usages of industry \( i \)'s output in industry \( j \)'s production, as well as the direct use of industry \( i \)'s output in the final consumption. The method by which downstreamness is measured is described in the following subsection. The measure of downstreamness at 4-digit NAICS level is calculated based on the 2002 BEA Make and Use Table.

The credit spread index and the excess bond premium index constructed by Gilchrist and Zakrajsek (2010) here serve as instruments for aggregate financial shocks. Moreover, TRACE collects corporate bond tick data from 2002 to 2015, and this dataset was used to construct bond yield changes at sectoral level. The first step in doing so was to normalize the data to
a monthly frequency tied to the last observation of each month. Industrial-specific financial shocks are calculated using TRACE data of bond yield that is measured as the mean of bond yields changes by industries.

### 3.3 Two empirical approaches

The propagation of financial shocks is tested using the following two approaches. The **first** method compares the relative strength of upstream versus downstream propagation, in a manner similar to the method employed by Acemoglu et al. (2015), which takes the following form:

$$
\tilde{m}_{it} = \beta^{si} shock_{it-1} + \beta^{up} UP_{it-1} + \beta^{down} DOWN_{it-1} + controls.
$$

(25)

$\tilde{m}_{it}$ is the change in industrial production change at 4-digit NAICS level. Control variables include aggregate industrial production change and business cycle variables such as the credit spread index. $UP_{it}$ and $DOWN_{it}$ stand for shocks working through the network. Specifically, $UP_{it}$ measures the shocks to customers of an industry that flow up the production chain, while $DOWN_{it}$ measures the shocks to suppliers of an industry that flow down the production chain. They are calculated as the following:

$$
UP_{it} = \sum_j \hat{h}_{ji} \cdot shock_{jt},
$$

$$
DOWN_{it} = \sum_j h_{ij} \cdot shock_{jt}.
$$

Thus, $UP_{it}$ is a measure of shocks to customers of industry $i$ weighted according to the entry of the Leontief inverse matrix $\hat{\mathcal{H}}$. $DOWN_{it}$ is a measure of shocks to suppliers of industry $i$ weighted likewise according to the entry of the Leontief inverse matrix $\mathcal{H}$. $shock_{it}$ is the idiosyncratic part of the bond yield changes. I regress monthly sectoral bond yield changes on Federal Funds rate change and Aaa bond spread change; the residual of this regression is the instrument of sectoral idiosyncratic financial shocks, $\text{shock}_{it}$. The focus of this regression is $\beta^{up}$ and $\beta^{down}$.
The second method tests which sectors, upstream sectors or downstream, are more sensitive to aggregate financial shocks. The regression is as follows,

$$\tilde{m}_{it} = \beta^{dn}downstreamness_i + \beta^{shock}shock_{t-1} + \beta^{dnshock}downstreamness_i \times shock_{t-1} + controls.$$  \hspace{1cm} (26)

In this exercise, I use the change of industrial production (at 4-digit NAICS level), the change of real value added (at 6-digit NAICS level) and the change of real value added over employment (at 6-digit NAICS level) as proxies for sectoral output change. The control variables include aggregate manufacturing output change, the change of Federal Funds rate and $shock_{it-1}$. I use the changes in the credit spread index and bond premium index as instruments for aggregate financial shocks ($shock_t$). The focus is $\beta^{dnshock}$.

Specifically, two measures of downstreamness are applied. The measure of upstreamness equals $\hat{h}_i$.  

$$Upstream : \hat{h}_i.$$  

A high value of $Upstream$ indicates that an industry’s production takes place at a relatively more upstream stage.

For robustness checks, a second measure follows the method of Antras and Chor (2011). Commodities are used as final production inputs or intermediate production inputs, $M = F + Z$, where $M$ is a vector of total output $m_i$ in each sector. $F$ is a vector of $y_i$, sectoral output that goes toward final consumptions and investment. $Z$ is a vector of the use of sectoral output as inputs to other sectors. $DuseTuse$ is a measure of the ratio of the direct use of the final good to the total use of an industry’s output.  

$$DuseTuse: \frac{\Omega^T F}{M - F}.$$  \hspace{1cm} (27)

---

3 $shock_{it-1}$ is only included in regressions using the 4-digit NAICS level category.

4 This measure is also used by Fally (2011) and Kalemli-Ozcan et al. (2012).

5 $M = F + \Omega^T F + (\Omega^T)^2 + \cdots$, where $\Omega^T F$ captures the direct use of $i$ as an input for the final use. The remaining terms are the indirect use of $i$ as input.
DuseTuse is between 0 and 1. A high value of DuseTuse would indicate that the contribution of an industry’s output to production processes occurs in relatively more downstream stages.

### 3.4 Results

Table 3 presents regression results of Equation 25. I regress the dependent variable on lags of independent variables in order to control for the endogeneity issue, using the fact that current real variables do not impact past financial conditions. Upstream effects that come from financial shocks to an industry’s customers strongly influence the output of the focal sector. Downstream effects that come from financial shocks to an industry’s suppliers are less significant than upstream effects. Thus, upstream propagation of financial shocks is stronger than downstream propagation.

Moreover, Tables 4 and 5 present results of regression 26. Both positive changes of credit spread and positive changes of bond excess return correspond to negative financial shocks. Thus, $\beta^{\text{shock}}$ is negative. Notably, $\beta^{dn\text{shock}}$ of the cross terms $\text{Downmeasure} \times \text{Shock}_{t-1}$ is significantly positive, for which reason firms that are further downstream are less sensitive to aggregate financial shocks. $\beta^{dn\text{shock}}$ of the cross term $\text{Upmeasure} \times \text{Shock}_{t-1}$ is significantly negative, which means that firms that are further upstream are relatively more sensitive to aggregate financial shocks. This exercise demonstrates that the upstream propagation effect is stronger than the downstream.

To sum up my conclusions here, empirical results suggest that financial shocks generate strong upstream propagation effect. Financial friction in trade and financial linkages of firms are necessary features in a network model to predicting this finding, such as CIA- and TC-network models.
4 A General Network Model With Trade Credit

The production network in reality is much more complicated than a three-firm chain network. Moreover, in order to study the propagation of financial shocks, it is necessary to micro-found the credit market and to endogenize interest rate $R_i$. For this reason, I generalize the simple TC-network model. Specifically, I consider a DSGE model with an input-output structure in which firms are linked financially. This section illustrates one way to incorporate financial frictions and the trade credit network into a dynamic input-output model that can be used to calibrate and study the U.S. economy in Section 5.

The input-output structure of the model follows that of Long and Plosser (1983) and Acemoglu (2012). The financial friction is introduced through the working capital requirements of the production sector. The financial intermediaries that face endogenously determined balance sheet constraints follow the framework of Gertler and Karadi (2011). Furthermore, firms are connected financially through the trade credit network, as discussed by Kiyotaki and Moore (1997). Figure 5 illustrates the economic structure assumed by this paper. The solid grey arrows represent the flow of goods while the dashed red arrows represent the flow of capital. Different from a representative or horizontal economy (as Panel a), the economy studied in this paper has an input-output structure and financial linkages of firms (as Panel b).

4.1 The model

4.1.1 Intermediate Goods Firms.

There are $N$ sectors in the economy. Each sector produces one type of product. Firms within each sector are homogeneous and competitive. The intermediate goods are consumed by other firms within and across sectors, and are used by wholesalers. Each sector produces
output using the following Cobb-Douglas production function,

\[ m_{it} = z_{it}^{\alpha_i} l_{it}^{\alpha_i} (k_{it-1})^{\beta_i} \left( \prod_{j=1}^{N} m_{ij,t}^{\omega_{ij}} \right)^{\gamma_i}, \]  

(28)

where \( z_{it} \) denotes technology. \( m_{ij,t} \) denotes the amount of product \( j \) used by sector \( i \). The exponent \( \omega_{ij} \) denotes the share of good \( j \) in the total intermediate input use of sector \( i \). The direct requirement matrix \( \Omega \) in Equation 23 has entry \( \omega_{ij} \). Assume constant returns to scale \( \alpha_i + \beta_i + \gamma_i = 1 \) and \( \sum_j \omega_{ij} = 1 \). Up to now, I have presented a standard input-output
network model. I will now introduce the working capital requirement and financial frictions into this model. The timeline is presented in Figure 6.

At the end of period $t$, an intermediate goods firm acquires capital $k_{it}$ from the capital market $i$ for the production in period $t+1$. The firm issues $S_{it}$ claims equal to $k_{it}$, and prices each claim at the capital price of $Q_{it}$ in order to acquire funds for capital from financial intermediaries at the beginning of the period. Thus, the total amount of funds obtained for the capital purchase is $Q_{it}k_{it}$, which equals $Q_{it}S_{it}$. Given that firms earn zero profits, firms resell the capital to the capital market and pay out the ex-post return to capital and the sales of capital to the financial intermediaries at the end of period $t+1$. Accordingly, the stochastic return of a given financial intermediary’s investment on a capital asset of firm $i$ is,

$$R_{ik,t+1} = \frac{u_{i,t+1} + (1 - \delta)Q_{it+1}}{Q_{it}},$$

(29)

where $u_{i,t+1}$ is the capital utilization rate at period $t+1$.

Furthermore, I assume that firms dealing in intermediate goods face a working capital requirement on labor and intermediate inputs. In particular, the labor expenditure and the intermediate input purchases need to be paid in full in advance of production. Firms therefore need additional banking credit. They do so via loans $L_{it}$ from banks at the beginning of period $t$, which pays a non-contingent interest rate $R_{iLt}$ at the beginning of period $t$. I simplify the model by further assuming that there is no friction in the process of obtaining funds. There is no information friction or moral hazard problems between firms and banks.

Additionally, suppliers provide liquidity to their customers in the form of trade credit. Firms in sector $i$ only need to pay $(1 - \theta_i)p_{jt}m_{ijt}$ to their suppliers in sector $j$ at the beginning of period $t$ and to clear their accounts payable $\theta_i p_{jt}m_{ijt}$ at the end of period $t$.

---

6As discussed by Kiyotaki and Moore (1997), a supply contract and a debt contract between two trading parties are bundled together. The deferment of part of the purchase can be considered as customers borrowing from suppliers. Alternatively, the suppliers borrow from customers because they are paid something in advance of product delivery. In this model, I simply assume that all firms are produced simultaneously and that products are delivered simultaneously in the network. The distinctions between borrower and lender are not important for my argument.
Moreover, I introduce a flexible trade credit adjustment feature into the model. Firm $i$ could adjust its level of accounts-payable $\theta_{ij}$ while supplier $j$ takes $\theta_{ij}$ as a given. Nonetheless, there is a quadratic trade credit adjustment cost

$$C(\theta_{ij}, \bar{\theta}_i) = \varsigma (\theta_{ij} - \bar{\theta}_i)^2$$

per dollar unit of input purchases, where $\bar{\theta}_i$ is the steady state trade credit of firms in sector $i$. The cost is zero when $\theta_{ij} = \bar{\theta}_i$. $\varsigma$ controls the size of the cost. Trade credit adjustment is more flexible when $\varsigma$ is small (i.e. $\partial C / \partial \varsigma > 0$).

Thus, each firm solves the following problem,

$$\max_{l_{it},(m_{ijt})^N_{j=1},(m_{jit})^N_{j=1},y_{it}} \sum_j [(1 - \theta_{jit})R_{iLt} + \theta_{jit}]p_{jit}m_{jit} + p_{it}y_{it} - w_{it}l_{it}R_{iLt} - u_{it}k_{it-1}$$

$$- \sum_j [(1 - \theta_{ijt})R_{iLt} + \theta_{ijt}]p_{ijt}m_{ijt} - \sum_j C(\theta_{ijt}, \bar{\theta}_i)p_{ijt}m_{ijt}$$

$$+ \Phi_{it} \left[(z_{it}l_{it})^{\alpha_i}k_{it-1}^{\beta_i}(\Pi_j m_{ijt}^{\omega_{ij}})^{\gamma_i} - \sum_j m_{jit} - y_{it}\right].$$

$\Phi_{it}$ is the Lagrangian multiplier and corresponds to the marginal benefit of producing one unit of product. The total working capital requirement for intermediate inputs is $\sum_{j=1}^N (1 - \theta_{ijt})p_{ijt}m_{ijt}$. Equivalently, firms in sector $i$ also receive $(1 - \theta_{ijt})p_{ijt}m_{ijt}$ from their customers in sector $j$ at the beginning of $t$ and have account receivables $\theta_{jit}p_{jit}m_{jit}$. The total working capital gain from trade is $\sum_{j=1}^N (1 - \theta_{ijt})p_{ijt}m_{ijt}$. Firms are linked through production and trade credit networks.

Managers make two-step decisions. Initially, they decide the level of trade credit for each intermediate input purchase, $\theta_{ijt}$. They then choose $\{l_{it}, m_{ijt}, m_{jit}, y_{it}\}$ given $\{\theta_{ijt}, \theta_{jit}\}$. A firm’s problem is solved through backward induction.

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30

It is well observed in reality that customers have stronger bargaining power on trade credit. Cite:
The first-order conditions of the problem that occurs in the context of the second, choosing, step are,

\[
\begin{align*}
\frac{\partial m_{jit}}{\partial l_{it}} : & \quad p_{jit}[(1 - \theta_{jit})R_{iLt} + \theta_{jit}] = \Phi_{it} \\ 
\frac{\partial l_{it}}{\partial l_{it}} : & \quad \alpha_i \Phi_{it} m_{it} = w_l R_{iLt} \\ 
\frac{\partial m_{ijt}}{\partial y_{it}} : & \quad [(1 - \theta_{ijt})R_{iLt} + \theta_{ijt}] p_{ijt} m_{ijt} = \Phi_{it} \gamma_i \omega_{ij} m_{it} \\ 
\frac{\partial y_{it}}{\partial y_{it}} : & \quad \Phi_{it} = p_{it}
\end{align*}
\tag{32-35}
\]

Notably, \( p_{ijt} = p_{jt}/[(1 - \theta_{ijt})R_{jLt} + \theta_{ijt}] \). The price of product \( j \) purchased by sector \( i \), \( p_{ijt} \), depends on \( \theta_{ijt} \). The higher the trade credit, the more expensive the product price becomes. Producers naturally value early payments, and a manager, recognizing this, chooses trade credit level \( \theta_{ijt} \) in the first step so as to minimize the unit intermediate input cost. The trade-off for adjusting trade credit is as follows. The benefit of increasing \( \theta_{ijt} \) is the reduction of banking loans and interest costs. The cost of increasing \( \theta_{ijt} \) is the increase in intermediate input price \( p_{ijt} \) given that \( \partial p_{ijt}/\partial \theta_{ijt} > 0 \). There is moreover a trade credit adjustment cost per dollar purchases, \( C(\theta_{ijt}, \bar{\theta}_i) \). Thus, the manager solves the following problem,

\[
\begin{align*}
\min_{\theta_{ijt}} & \quad \left[(1 - \theta_{ijt})R_{iLt} + \theta_{ijt}\right] p_{ijt} + C(\theta_{ijt}, \bar{\theta}_i) p_{ijt} \\
\text{s.t.} & \quad p_{ijt} = p_{jt}/[(1 - \theta_{ijt})R_{jLt} + \theta_{ijt}]
\end{align*}
\]

Therefore, the optimal level of trade credit is,

\[
\theta_{ijt} = f(\varsigma, \bar{\theta}_i, R_{it}, R_{jt}) \tag{36}
\]

with

\[
\begin{align*}
\partial \theta_{ijt}/\partial R_{it} > 0, \\
\partial \theta_{ijt}/\partial R_{jt} < 0, \\
|\partial \theta_{ijt}/\partial \varsigma| < 0, \\
\theta_{ijt} = \bar{\theta}_i, \text{ when } R_{it} = R_{jt}.
\end{align*}
\]
Further, trade credit is more sensitive to the relative financial condition of the two trading parties when $\varsigma$ is low. In the extreme case that $\varsigma = 0$, trade credit becomes fully flexible. Under these conditions, $\theta_{ijt} = 1$ when $R_{iLt} > R_{jLt}$ and $\theta_{ijt} = 0$ when $R_{iLt} < R_{jLt}$.

**Proposition 5.** Trading parties share liquidity through the trade credit mechanism. $\theta_{ijt}$ is an increasing function of $R_{iLt}$ and a decreasing function of $R_{jLt}$. Sectoral correlation is high when trade credit adjustment is flexible.

When firm $i$ finds that bank loans are becoming costly, it increases trade credit. When its suppliers are suffering financially, firm $i$ shrinks its accounts-payable. This response consistent with the finding by Gao (2014) that trade credit plays an important role as an inter-firm financing channel by allowing firms to share liquidity with each other. It is also in line with the empirical finding that an increase in the use of trade credit along the product chain that links two sectors results in an increase in correlation between them (Raddatz 2008). In addition, this model predicts that sectoral correlation is even higher when trade credit adjustment is flexible. An idiosyncratic liquidity shock could thus spill over to surrounding firms vigorously.

### 4.1.2 Retailers and Wholesale Firms.

Wholesale firms in the economy produce wholesale product $Y_{wt}$, which is a composite of the products produced by each sector. The wholesale output and the price are represented by

\[ Y_{wt} = \prod_{i=1}^{N} \zeta_i^{-\varsigma_i} y_{it} \varsigma_i, \quad P_{wt} = \prod_{i=1}^{n} p_{it} \varsigma_i, \]

where $y_{it}$ is the amount of products produced by sector $i$ and used in the production of wholesale goods. $\varsigma_i$ governs the share of output $i$ used in the production of final goods.

Consumption goods are sold by a set of monopolistically competitive retailers uniformly distributed from 0 to 1, who can costlessly differentiate the single final good assembled by

\[ p_i \text{ and } P_m \text{ are real prices.} \]
One unit of wholesale output \( Y_w \) is required to make a unit of retail output with marginal cost \( P_w \). The final output composite is,

\[
Y_t = \left( \int_0^1 Y_{rt}^{\epsilon - 1} \, dr \right)^{\frac{1}{\epsilon}},
\]

where \( Y_{rt} \) is the retail output of retailer \( r \). Retailers face nominal rigidities following CEE. They could freely adjust their price with probability \( 1 - \gamma \) each period; the problem is identifying the optimal price \( P_{rt}^* \). By tedious but straightforward derivation in Appendix C.1, I have

\[
P_{rt}^* = \frac{\epsilon}{\epsilon - 1} \sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} \left( \frac{P_{i+1}}{P_i} \right)^{\epsilon - 1} \left( \prod_{k=1}^i \pi_{t+k-1}^{(1-\epsilon)} \right) Y_{t+i}.
\]

Thus, the innovation of the aggregate price level is,

\[
P_t^{1-\epsilon} = (1 - \gamma) P_{rt}^{1-\epsilon} + \gamma (P_{t-1}^{\gamma P_{t-1}})^{1-\epsilon}.
\]

**4.1.3 Capital Producer.**

At the end of period \( t \), competitive capital producers in each sector buy capital from their respective capital markets, and repair and build new capital. Capital producers make new capital using input of final output and are subject to adjustment costs. They sell new capital \( k_{it} \) to firms in sector \( i \) at the price \( Q_{it} \), so their problem is to,

\[
\max_{I_{it}} Q_{it} I_{it} - \left[ 1 + \frac{\eta_t}{2} \left( \frac{I_{it}}{I_{i,t-1}} - 1 \right)^2 \right] I_{it}.
\]

Thus, the price of capital goods is equal to the marginal cost of investment goods production as follows,

\[
Q_{it} = 1 + \frac{1}{2} \eta_t \left( \frac{I_{it}}{I_{i,t-1}} - 1 \right)^2 + \frac{I_{it}}{I_{i,t-1}} \eta_t \left( \frac{I_{it}}{I_{i,t-1}} - 1 \right) - \mathbb{E}_t \left[ \beta \Lambda_{t+1} \left( \frac{I_{i,t+1}}{I_{it}} - 1 \right)^2 \eta_t \left( \frac{I_{i,t+1}}{I_{it}} - 1 \right) \right].
\]

---

\(^9\)In order to introduce price friction in a simple way.
The capital innovation is:

\[ k_{it} = e^{\psi_t} (1-\delta)k_{it-1} + I_{it}. \]

### 4.1.4 Households.

There is a continuum of identical households with a fraction \(1-u\) of workers and a fraction \(u\) of bankers. Over time, an individual switch between a worker and a banker with probability \((1-\tau)\). In other words, a banker at time \(t\) stays as a banker at time \(t+1\) with probability \(\tau\).

Workers supply labor \(L_t\) to the production sector and return their wages to households. Bankers manage financial intermediaries and transfer profits back to households. Households consume \(C_t\) and save. They save by depositing funds in banks or by purchasing government debt. Both deposits and government debt are one-period riskless assets that pay the real return of \(R_t\). I consider these two assets perfect substitutes and denote them by \(B_t\). The households’ welfare function is,

\[
\max \mathbb{E}_0 \sum_{i=0}^{\infty} \beta^i \left[ \ln(C_{t+i} - hC_{t+i-1}) - \frac{X}{1+\psi} L_{t+i}^{1+\psi} \right].
\]  
(39)

The budget constraint they face is,

\[ C_t = w_t L_t + \Pi_t + T_t + R_{t-1}B_t - B_{t+1}, \]  
(40)

where \(w_t\) is real wage, \(\Pi_t\) represents profits distributed from bankers and capital producing firms, \(T_t\) represents government transfers.

When \(\varrho_t\) denotes the marginal utility of consumption, the households’ first order conditions for consumption, saving and labor supply are:

\[ \varrho_t = (C_t - hC_{t-1})^{-1} - \beta h \mathbb{E}_t(C_{t+1} - hC_t)^{-1}, \]

\[ \mathbb{E}_t \beta \Lambda_{t,t+1} R_{t+1} = 1, \]
\[ q_t w_t = \chi L^t, \]
and \[ \Lambda_{t,t+1} = \frac{\rho_{t+1}}{\rho_t}. \]

### 4.1.5 Financial Intermediaries.

The structure of financial intermediaries generally follows that of Gertler and Karadi (2011), but with the following modifications. Financial intermediaries (banks) are segmented into \( N \) groups. Each production sector \( i \) is connected to a unique banking group \( i \). Hence, banks of group \( i \) cannot finance firms in sector \( j \neq i \). I further assume that there is no lending among banks. Each period, banks use their own net worth \( N_{it} \) and the deposit \( B_{it} \) from households in order to finance their purchases of financial claims \( S_{it} \) and loans \( L_{it} \). The intermediary balance sheet of banks in group \( i \) is

\[ Q_{it} S_{it} + L_{it} = N_{it} + B_{it}. \]  \( (41) \)

The return of \( S_{it} \) is realized by the end of period \( t + 1 \) with a stochastic return \( R_{ikt+1} \). \( L_{it} \) is matured by the end of period \( t \) with a non-contingent return \( R_{iL_t} \). Also, banks pay a non-contingent return \( R_t \) to households at period \( t + 1 \). Their net worth then evolves according to

\[ N_{it+1} = R_{ik,t+1} Q_{it} S_{it} + R_{iL,t} L_{it} - R_t B_{it+1} \]  \( (42) \)

The financial intermediary’s objective is to maximize:

\[ V_{it}(N_t) = \max \mathbb{E}_t \sum_{j=0}^{\infty} (1 - \tau)^j \beta^j \Lambda_{t,t+1+j} (N_{i,t+1+j}), \]  \( (43) \)

given that the probability of a banker becoming a worker in the next period is \((1 - \tau)\). For the intermediary to operate, the risk premium must be positive, i.e. \( E_t^{\beta^j} \Lambda_{t,t+1} (R_{ikt+1+i} - R_{t+i}) \geq 0 \) and \( E_t^{\beta^j} \Lambda_{t,t+1} (R_{iL,t+i} - R_{t+i}) \geq 0 \). However, in order to prohibit intermediaries expanding

\[ ^{10} \text{This is consistent with the finding by Chodorow-Reich (2014) that bank-borrower relationships are sticky.} \]

\[ ^{11} L_{it} \text{ could be negative, in which case working capital from trade among firms would be sufficient to pay labor costs. The remainder would then be deposited into the bank with an interest rate of } R_{iL_t}. \]
their assets indefinitely when risk premium is positive, I introduce the following incentive constraint,

$$V_{it} \geq \lambda_i(Q_{it}S_{it} + L_{it}). \quad (44)$$

Bankers will lose their expected terminal wealth $V_{it}$ if they divert assets, while their gain from such an action is $\lambda_i(Q_{it}S_{it} + L_{it})$.

The binding incentive constraint at equilibrium implies that the leverage ratio $(Q_{it}S_{it} + L_{it})$ denoted by $\phi_{it}$ equals,

$$\phi_{it} = \eta_{it} \lambda_i - d_{it} \nu_{kit} - (1 - d_{it}) \nu_{lit}, \quad (45)$$

with

$$\nu_{kit} = \mathbb{E}_t[(1 - \tau) \beta \Lambda_{t,t+1} (R_{ikt,t+1} - R_t) + \beta \Lambda_{t,t+1} \tau x_{kit,t+1} \nu_{kit+1}],$$

$$\nu_{lit} = \mathbb{E}_t[(1 - \tau) \beta \Lambda_{t,t+1} (R_{iLt,t} - R_t) + \beta \Lambda_{t,t+1} \tau x_{lit,t+1} \nu_{lit+1}],$$

and

$$\eta_{it} \equiv \mathbb{E}_t[(1 - \tau) \Lambda_{t,t+1} R_t + \beta \Lambda_{t,t+1} \tau z_{it,t+1} \eta_{it+1}],$$

where $d_{it} = \frac{Q_{it}S_{it}}{Q_{it}S_{it} + L_{it}}$ is the share of risky assets, $x_{kit,t+j} \equiv \frac{Q_{it+j}S_{it+j}}{Q_{it}S_{it}}$ and $x_{lit,t+j} \equiv \frac{L_{it+j}}{L_{it}}$ are the gross growth rate in assets between $t$ and $t + j$, and $z_{it,t+j} \equiv \frac{N_{it+j}}{N_{it}}$ is the gross growth rate of net worth. $\nu_{it}$ is the expected discounted marginal gain that a unit enjoys by expanding its assets, when net worth remains constant, and is an increasing function of the risk premium. $\eta_{it}$ is the expected discounted value of having an additional unit of net worth, when the asset remains constant. As derived in Appendix [C.3], the first order conditions of the problem facing banks imply the no-arbitrage condition,

$$\mathbb{E}_t(\Lambda_{t,t+1} R_{ikt,t+1}) = \mathbb{E}_t(\Lambda_{t,t+1}) R_{iLt}.$$ 

The evolution of bankers’ net worth can be expressed as,

$$N_{it} = [(R_{ikt} - R_{i-1})d_{it-1} \phi_{it-1} + (R_{iLt,t-1} - R_{i-1})(1 - d_{it-1}) \phi_{it-1} + R_{it-1}]N_{it-1}. \quad (46)$$
Notably, the sensitivity of \( N_{it} \) to the excess return is increasing in the leverage ratio \( \phi_{it-1} \). Therefore, \( \phi_i \) is an important factor affecting the sectoral sensitivity to financial shocks.

The total net worth of bankers in group \( i \) includes the net worth of existing bankers \( N_{iet} \) together with the net worth of new bankers \( N_{int} \),

\[
N_{it} = N_{iet} + N_{int},
\]

where \( N_{eit} = \tau[(R_{ikt} - R_{t-1})d_{it-1}\phi_{it-1} + (R_{il,t-1} - R_{t-1})(1 - d_{it-1})\phi_{it-1} + R_{t-1}]N_{it-1} \). A financial shock represents an unexpected contraction of the existing bankers’ net worth.

Assuming further that a fraction of \( \omega_i/(1 - \tau) \) of the total assets of exiting bankers \(((1 - \tau)(Q_{it}S_{it-1} + L_{it-1})) \) is transferred to new bankers, I have

\[
N_{nit} = \omega_i(Q_{it}S_{it-1} + L_{it-1}).
\]

4.1.6 Monetary policy

Monetary policy follows a simple Taylor rule with interest rate smoothing. The nominal interest rate follows,

\[
i_t = (1 - \rho)[i + \kappa_\pi\pi_t + \kappa_y(\log Y_t - \log Y_t^*)] + \rho i_{t-1} + \epsilon_t, \tag{47}
\]

where \( i \) denotes the steady state nominal rate, and \( Y_t^* \) is the natural level of output.

4.1.7 Equilibrium

The competitive equilibrium in the general model is defined as follows:

A competitive equilibrium consists of a collection of quantities \( \{m_{it}, l_{it}, k_{it}, m_{ijt}, \theta_{ijt}, u_{it}, Y_{wt}, Y_t, I_{it}, C_t, S_{it}, N_{it}, L_{it}, B_{it}\} \) and a sequence of prices \( \{R_{ik,t}, R_{id,t}, i_t, Q_{it}, P_{it}, p_{ijt}, w_t, P_{wt}, P_t\} \) for \( i \in 1, ..., N \) and \( j \in 1, ..., N \), such that
1. Intermediate firms maximize profits subject to working capital requirements.

2. Wholesalers and retailers maximize profits.

3. Capital producers maximize profit \(^{(38)}\).

4. Households maximize utility \(^{(39)}\) subject to their budget constraints \(^{(40)}\).

5. Financial intermediaries maximize expected profit \(^{(43)}\) subject to incentive constraint \(^{(44)}\).

4.2 The propagation of financial shocks

Figure 7: Supply chain of a 5-firm circle economy

Factors that are specific to certain firms, such as intermediate input share, capital share, leverage, and trade credit, all have strong impacts on the sensitivity of output to shocks. In order to focus on the propagation effect along the production chain, I simulate a 5-firm model with the following production functions,

\[
\begin{align*}
  m_{1,t} &= z_1 l_{1,t}^\alpha k_{1,t-1}^{\beta} m_{15,t}^{(1-\alpha-\beta)}, \\
  m_{2,t} &= z_2 l_{2,t}^\alpha k_{2,t-1}^{\beta} m_{21,t}^{(1-\alpha-\beta)}, \\
  m_{3,t} &= z_3 l_{3,t}^\alpha k_{3,t-1}^{\beta} m_{32,t}^{(1-\alpha-\beta)}, \\
  m_{4,t} &= z_4 l_{4,t}^\alpha k_{4,t-1}^{\beta} m_{43,t}^{(1-\alpha-\beta)}, \\
  m_{5,t} &= z_5 l_{5,t}^\alpha k_{5,t-1}^{\beta} m_{54,t}^{(1-\alpha-\beta)}.
\end{align*}
\]

Firm \(i\) uses intermediate inputs \(i-1\) (as illustrated in Figure 7). The only difference among
the firms is their relative location along the supply chain. Consider an idiosyncratic shock to Firm 3. If the shock propagates clockwise (counter-clockwise) along the supply chain, the propagation is downstream (upstream).

I simulate the five-firm model with calibrations that use the U.S. data. Specifically: labor share $\alpha$ is $0.33$; the capital share is $(1 - \beta)$ is $0.22$; and the leverage ratio ($\phi$) is targeted to be $2$. Trade credit equals $0.46$. The five firms have identical final output share $\zeta$, which equals $0.2$. Other conventional parameters follow the calibration by Gertler and Karadi (2011) and are listed in Table 9. Figure 10 illustrates the impulse-response of sectoral output after $Ne_3$ shock at $t = 1$ with fixed trade credit levels. $Ne_3$ shock is a negative 1% change in the net worth of the existing bankers linked to Firm 3, with zero autocorrelation. I define the short term sectoral output change as the contemporaneous output change when the shock is felt, while the long term sectoral output change is defined as the largest output change after the shock.

Figure 8 illustrates the short-term effect of a negative $Ne_3$ shock. The left panel plots the short term sectoral output changes. The bar representation in the right panel illustrates the conclusion that the financial shock propagation is counter-clockwise (upstream propagation) in the CIA-network economy, while it is clockwise (downstream propagation) in the DP-network economy. Moreover, given that the average trade credit in the US economy is lower than 0.5, the upstream propagation effect is stronger than the downstream propagation effect in the TC-network model.

Moreover, Figure 9 plots sectoral output changes after $Ne_3$ shock with different levels of trade credit adjustment cost. When the trade credit adjustment cost is low, there is the flexibility to adjust trade credit, and sectoral correlation is exceedingly high. The right panel plots the IRF of the trade credit. The trade credit of Firm 3 ($\theta_3$) increases, while the trade credit of Firm 4 ($\theta_4$) decreases, because $R_{3L}$ is larger than $R_{2L}$ and $R_{4L}$. Surrounding firms share liquidity.
In addition to the upstream propagation effect controlled by the financial friction in trade in CIA- and TC-network economies, there is a downstream propagation effect through the innovation of bank wealth. The real interest rate declines after a negative financial shock because of the decrease in the demand for deposits and the response of monetary policy. Firms that are not affected by financial shocks enjoy a positive effect, in that their financial intermediaries can borrow money from households at lower cost. Thus, both the banking interest rate and the product price decrease. The intermediate input price decreases for all firms except Firm 4. The price of Product 3 increases, which generates in turn a negative supply impact to Firm 4. In the long run, this effect is stronger than the upstream spill-over effect from the financial friction in trade. Consequently, in the long run, the downstream propagation effect of financial shocks is stronger than the upstream effect. As illustrated in Figure 10, upstream propagation is strong in the first 6 quarters after the shock in the CIA-network model, and in the first 4 quarters in the TC-network model. Downstream propagation is strong in both short and long term in the DP-network model.

Therefore, as was predicted by the simple model in Section 2, the upstream propagation effect is stronger in the short term in those models that take into account financial friction in trade and financial linkages among firms. These conclusions are also consistent with my empirical finding regarding the short term effect in Section 3. Moreover, all three types of models predict strong downstream propagation during supply-side shocks and upstream propagation during demand-side shocks, as discussed in Appendix.

5 The Role of Firm Level Heterogeneity

With these considerations in mind, the forms of firm-level heterogeneity are the most powerful driver of the sectoral output fluctuations will now be discussed.
5.1 Calibration

I have calibrated the general model using the U.S. data at the 2-digit NAICS level. I ex-
clude the government sector, as well as the finance and insurance subsectors from the FIRE
(finance, insurance and real estate) sector, for the simple reason that those sectors cannot
be formulated by the Cobb-Douglas production function presented in Section 4.

*Conventional Parameters.* Conventional parameters (such as the discount factor and the
Calvo parameter) follow the calibration by Gertler and Karadi (2011) and are listed in Table
9.

The input-output structure \( \omega_{ij} \) is calibrated using the BEA input-output table. Labor share
\( \alpha_i \), capital share \( \beta_i \) and intermediate input share \( \gamma_i \) are listed in Table 10. \( \alpha_i \) and \( \beta_i \) are
calibrated using the BEA GDP by Industry Value-added Components Table (1998-2013) and
the calibration method follows Su (2014) (refer to Appendix).

*Banking Parameters.* \( \omega_i \) (the proportional transfer to the entering bankers) and \( \lambda_i \) (the
fraction of capital that can be diverted) are calibrated to match \( \phi_i \) (the leverage of each
sector) and \( R_{iLt} \) (the banking lending rate). I assume the steady state \( R_{iLt} \) to be identical
across sectors\(^\text{12}\) It is calibrated to hit the steady state credit spread. The leverage level of
each sector is measured using the Compustat Data, which is listed in Table 10. The survival
rate of bankers \( \theta = 0.972 \) adopts the value set by Gertler and Karadi (2011), which hits the
average horizon of bankers within a given decade.

*Trade credit.* I present the summary of trade credit statistics in Table 6, 7 and 8. The
National Survey of Small Business Finances (NSSBF) is the survey of U.S. small businesses
conducted by the Board of Governors and the U.S. Small Business Administration. Table 6
shows that 45.9% of small business purchases used trade credit in 1998, while the percent-

\(^{12}\)No strong empirical evidence shows that sectoral interest rates differ significantly.
age climbs to 70% in 2003. Regarding large business entities, Table 7 shows that accounts payable over cost of goods sold varies from 0.21-1 across sectors, based on Compustat data (measured as the median of each sector in each time period, and averaged from 2002 to 2006). Additionally, Quarterly Financial Report (QFR) has collected quarterly aggregate statistics on the financial results and positions of U.S. corporations. QFR is more comprehensive than NSSBF and Compustat, but it only covers the following four industry sectors, Mining, Manufacturing, Wholesale and Retail. All four of these measures of the standardized trade credit are relatively stable over the QFR sample period (2000q4-2014q4). Moreover, these measures by QFR approximate the measures using Compustat data (compare Table 7 and Table 8). Therefore, I calibrate \( \bar{\theta}_i \) using the Compustat measure and assume that it is constant.

5.2 Quantitative predictions

5.2.1 Amplification effect of the network structure

Financial friction in trade generates an amplification effect on the aggregate impact of shocks. Figure 11 plots the impulse responses of aggregate output \( Y \), consumption \( C \), investment \( I \) and premium \( (E(R_k) - R) \) after an aggregate banking net worth \( Ne \) shock, an aggregate technology shock \( z \), a capital quality shock \( \xi \), and monetary policy shock \( i \). Ne shock is an unanticipated negative 10% reduction of the existing banker’s net worth of each sector. The technology z shock is a negative 10% innovation in TFP of each sector, which is an AR1 process with autocorrelation 0.95. \( \xi \) shock is a negative 10% change in capital quality of each sector and is AR1 with autocorrelation 0.66. Monetary policy \( i \) shock is an unanticipated 100 basis-point increase in the short term interest rate.

---

13 The financial and insurance sectors have accounts payable over cost of goods sold at a level of 60. The balance sheets of these financial sectors are complicated, and the definition of accounts-payable in those sectors differs from the definition used in other production sectors.

14 I standardize accounts payable and accounts receivable based on total sales and total assets.

15 \( (E(R_k) - R) \) is calculated as the average premium across sectors. \( I \) is the total investment in the economy.
Compared with a representative firm model (indicated by the dashed black line), a TC-network economy (indicated by the solid red line) is more responsive to all manner of shocks.\(^{16}\)

5.2.2 The role of sector specific factors

With regard to the question of which sector-specific factor is more important for a certain type of shocks, Table 10 presents the value of intermediate input share, labor share, capital share, trade credit and leverage level of the 14 sectors. Table 11 presents the correlation between each pair of these sector-specific factors. These correlations are moderate (\(< 0.5\)) on average, apart from the fact that the correlation between capital share and trade credit is relatively high (0.69).

Again, I simulate the model with aggregate banking net worth shock \((Ne)\), aggregate technology shock \(z\), monetary policy shock \(i\) and capital quality shock \(\xi\). I correlate sectoral output change with firm-specific factors in the short and long term. The short-term effect concerns the contemporaneous impact on sectoral outputs, while the long-term concerns the strongest response of sectoral output in the wake of the shock, which generally occurs five quarters later (as illustrated in Figure 12). Table 12 lists the correlation of short-term sectoral output changes with sector-specific factors, while Table 13 lists the correlation of long-term sectoral output changes with sector-specific factors.

The most influential factors with respect to each type of shock are listed in Table 2.

To begin with, firms with a larger capital share \(\beta_i\) are less affected by financial shocks.

\begin{table}[h]
\centering
\begin{tabular}{lll}
\hline
shocks & short term & long term \\
\hline
Financial shock: & capital share & capital share, leverage level \\
Technology shock: & intermediate share & intermediate share \\
Monetary Policy shock: & capital share, & trade credit, influence factor \\
Capital quality shock: & capital share & capital share \\
\hline
\end{tabular}
\end{table}

\(^{16}\)The quantitative difference of the IRF of the three types of models (DP, CIA and TC) is not big.
A negative financial shock decreases the price of capital $Q_i$ and increases the bank credit rate. The negative impact on capital is relatively weaker than is the impact on labor and intermediate inputs. Thus, firms with a larger capital input share are less affected by a financial shock. Second, firms with larger financial leverage $\phi_i$ are less affected by financial shocks in the long term. According to Equation 46, the sensitivity of $N_{it}$ to the excess return $(R_{ikt} - R_{t-1})$ is an increasing function of the leverage ratio $\phi_i$. After the financial shock and during the high excess return periods, the banking wealth $N_{it}$ bounces back faster if $\phi_i$ is high. Third, intermediate input share $\gamma_i$ governs the downstream propagation power (by Proposition 2). The larger a firm’s intermediate input share, the more sensitive its output is to supply side shocks. Therefore, sectors with a larger intermediate input share are proportionately more sensitive to technology shocks. Furthermore, the response of financial intermediaries is closely related to subsequent changes in monetary policy. Thus capital share is an important factor in the impact of monetary shocks. Monetary policy also impact households’ saving and consumption behaviors. As a result, influence factor $v_{it}$ (in Equation 3) is another important factor in the impact of monetary shocks, especially in the long term. Finally, the capital share is obviously significant for the impact of capital quality shocks.

6 Credit Policy Implication

Suppose that, during a severe financial crisis, the central bank is willing to facilitate lending. Let $S^p_{it}$ and $L^p_{it}$ be the value of assets intermediated by the financial intermediaries, and let $S^g_{it}$ and $L^g_{it}$ be the value of assets intermediated by the central bank. Under these circumstances, the total value of intermediated assets of sector $i$ is,

\begin{align*}
S_{it} &= S^p_{it} + S^g_{it}, \\
L_{it} &= L^p_{it} + L^g_{it}.
\end{align*}

To facilitate lending, the central bank issues government debt to households that pay the riskless rate $R_t$, and it lends funds to non-financial firms at the banking credit rate $R_{itLt}$. 

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Nonetheless, unlike the financial intermediaries, the central bank has no frictions on issuing debt and purchasing private assets. Whereas there are $N$ types of assets according to the issuing sector, the central bank is flexible in targeting liquidities. The central bank could then either lend funds to all sectors proportionally (corresponding to their market size), or could target liquidity to specific sectors. The total value of assets intermediated by the central bank is,

$$CP_t = \sum_{i} CP_{it} = \sum_{i} (Q_{it}S_{it}^g + L_{it}^g)$$

(48)

Suppose the credit policy follows that

$$CP_{t+1} = \rho_{CP}CP_t + \epsilon_{t+1}^{CP}, \quad \text{with} \quad \epsilon_t^{CP} = -\vartheta \epsilon_t^{Ne},$$

(49)

where $\vartheta$ is a constant and $\epsilon_t^{Ne}$ is the banking net worth shock. Accordingly, the total value of assets intermediated by the central bank would be proportional to the shock to the banking net worth, and $CP_t$ would follow an AR1 process. Moreover, following Gertler and Karadi (2011), this central bank intermediation involves the loss of efficiency. There is an efficiency cost of $o$ for every unit of central bank credit that is supplied. I compare credit policies by targeting liquidity to different sectors. Let $\tilde{Y}_{CPoni}$ denote the aggregate output change after an aggregate financial shock ($Ne$ shock) and a sectoral specific credit policy shock (i.e. $CP_t = CP_{it}, CP_{jt} = 0, \forall j \neq i$). The comparison result of $\tilde{Y}_{CPoni}$ across $i$ makes clear which sector $i$ the central bank should facilitate lending during the crisis.

I simulate the model with $\vartheta = 5\%$ and $o = 0.01$. Figure 13 plots the response of $Y$, $C$, $I$ and the premium after a -1% $Ne$ shock and a credit policy targeting the real estate sector. Compared with an economy that lacks credit policies, this economy experiences weaker output and suffers from capital and investment contractions as well as a smaller premium increase. Given that government could target liquidity in order to different sectors, the question arises as to which sector it should target following aggregate financial shocks. Figure 14 plots $\tilde{Y}_{CPoni}$ in three types of economies after the same aggregate financial shock and the same level of $\epsilon_t^{CP}$. $\tilde{Y}_{CPoni}$ barely fluctuates across $i$ in a DP-network model in both the short
(contemporaneous impact) and long term. However, the aggregate output response varies across targeted sectors in the CIA- and TC-network economies. It is therefore has a stronger impact to target liquidity to downstream sectors than to upstream sectors when there is financial friction in trade. Two factors, the downstreamness and the influence factor $v_i$ of a sector, are important in the impact of a given credit policy. Since financial shocks propagate upstream in CIA- and TC-network models, it is not surprising that credit policy has a larger impact by targeting liquidity to downstream sectors. It also has a stronger impact by target liquidity to sectors that make a large contribution to the aggregate output. In general, downstream sectors contribute a larger proportion of output to the final goods production. In the CIA- and TC-network economies, both the propagation effect and the final share effect work in the same direction, for which reason credit policy should target liquidity to the downstream sector. In the DP-network economy, the propagation effect and the final share effect work in opposite directions. Because financial shocks propagate downstream, the liquidity of upstream firms is more important, whereas the final share effect argues for targeting liquidity to downstream firms. As a consequence, the difference of $\tilde{Y}_{C_{Poni}}$ across targeted sectors is small in a DP-network model.

A key conclusion here, then, is that a central bank should facilitate lending to downstream sectors during the crisis when the economy has financial friction in trade.

### 7 Conclusions

This paper focuses on the propagation of financial shocks in a network economy with financial linkages of firms. Unlike supply and demand shocks, the propagation of financial shocks depends on the payment schedule of intermediate inputs. Financial frictions in trade and financial linkages of firms are necessary features for generating strong upstream propagation of financial shocks, which is revealed by the data. Consequently, credit policy has a stronger impact by targeting liquidity to downstream sectors.
This paper represents the first theoretical study, which introduces a trade credit network into an input-output model within a general equilibrium framework. Financial market loans and trade credit are the two most important sources of external finance for firms. Through trade credit mechanism, firms share liquidity with surrounding firms. Idiosyncratic shocks spread out through trade and financial linkages of firms. There are several possible extensions of this paper. For example, more interesting results might be observed in a model with a micro-founded trade credit structure. If one allows trade credit default, a strong liquidity shock to one firm may cause a cascade of defaults throughout the trade credit network. It can set off an avalanche of production failure and generate a persistent aggregate output contraction. Another extension of this work would be adding inter-bank lending. In reality, there is both a complicated production network and an entangled financial network. An idiosyncratic financial shock not only transfers through the input-output network, but also spills over in the banking network. This paper will motivate both theoretical and empirical studies on the role of financial fictions in the input-output economy.

References


Figure 8: Sectoral output changes after firm 3 financial shock (different types of economies) (firm number on x-axis)

Figure 9: Sectoral output changes after firm 3 financial shock (different adjustment cost $\varsigma$) (firm number on x-axis)
Figure 10: IRF of sectoral output changes after $N e_3$ shock.

Figure 11: Aggregate variables response to banking net worth ($Ne$), technology ($z$), capital quality ($\xi$) and monetary ($i$)

Figure 12: Sectoral output changes after aggregate financial shock $Ne$ (tc-network)

![Output (aggregate financial shock)](image)

Figure 13: Aggregate variables response to banking net worth ($Ne$) and credit policy ($CP$) shocks

![Aggregate variables response](image)

$Ne$ shock is a negative 1% change in the banking net worth of each sector. Credit policy is a 5% of the $Ne$ shock targeting one of the sectors with a quarterly autoregressive factor of 0.8.
Ne shock is a negative 1% change in the banking net worth of each sector. Credit policy is a 5% of the Ne shock targeting one of the sectors with a quarterly autoregressive factor of 0.8. In all types of economies, without credit policy, the change of Y in the short term is -9.3e-4. The change of Y in the long term is -2.4e-3.
Table 3: Upshock VS Downshock - Industrial Production (2002.09-2012.12) monthly

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
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<tbody>
<tr>
<td>( \text{UP}_{it-1} )</td>
<td>-0.0054***</td>
<td>-0.0058***</td>
</tr>
<tr>
<td></td>
<td>[0.0018]</td>
<td>[0.0019]</td>
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<tr>
<td>( \text{DOWN}_{it-1} )</td>
<td>0.0009</td>
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Note: Subsectors are at 4-digit NAICS level.
Table 4: Downstreamness - Industrial Production (2002.09-2012.12) monthly

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<th>$\Delta$ credit spread (1)</th>
<th>$\Delta$ bond premium (2)</th>
<th>(3)</th>
<th>(4)</th>
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<td><strong>Upmeasure$_i \times \text{Shock}_t$</strong></td>
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<td></td>
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<td>[0.0002]</td>
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<td>Downmeasure$_i \times \text{Shock}_t$</td>
<td>0.0056</td>
<td>0.0027**</td>
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<td>Downmeasure$_i$</td>
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<td>-0.1200***</td>
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<td>-0.1272***</td>
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<td>0.0034**</td>
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<td>[0.0009]</td>
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<td>0.9654***</td>
<td>0.9455***</td>
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Note: Subsectors are at 4-digit NAICS level.
Table 5: Downstreamness - NBER CES (1973-2009) yearly

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<th>$\tilde{m}_{it}$ real value added per person</th>
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<td>$\text{Upmeasure}<em>i \times \text{Shock}</em>{t-1}$</td>
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<tr>
<td></td>
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<td>0.0805**</td>
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<td></td>
<td>[0.0393]</td>
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<td>0.0584**</td>
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Note: Subsectors are at 6-digit NAICS level.
## Table 6: NSSBF data

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<th>Year</th>
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<th>Std</th>
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<td>1998</td>
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## Table 7: Trade Credit (Compustat)

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<th>AP/S</th>
<th>AR/S</th>
<th>AP/TA</th>
<th>AR/TA</th>
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<tbody>
<tr>
<td>Agriculture</td>
<td>0.43</td>
<td>0.30</td>
<td>0.41</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
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<td>0.50</td>
<td>0.60</td>
<td>0.05</td>
<td>0.05</td>
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<td>0.36</td>
<td>0.51</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>Construction</td>
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<td>0.34</td>
<td>0.88</td>
<td>0.10</td>
<td>0.20</td>
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<td>Manufacturing</td>
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<td>0.32</td>
<td>0.57</td>
<td>0.07</td>
<td>0.13</td>
</tr>
<tr>
<td>Wholesale trade</td>
<td>0.44</td>
<td>0.33</td>
<td>0.41</td>
<td>0.18</td>
<td>0.21</td>
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<tr>
<td>Retail trade</td>
<td>0.42</td>
<td>0.28</td>
<td>0.09</td>
<td>0.14</td>
<td>0.05</td>
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<td>0.37</td>
<td>0.04</td>
<td>0.09</td>
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<td>0.22</td>
<td>0.06</td>
<td>0.07</td>
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## Table 8: Trade Credit (QFR)

<table>
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<th>AR/S</th>
<th>AP/TA</th>
<th>AR/TA</th>
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<tr>
<td>Mining</td>
<td>0.49</td>
<td>0.64</td>
<td>0.05</td>
<td>0.06</td>
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<tr>
<td>Manufacturing</td>
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<td>0.45</td>
<td>0.07</td>
<td>0.10</td>
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<tr>
<td>Wholesale trade</td>
<td>0.33</td>
<td>0.36</td>
<td>0.20</td>
<td>0.22</td>
</tr>
<tr>
<td>Retail trade</td>
<td>0.28</td>
<td>0.14</td>
<td>0.16</td>
<td>0.08</td>
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Table 9: Conventional Parameters (Gertler and Karadi (2011))

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<td>$\gamma_P$</td>
<td>0.241 or 0</td>
<td>measure of price indexation</td>
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<tr>
<td>$\kappa_y$</td>
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<td>output gap coefficient of the taylor rule</td>
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Table 10: Sectoral Level Parameters

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<th>$\beta_i$</th>
<th>$\gamma_i$</th>
<th>$\zeta_i$</th>
<th>$\theta_i$</th>
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<td>0.37</td>
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<td>1.00</td>
<td>1.53</td>
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<td>0.64</td>
<td>0.21</td>
<td>0.49</td>
<td>1.50</td>
</tr>
<tr>
<td>Wholesale</td>
<td>0.37</td>
<td>0.33</td>
<td>0.31</td>
<td>0.04</td>
<td>0.44</td>
<td>2.30</td>
</tr>
<tr>
<td>Retail</td>
<td>0.41</td>
<td>0.27</td>
<td>0.32</td>
<td>0.10</td>
<td>0.42</td>
<td>1.99</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.35</td>
<td>0.16</td>
<td>0.50</td>
<td>0.02</td>
<td>0.30</td>
<td>2.33</td>
</tr>
<tr>
<td>Information</td>
<td>0.25</td>
<td>0.30</td>
<td>0.45</td>
<td>0.04</td>
<td>0.64</td>
<td>1.58</td>
</tr>
<tr>
<td>real estate</td>
<td>0.24</td>
<td>0.48</td>
<td>0.28</td>
<td>0.15</td>
<td>0.64</td>
<td>2.27</td>
</tr>
<tr>
<td>PBS</td>
<td>0.50</td>
<td>0.13</td>
<td>0.37</td>
<td>0.06</td>
<td>0.37</td>
<td>1.53</td>
</tr>
<tr>
<td>Education</td>
<td>0.53</td>
<td>0.08</td>
<td>0.40</td>
<td>0.17</td>
<td>0.25</td>
<td>1.77</td>
</tr>
<tr>
<td>Arts</td>
<td>0.38</td>
<td>0.18</td>
<td>0.44</td>
<td>0.07</td>
<td>0.21</td>
<td>1.92</td>
</tr>
<tr>
<td>Other services</td>
<td>0.49</td>
<td>0.13</td>
<td>0.38</td>
<td>0.04</td>
<td>0.37</td>
<td>2.33</td>
</tr>
</tbody>
</table>
Table 11: The correlation between sector specific factors

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_i$</th>
<th>$\beta_i$</th>
<th>$\theta_i$</th>
<th>$\phi_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstreamness</td>
<td>-0.34</td>
<td>-0.32</td>
<td>-0.51</td>
<td>0.12</td>
</tr>
<tr>
<td>intermediate share</td>
<td>-0.47</td>
<td>-0.15</td>
<td>-0.26</td>
<td></td>
</tr>
<tr>
<td>capital share</td>
<td>0.69</td>
<td>0.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>trade credit</td>
<td></td>
<td></td>
<td>-0.19</td>
<td></td>
</tr>
</tbody>
</table>

This table presents the correlation between each two of the sector specific factors of the 14 industries (at the 2-digit NAICS level).

Table 12: the correlation of sectoral output changes after negative aggregate shocks and sector-specific factors (short term)

<table>
<thead>
<tr>
<th>shocks</th>
<th>downstreamness</th>
<th>intermediate share</th>
<th>capital share</th>
<th>trade credit</th>
<th>leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>finance</td>
<td>-0.62</td>
<td>-0.06</td>
<td>0.88</td>
<td>0.67</td>
<td>0.28</td>
</tr>
<tr>
<td>technology</td>
<td>0.51</td>
<td>-0.79</td>
<td>-0.12</td>
<td>-0.28</td>
<td>0.02</td>
</tr>
<tr>
<td>monetary</td>
<td>-0.59</td>
<td>0.00</td>
<td>0.83</td>
<td>0.72</td>
<td>-0.04</td>
</tr>
<tr>
<td>capital quality</td>
<td>0.42</td>
<td>0.16</td>
<td>-0.92</td>
<td>-0.67</td>
<td>-0.45</td>
</tr>
<tr>
<td>government expenditure</td>
<td>-0.44</td>
<td>-0.11</td>
<td>0.92</td>
<td>0.72</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table 13: the correlation of sectoral output changes after negative aggregate shocks and sector-specific factors (long term)

<table>
<thead>
<tr>
<th>shocks</th>
<th>downstreamness</th>
<th>intermediate share</th>
<th>capital share</th>
<th>trade credit</th>
<th>leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>finance</td>
<td>-0.32</td>
<td>-0.18</td>
<td>0.86</td>
<td>0.40</td>
<td>0.64</td>
</tr>
<tr>
<td>technology</td>
<td>0.40</td>
<td>-0.96</td>
<td>0.34</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>monetary</td>
<td>-0.57</td>
<td>0.20</td>
<td>0.50</td>
<td>0.75</td>
<td>-0.55</td>
</tr>
<tr>
<td>capital quality</td>
<td>0.44</td>
<td>0.15</td>
<td>-0.93</td>
<td>-0.67</td>
<td>-0.38</td>
</tr>
<tr>
<td>government expenditure</td>
<td>-0.44</td>
<td>-0.11</td>
<td>0.92</td>
<td>0.72</td>
<td>0.27</td>
</tr>
</tbody>
</table>
A Appendix: Simple Network Economics

A.1 Sectoral output

The equilibrium sectoral outputs of the DP-network economy are,

\[ m_1^{dp} = R_1^{-1} \zeta_1 z_1 v_1^{dp} \]
\[ m_2^{dp} = R_2^{-\beta_2} R_1^{-(1-\beta_2)} \zeta_2 z_2^{-1-\beta_2} v_2^{dp} \]
\[ m_3^{dp} = R_3^{\beta_2} R_2^{-\beta_2(1-\beta_3)} R_1^{-1-\beta_2(1-\beta_3)} \zeta_3 z_3^{-1-\beta_2} z_1^{-1-\beta_2} v_3^{dp} , \]

The equilibrium sectoral outputs of the CIA-network economy are,

\[ m_1^{cia} = \left[ \frac{\zeta_1}{R_1} + \frac{(1-\beta_2)\zeta_2}{R_2} + \frac{(1-\beta_2)(1-\beta_3)\zeta_3}{R_3} \right] \zeta_1 \]
\[ m_2^{cia} = \left[ \frac{\zeta_2}{R_2} + \frac{(1-\beta_3)\zeta_3}{R_3} \right] \zeta_2 z_2^{-1-\beta_2} \]
\[ m_3^{cia} = R_3^{-1} \zeta_3 z_2^{-1-\beta_2} z_1^{-1-\beta_2(1-\beta_3)} \zeta_3 \]

The equilibrium sectoral outputs of the TC-network economy are,

\[ m_1^{tc} = \frac{z_1}{R_1} \left[ \zeta_1 + (v_1^{tc} - \zeta_1)[\theta_2 + (1-\theta_2)R_1] \right] \]
\[ = \frac{z_1}{R_1} \left\{ \zeta_1 + (1-\beta_2) \left[ \frac{\theta_2 + (1-\theta_2)R_1}{\theta_2 + (1-\theta_2)R_2} \right] \left[ \zeta_2 + (1-\beta_3) \frac{\zeta_3}{\theta_3 + (1-\theta_3)R_3} \right] \right\} \]
\[ m_2^{tc} = z_2 z_1^{-1-\beta_2} \beta_2 (1-\beta_2)^{1-\beta_2} R_1^{-1-\beta_2} R_2^{-\beta_2} \left[ \frac{\theta_2 + (1-\theta_2)R_1}{\theta_2 + (1-\theta_2)R_2} \right]^{-1-\beta_2} \left\{ \zeta_2 + (v_2^{tc} - \zeta_2)[\theta_3 + (1-\theta_3)R_2] \right\} \]
\[ m_3^{tc} = z_3 z_2^{-1-\beta_2} z_1^{-1-\beta_2} \beta_3 (1-\beta_3)^{1-\beta_3} R_1^{-1-\beta_2(1-\beta_3)} R_2^{-\beta_2} \left[ \frac{\theta_2 + (1-\theta_2)R_1}{\theta_2 + (1-\theta_2)R_2} \right]^{-1-\beta_2} \left[ \zeta_2 + (1-\beta_3) \frac{\zeta_3}{\theta_3 + (1-\theta_3)R_3} \right] \]
\[ R_1^{-1-\beta_2(1-\beta_3)} R_2^{-\beta_2(1-\beta_3)} R_3^{-\beta_3} \left[ \frac{\theta_2 + (1-\theta_2)R_1}{\theta_2 + (1-\theta_2)R_2} \right]^{(1-\beta_2)(1-\beta_3)} \left[ \frac{\theta_3 + (1-\theta_3)R_2}{\theta_3 + (1-\theta_3)R_3} \right]^{1-\beta_3} \]
A.2 Propagation of Shocks

A.2.1 Demand shocks

Government expenditure is financed through lump-sum taxation \( T = \sum p_i g_i \) on households. Households’ utility is \( \log(C) - L \), where \( C = c_1 \xi_1 c_2 \xi_2 c_3 \xi_3 \) and \( \sum p_i c_i = wL - T \). Assume \( \beta_1 = 1 \), so that all the production functions are Cobb Douglas. By the zero profit condition,

\[
p_1 = \frac{w}{z_1}
\]

\[
p_2 = \frac{1}{z_2} \left( \frac{1}{\beta_2} \right)^{\beta_2} \left( \frac{1}{1 - \beta_2} \right)^{(1-\beta_2)} \frac{w^{\beta_2} p_1^{(1-\beta_2)}}{z_1 \Psi_1^{\beta_2}}
\]

\[
p_3 = \frac{1}{z_3} \left( \frac{1}{\beta_3} \right)^{\beta_3} \left( \frac{1}{1 - \beta_3} \right)^{(1-\beta_3)} \frac{w^{\beta_3} p_2^{(1-\beta_3)}}{z_2 \Psi_2^{\beta_3}}
\]

Normalize \( w \) to be 1. I have sectoral prices, \( p_1, p_2, p_3 \) are independent of the demand shock. Consumption \( \{c_1, c_2, c_3\} \) is independent of the demand shock. \( m_3 \) doesn’t change. \( \Delta m_2 \) equals \( \Delta g_2 \).

A.2.2 Financial shocks

In the CIA-network model, the elasticity of sectoral output with respect to the interest rate of firm 2 \( (R_2) \) are,

\[
\frac{\partial m_1^{cia}}{\partial R_2} \frac{R_2}{m_1^{cia}} = -(1 - \beta_2) \frac{\xi_2 z_1 \Psi_1}{R_2 m_1^{cia}}
\]

\[
\frac{\partial m_2^{cia}}{\partial R_2} \frac{R_2}{m_2^{cia}} = -\frac{\xi_2 z_2 z_1^{1-\beta_2} \Psi_2}{R_2 m_2^{cia}}
\]

\[
\frac{\partial m_3^{cia}}{\partial R_2} \frac{R_2}{m_3^{cia}} = 0,
\]
In the TC-network model, the elasticity of sectoral output with respect to the interest rate of firm 2 \(R_2\) are,

\[
\frac{\partial m_{tc}^1}{\partial R_2} \frac{R_2}{m_{tc}^1} = -\frac{(1 - \beta_2)[\theta_2 + (1 - \theta_2)R_1]}{\zeta_1 + (1 - \beta_2)[\theta_2 + (1 - \theta_2)R_1]} \left( \frac{\zeta_2}{R_2} + \frac{(1 - \beta_3)\zeta_3 \theta_3}{R_2 R_3} \right)
\]

\[
\frac{\partial m_{tc}^2}{\partial R_2} \frac{R_2}{m_{tc}^2} = -\frac{\zeta_2 + (1 - \beta_3)\zeta_3 \theta_3 / R_3}{\zeta_2 + (1 - \beta_3)\zeta_3 \theta_3 / R_3} \cdot \frac{\theta_3}{[\theta_3 + (1 - \theta_3)R_2]}
\]

\[
\frac{\partial m_{tc}^3}{\partial R_2} \frac{R_2}{m_{tc}^3} = -(1 - \beta_3) \frac{\theta_3}{[\theta_3 + (1 - \theta_3)R_2]}
\]

### B Appendix: Trade Credit

Trade credit is a short-term loan a supplier provides to its customer upon a purchase of its product. It is the single most important source of external finance for firms (Boissay 2007). Two types of trade credit rules are common. One type is a one part contract, Net-30. Suppliers give buyer 30-day interest free loans. The other type is a two part contract, 2/10 Net 30. If customers pay within 10 days of delivery, then they qualify a 2% discount; otherwise they can pay up to 30 days after delivery.

Be aware that price may be in the form of intrinsic interest. Empirical evidence shows that the implicit interest rate in a trade credit agreement is usually very high as compared with the rates on bank credit. Petersen and Rajan (1994) for example, conservatively estimate trade credits cost and find it more expensive than 99.8% of the loans in the sample. The high interest rate on trade credit arises because of an insurance premium and a default premium. Suppliers provide financing help through trade credit or defer payments when firms have already exhausted their bank credit line.

Several theories have been put forth to explain why suppliers provide credit to customers even though those firms cannot get additional banking credit. On the one hand, suppliers would like to keep business relationship as the cost of losing customers is high. On the other hand, suppliers may have an information advantage over banks and have a comparative ad-
Table 14: reg QRF data

<table>
<thead>
<tr>
<th>Accounts Payable / Net Sales</th>
<th>Aaa bond spread</th>
<th>0.0025***</th>
<th>0.0025***</th>
<th>0.0026***</th>
<th>0.0026***</th>
<th>Bbb bond spread</th>
<th>0.0018***</th>
<th>0.0019***</th>
<th>0.0019***</th>
<th>0.0019***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inventories</td>
<td>0.0357</td>
<td></td>
<td></td>
<td></td>
<td>Inventories</td>
<td>0.0389</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Net working capital</td>
<td>-0.0382**</td>
<td>-0.0606***</td>
<td>-0.0683***</td>
<td></td>
<td>Net working capital</td>
<td>-0.0387**</td>
<td>-0.0635**</td>
<td>-0.0710***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Short term bank loan</td>
<td>0.1514</td>
<td>0.0967</td>
<td></td>
<td></td>
<td>Short term bank loan</td>
<td>0.1150</td>
<td>0.0607</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Long term bank loan</td>
<td>-0.3039**</td>
<td>-0.1044**</td>
<td></td>
<td></td>
<td>Long term bank loan</td>
<td>-0.1057**</td>
<td>-0.1063**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>0.3070***</td>
<td>0.3092***</td>
<td>0.3184***</td>
<td>0.3158***</td>
<td>Constant</td>
<td>0.3074***</td>
<td>0.3103***</td>
<td>0.3197***</td>
<td>0.3170***</td>
</tr>
</tbody>
</table>

Accounts payables are normalized by net sales, because total cost of goods sold is not available in QRF. Dependent variables are normalized by total asset.

vantage in liquidating collateral that the borrower may put up to secure the loan. Moreover, the transaction argument states that trade credit reduces the transaction cost of paying bills. Also, there are papers study the role of trade credit in mitigating supply chain moral hazard. This paper does not focus on explaining the existence and the optimal choice of trade credit, but it studies the impact of trade credit on the propagation of shocks.

Although trade credit is relatively stable over time, the primary regression exercise in Table 14 shows that firms increase the amount of trade credit slightly when the bank borrowing cost is high, when net working capital is low, when bank loan is low. It is consistent with the theory that suppliers extend credit when customers are in financial trouble. Calomiris (1995) for example shows that the extension of trade credit increases during financial crisis. This pattern would reinforce the upstream propagation effect during all kinds of shocks.

Moreover, a trade credit debtor in bankruptcy almost surely default on the claims held by its trade credits (Boissay and Gropp(2007)). As shocks to the liquidity of some firms caused by the default of the customers, may in turn cause default or postponement of accounts payables on their suppliers and propagate upstream through the production chain. A distress at a single firm may induce a cascade of defaults throughout the production chain. The trade credit propagation mechanism will amplify the impact of idiosyncratic shocks on aggregate output. Thus, this mechanism creates a big multiplier effect and an extensive margin. Although this feature is not included in this paper, it is worth investigating in my future research. Even without trade credit defaults, my model predicts that financial linkages of firms enhance the upstream propagation of shocks.
C Appendix: General Model

C.1 Derivation of Philips’ Curve

For retailer $f$ the problem is,

$$\max \sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} \left( \frac{P_{ft}^*}{P_{t+i}} - P_{mt+i} \right) Y_{ft+i}$$

because $Y_{ft} = (\frac{P_{ft}}{P_t})^{-\epsilon} Y_t$

$$\max \sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} \left[ P_{ft}^{(1-\epsilon)} P_{t+i}^{(1-\epsilon)} - P_{mt+i} P_{ft}^{(1-\epsilon)} P_{t+i}^{(1-\epsilon)} \right] Y_{t+i}$$

F.O.C.$[(1-\epsilon)P_{ft}^{(-\epsilon)} P_{t+i}^{(-\epsilon)} + \epsilon P_{mt+i} P_{ft}^{(-\epsilon-1)} P_{t+i}^{(-\epsilon-1)}] Y_{t+i}$

$$\sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} [ P_{ft} P_{t+i}^{(1-\epsilon)} - \frac{\epsilon}{\epsilon - 1} P_{mt+i} P_{t+i}^{(1-\epsilon)} ] Y_{t+i}$$

$$P_{ft}^* = \frac{\epsilon}{\epsilon - 1} \frac{P_{mt+i}}{P_{t+i}} \left( P_{t+i}^{(1-\epsilon)} \right)^{\frac{\epsilon}{\epsilon - 1}} Y_{t+i}$$

or

$$\frac{P_{ft}^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{P_{mt+i}}{P_{t+i}} \left( \frac{P_{t+i}}{P_t} \right)^{\frac{\epsilon-1}{\epsilon}} Y_{t+i}$$

$$F_t = Y_t P_{mt} + \beta \gamma ( \Lambda_{t,t+1} P_{t+1}^{(1-\epsilon)} F_{t+1} )$$

$$Z_t = Y_t + \beta \gamma ( \Lambda_{t,t+1} P_{t+1}^{(1-\epsilon)} Z_{t+1}$$

If the nominal rigidities follows CEE. Each period a firm is able to freely adjust its price with probability $(1 - \gamma)$. In between these periods, the firm is able to index its price to the lagged rate of inflation.
For retailer $f$ the problem is,

$$\max \sum_{i=0}^{\infty} \beta_{i}^{t+i} \lambda_{t+i}(P_{ft}^{t} (\Pi_{k=1}^{i} \pi_{t+k-1}) - P_{mt+i}) Y_{t+i}$$

because $Y_{ft} = \left(\frac{P_{ft}}{P_{t}}\right)^{\epsilon} Y_{t}, Y_{ft} = \left(\frac{P_{ft}(\Pi_{k=1}^{i} \pi_{t+k-1})}{P_{t+i}}\right)^{\epsilon} Y_{t+i}$

$$\max \sum_{i=0}^{\infty} \beta_{i}^{t+i} \lambda_{t+i} \left[P_{ft}^{\epsilon-1} (\Pi_{k=1}^{i} \pi_{t+k-1}) - P_{mt+i} P_{ft}^{\epsilon-1} (\Pi_{k=1}^{i} \pi_{t+k-1}) P_{t+i}\right] Y_{t+i}$$

F.O.C. $[(1 - \epsilon) P_{ft}^{\epsilon-1} (\Pi_{k=1}^{i} \pi_{t+k-1}) + \epsilon P_{mt+i} P_{ft}^{\epsilon-1} (\Pi_{k=1}^{i} \pi_{t+k-1}) P_{t+i}] Y_{t+i}$

$$\sum_{i=0}^{\infty} \beta_{i}^{t+i} \lambda_{t+i} \left[P_{ft}^{\epsilon-1} (\Pi_{k=1}^{i} \pi_{t+k-1}) - \frac{\epsilon}{\epsilon - 1} P_{mt+i} (\Pi_{k=1}^{i} \pi_{t+k-1}) P_{t+i}\right] Y_{t+i}$$

$$P_{ft} = \frac{\epsilon}{\epsilon - 1} \sum_{i=0}^{\infty} \beta_{i}^{t+i} \lambda_{t+i} P_{mt+i} \left(P_{t+i}\right)^{\epsilon} (\Pi_{k=1}^{i} \pi_{t+k-1}) Y_{t+i}$$

or

$$P_{t}^{*} = \frac{\epsilon}{\epsilon - 1} \sum_{i=0}^{\infty} \beta_{i}^{t+i} \lambda_{t+i} \left(P_{t+i}\right)^{\epsilon-1} (\Pi_{k=1}^{i} \pi_{t+k-1}) Y_{t+i}$$

$$F_{t} = Y_{t} P_{mt} + \beta \gamma \lambda_{t+1} \pi_{t+1}^{\epsilon} \pi_{t}^{\gamma \pi} F_{t+1}$$

$$Z_{t} = Y_{t} + \beta \gamma \lambda_{t+1} \pi_{t+1}^{\epsilon-1} \pi_{t}^{(1-\epsilon) \gamma \pi} Z_{t+1}$$

Aggregate price level:

$$P_{t}^{1-\epsilon} = \int_{0}^{1} P_{ft}^{1-\epsilon} df$$

$$P_{t}^{1-\epsilon} = \int_{0}^{1-\gamma} P_{ft}^{(1-\epsilon)} df + \int_{1-\gamma}^{1} (P_{ft-1}^{\gamma \pi})^{(1-\epsilon)} df$$

$$P_{t}^{1-\epsilon} = (1 - \gamma) P_{ft}^{(1-\epsilon)} df + \int_{1-\gamma}^{1} (P_{ft-1}^{\gamma \pi})^{(1-\epsilon)} df$$

$$P_{t}^{1-\epsilon} = (1 - \gamma) P_{ft}^{(1-\epsilon)} + \gamma (P_{t-1}^{\gamma \pi})^{(1-\epsilon)}$$

66
Denote

$$\pi_t^* \equiv \frac{P_{ft}^*}{P_{t-1}}$$

$$\pi_t^{1-\epsilon} = (1-\gamma)\pi_t^{(1-\epsilon)} + \gamma(\pi_{t-1}^{p^*})^{(1-\epsilon)}$$

$$\pi_t^* = \frac{\epsilon}{1-\epsilon} Z_t \pi_t$$

C.2 Output loss from price dispersion

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} Y_t$$

$$Y_m = \int Y_{ft} df = \int \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} Y_t df = Y_t \int \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} df$$

$$D_t \equiv \int \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} df$$

$$D_t = \int_0^{1-\gamma} \left(\frac{P_{ft}}{P_{t-1}}\right)^{-\epsilon} df + \int_0^{\gamma} \left(\frac{P_{ft-1}^* \pi_{t-1}^{p^*}}{P_{t-1}}\right)^{-\epsilon} \left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon} df$$

$$D_t = (1-\gamma)\pi_t^{*\epsilon} - \epsilon + \gamma D_{t-1} \pi_t^{p^*\epsilon} - \epsilon$$

$$Y_m = D Y$$

C.3 Financial Intermediary

The intermediary balance sheet of banks in group $i$ is

$$Q_{it}S_{it} + L_{it} = N_{it} + B_{it}. \quad (50)$$

Their net worth evolves according to:

$$N_{it+1} = (R_{iL,t} - R_t)Q_{it}S_{it} + (R_{itL,t} - R_t) L_{it} + R_t N_{it} \quad (51)$$

\footnote{L_{it} could be negative. In that case, the working capital gain from trade is sufficient to fund labor cost and part of the capital cost. Firm will deposit it into banks with an interest rate of $R_{iL,t}$. Banks are indifferent between investing risky asset and investing safe asset.}
Thus,

\[ N_{it} = [(R_{ikt} - R_{t-1})d_{it-1}\phi_{t-1} + (R_{iLt} - R_{t-1})(1 - d_{it-1})\phi_{t-1} + R_{t-1}]N_{it-1}. \]

Denote \( z_{it-1,t} = \frac{N_{it}}{N_{it-1}} \) is the gross growth rate of net worth. Financial intermediary’s objective is to maximize:

\[ V_{it}(N_{it}) = \max E_t \sum_{j=0}^{\infty} (1 - \tau)^{j+1} \Lambda_{it+1+j}(N_{it+1+j}), \tag{52} \]

Thus, it can be written as,

\[ V_{it}(N_{it}) = \nu_{kit}Q_{it}S_{it} + \nu_{lit}L_{it} + \eta_{it}N_{it} \]

with

\[ \nu_{kit} = E_t[(1 - \tau)\beta \Lambda_{it+1}(R_{ikt} - R_{t}) + \beta \Lambda_{it+1}\tau x_{kit,t+1}\nu_{kit+1}], \]

\[ \nu_{lit} = E_t[(1 - \tau)\beta \Lambda_{it+1}(R_{iLt} - R_{t}) + \beta \Lambda_{it+1}\tau x_{lit,t+1}\nu_{lit+1}], \]

and

\[ \eta_{it} = E_t[(1 - \tau)\Lambda_{it+1}R_{t+1} + \beta \Lambda_{it+1}\tau z_{it,t+1}\eta_{it+1}]. \]

\( x_{kit,t+j} = \frac{Q_{i,t+j}S_{i,t+j}}{Q_{it}S_{it}} \) and \( x_{lit,t+j} = \frac{L_{i,t+j}}{L_{it}} \) are the gross growth rate in assets between \( t \) and \( t + j \), and \( z_{it,t+j} = \frac{N_{it+j}}{N_{it}} \) is the gross growth rate of net worth. \( \nu_{it} \) is the expected discounted marginal gain of expanding assets by a unit, holding net worth constant. It is an increasing function of the risk premium. \( \eta_{it} \) is the expected discounted value of having an additional unit of net worth, holding asset constant.

Given the binding incentive constraint,

\[ V_{it}(N_{it}) \geq \lambda_i(Q_{it}S_{it} + L_{it}), \]

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I have leverage ratio

\[ \phi_{it} = \frac{\eta_{it}}{\lambda_i - d_i \nu_{kit} - (1 - d_i) \nu_{lit}}. \] (53)

**C.4 Equilibrium Conditions**

**Household:**

\[ \varrho_t = (C_t - h C_{t-1})^{-1} - \beta h \mathbb{E}_t(C_{t+1} - h C_t)^{-1} \]

Euler equation:

\[ \mathbb{E}_t \beta \Lambda_{t,t+1} R_{t+1} = 1 \]

Stochastic discount rate:

\[ \Lambda_{t,t+1} \equiv \frac{\varrho_{t+1}}{\varrho_t} \]

Labor market equilibrium:

\[ \varrho_t w_t = \chi L_t^\psi \]

**Financial intermediary:**

Value of bank’s capital

\[ \nu_{kit} = \mathbb{E}_t[(1 - \tau) \beta \Lambda_{t,t+1}(R_{ikt} - R_t) + \beta \Lambda_{t,t+1} \tau x_{kit,t+1} \nu_{kit+1}], \]

\[ \nu_{lit} = \mathbb{E}_t[(1 - \tau) \beta \Lambda_{t,t+1}(R_{iLt} - R_t) + \beta \Lambda_{t,t+1} \tau x_{lit,t+1} \nu_{lit+1}], \]

Value of banks’ net wealth

\[ \eta_{it} = \mathbb{E}_t[(1 - \tau) \Lambda_{t,t+1} R_{t+1} + \beta \Lambda_{t,t+1} \tau z_{it,t+1} \eta_{it+1}]. \]

Optimal leverage

\[ \phi_{it} = \frac{\eta_{it}}{\lambda_i - d_i \nu_{kit} - (1 - d_i) \nu_{lit}} \]

Growth rate of banks’ capital

\[ z_{it,t+1} = [(R_{ikt} - R_{t-1})d_{it-1}\phi_{it-1} + (R_{iLt} - R_{t-1})(1 - d_{it-1})\phi_{it-1} + R_{t-1}] \]
Growth rate of banks’ net wealth

\[ x_{kit-1,t} = \frac{d_{it}\phi_{it}}{d_{it-1}\phi_{it-1}} z_{it,t+1} \]

\[ x_{lit-1,t} = \frac{(1 - d_{it})\phi_{it}}{(1 - d_{it-1})\phi_{it-1}} z_{it,t+1} \]

Aggregate capital, net worth

\[ Q_{it}S_{it} + L_{it} = \phi_{it}N_{it} \]

Risky asset share

\[ d_{it} = \frac{Q_{it}S_{it}}{Q_{it}S_{it} + L_{it}} \]

Banks’ net worth

\[ N_{it} = N_{eit} + N_{nit} \]

Existing banks’ net worth accumulation

\[ N_{eit} = \tau[(R_{ikt} - R_{t-1})d_{it-1}\phi_{it-1} + (R_{ilt-1} - R_{t-1})(1 - d_{it-1})\phi_{it-1} + R_{t-1}]N_{it-1}e_{Nei} \]

New banks’ net worth

\[ N_{nit} = \omega(Q_{it}S_{it-1} + L_{it-1}) \]

The return of the capital

\[ R_{ik,t+1} = e^{\psi_{k,t+1}} \frac{w_{it+1} + (1 - \delta)Q_{it+1}}{Q_{it}} \]

**Production sector**

Intermediate goods production function:

\[ m_{it} = z_{it}^{\alpha_{i}}k_{it}^{\alpha_{i}}(e^{\psi_{t}k_{it-1}})^{\beta_{i}}(\Pi_{t=1}^{n}m_{ij,t}^{\omega_{ij}})^{1-\alpha_{i}-\beta_{i}} \]

Labor demand

\[ w_{it}l_{it} = \alpha_{i}p_{l,t}m_{it} \]
Capital demand
\[ u_{it} e^{\psi_t} k_{it-1} = \beta_i p_{it} m_{it} \]

Intermediate goods demand
\[ \left[ \frac{[(1 - \theta_{ijt}) R_{iLt} + \theta_{ijt}] + \varsigma (\theta_{ijt} - \bar{\theta}_i)^2}{(1 - \theta_{ijt}) R_{jLt} + \theta_{ijt}} \right] p_{jt} m_{ijt} = \gamma_{ij} \omega_{ij} p_{it} m_{it} \]

Trade credit:
\[ 2 \varsigma (\theta_{ijt} - \bar{\theta}_i) [(1 - \theta_{ijt}) R_{jLt} + \theta_{ijt}] + (R_{jLt} - 1) \varsigma (\theta_{ijt} - \bar{\theta}_i)^2 = (R_{iLt} - R_{jLt}) \]

Wholesaler:
\[ Y_m = \Pi^n_{i=1} \zeta_i^{-c_i} m_i^{c_i} \]
\[ m_i = \zeta_i \left( \frac{p_{xi}}{P_m} \right)^{-1} Y_m \]

Capital goods producer
\[ Q_{it} = 1 + \frac{1}{2} \eta_t \left( \frac{I_{it}}{I_{it-1}} - 1 \right)^2 + \frac{I_{it}}{I_{it-1}} \eta_t \left( \frac{I_{it}}{I_{it-1}} - 1 \right) - \mathbb{E}_t [\beta \Lambda_{t+1} (I_{it+1} / I_{it})^2 \eta_t (I_{it+1} / I_{it} - 1)] \]

Retailer:
\[ Y = Y_m \]

Optimal price choice:
\[ F_t = Y_t P_{mt} + \beta \gamma \Lambda_{t,t+1} \Pi^t F_{t+1} \]
\[ Z_t = Y_t + \beta \gamma \Lambda_{t,t+1} \Pi^{t-1} Z_{t+1} \]

Price index
\[ 1 = \gamma \Pi^{t-1} + (1 - \gamma) \left( \frac{\epsilon}{\epsilon - 1} \frac{F_t}{Z_t} \right)^{1-\epsilon} \]

Capital accumulation equation
\[ k_{it} = e^{\psi_t (1 - \delta)} k_{it-1} + I_{it} \]
Goods market clearing:

\[ \sum_{j=1}^{n} m_{ij} + y_i = m_i \]

\[ Y_t = C_t + G_t + \sum_i [I_{it} + \frac{\eta_i}{2} \left( \frac{I_{it}}{I_{it-1}} - 1 \right)^2 I_t] \]

Labor market clearing:

\[ \sum_{i=1}^{n} l_i = L_t \]

Fisher equation

\[ i_t = R_{t+1} E_t (\pi_{t+1}) \]

Markup:

\[ X = 1/P_m \]

Interest rate rule:

\[ i_t = i_{t-1} \left( \frac{1}{\beta} \right)^{\kappa_\nu} \left( \frac{X}{c} \right)^{(1-\rho_\nu)} e_i \]

C.5 Key variables in the general model

influence vector

1. without trade friction and all key parameters are identical across bankers: \( v \) is time independent.

\[ p_{it} = \frac{\zeta_i Y_t}{y_{it}} \quad m_{jit} = \frac{\gamma_{ji} p_{jt} m_{jt}}{p_{it}} \]

\[ m_{it} = \sum m_{jit} + y_{it} \]

\[ p_{it} m_{it} = \sum p_{it} m_{jit} + p_{it} y_{it} \]

\[ p_{it} m_{it} = \sum \gamma_{ji} p_{jt} m_{jt} + \zeta_i Y_t \]

\[ \frac{\zeta_i Y_t}{y_{it}} m_{it} = \sum \gamma_{ji} \frac{\zeta_i Y_t}{y_{jt}} m_{jt} + \zeta_i Y_t \]

\[ v_{it} \equiv \frac{p_{it} m_{it}}{Y_t} = \frac{\zeta_i m_{it}}{y_{it}} \]
\[ v_{it} Y_t = \zeta_i Y_t + \sum \gamma_{ji} v_{jt} Y_t \]
\[ v_{it} = \zeta_i + \sum \gamma_{ji} v_{jt} \]
\[ v = \zeta + \Gamma v \]
\[ v = [I_N - \Gamma]^{-1} \zeta \]

2. with friction in trade: \( v \) is time varying.

**Intermediate input allocation**

\[ m_{jit} \equiv \varrho_{jit} m_{it} \quad \varrho_{it} = \sum \varrho_{jit} \quad y_{it} = (1 - \varrho_{it}) m_{it} \]
\[ \varrho_{ji} = \frac{\gamma_{ji} v_j}{v_i} \]

**Labor allocation**

\[ \alpha_i p_{it} m_{it} = w_t l_{it} \quad \sum \alpha_i p_{it} m_{it} = \sum w_t l_{it} \]
\[ \frac{\alpha_i p_{it} m_{it}}{l_{it}} = w_t = \sum \frac{\alpha_i p_{it} m_{it}}{L_t} \]
\[ l_{it} \equiv x^l_{it} L_t \]
\[ x^l_{it} = \frac{\alpha_i p_{it} m_{it}}{\sum \alpha_i p_{it} m_{it}} = \frac{\alpha_i v_{it}}{\sum \alpha_i v_{it}} \]

which is time independent if \( v \) is.

**Capital allocation**

\[ \beta_i p_{it} m_{it} = u_{it} e^{\psi_i k_{it-1}} \]
\[ \frac{e^{\psi_i k_{it-1}}}{e^{\psi_j k_{jt-1}}} = \frac{u_{jt} \beta_i v_{it}}{u_{it} \beta_j v_{jt}} \]
\[ x^k_{it-1} = \frac{\beta_i v_{it}/u_{it}}{\sum_i \beta_i v_{it}/u_{it}} \]

In the DP-network model (without friction in trade), \( u_{it} = u_{jt} \) and \( x^k_{it} \) is time independent. In the CIA- and TC-network models (with friction in trade), it is time varying.
Proof:
First consider steady state, solve the following system of equations in financial sector:

\[ \nu_{ki} = (1 - \tau)\beta(R_{ik} - R) + \beta\tau x_{ki}\nu_{ki} \]
\[ \nu_{li} = (1 - \tau)\beta(R_{iL} - R) + \beta\tau x_{li}\nu_{li} \]
\[ \eta_i = (1 - \tau)\beta R + \beta\tau z_i\eta_i \]
\[ \phi_i = \frac{\eta_i}{\lambda - d_i\nu_{ki} - (1 - d_i)\nu_{li}} \]
\[ z_i = (R_{ik} - R)d_i\phi_i + (R_{iL} - R)(1 - d_i)\phi_i + R \quad z_i = x_i \]
\[ N_{ni} = \omega(k_i + L_i) \]
\[ N_{ei} = \tau z_i N_i \]
\[ N_i = N_{ei} + N_{ni} \]

At equilibrium, \( R_{ik} = R_{iL}, x_{ki} = x_{li} = z_i \), so \( \nu_{ki} = \nu_{li} \). Thus,

\[ \nu_i = \frac{(1 - \tau)\beta(R_{ik} - R)}{1 - \beta\tau(R_{ik} - R)\phi - \beta\tau R} \]
\[ \eta_i = \frac{(1 - \tau)\beta R}{1 - \beta\tau(R_{ik} - R)\phi - \beta\tau R} \]
\[ \phi_i = \frac{\eta_i}{\lambda - \nu_i} \]
\[ N_i - \omega\tau(k_i + L_i) = \tau[(R_{ik} - R)\phi_i + R]N_i \]

using \( k_i + L_i = \phi_i N_i \), so we have \( 1 - \omega\tau\phi = \tau[(R_{ik} - R)\phi_i + R] \).

So there are four unknowns \( \nu_i, \eta_i, \phi_i, R_{ik} \) and four equations given \( R \). As long as parameters \( \lambda, \omega, \tau \) are the same across banks, the four unknowns are equal across sectors, \( R_{ki} = R_{kj} \Rightarrow u_i = u_j \)

Summary:
From the production function of firms, the log deviation of \( m_i \) from its steady state is,

\[
\tilde{m}_{it} = \alpha_i \tilde{z}_{it} + \alpha_i x^l_{it} \tilde{L}_t + \beta_i e^u_{it} + \beta_i x^k_{it} \tilde{K}_{t-1} + \sum \omega_{ij} \gamma_i \varrho_{ijt} \tilde{m}_{jt},
\]  

where \( \varrho_{ijt} = \frac{m_{ijt}}{m_{jt}} \), and I denote labor allocation as \( x^l_{it} \equiv \frac{l_{it}}{L_t} \) and capital allocation as \( x^k_{it} \equiv \frac{k_{it}}{K_t} \). In DP-network economy with homogeneous banking parameters (\( \lambda \) incentive constraint, \( \tau \) banker survival rate, \( \omega \) transfer ratio, \( \phi \) leverage), \( v_i, x^l_i, x^k_i, \varrho_{ij} \) are time independent. Thus, supply and financial shocks create strong downstream propagation\(^{18}\). However, in a DP-network economy with heterogeneous banking parameters (\( \lambda, \omega \), \( v_i, \varrho_{ij} \) are time independent but \( x^k_i \) and \( x^l_i \) are time dependent. The heterogeneity of financial intermediaries connected with each sector influence the sensitivity of sectoral output to shocks. Moreover, with financial linkages of firms (such as CIA-network and TC-network economies), \( v_{it}, x^l_{it}, x^k_{it}, \varrho_{it} \) are time dependent regardless of the conditions of banking parameters. All three types of shocks generate both upstream and downstream propagation.

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\(^{18}\)The last term in Equation 54 plays big role in the propagation of shocks.