New and Old Housing Markets, Term Structure and the Macroeconomy *

(Job Market Paper)

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Abstract

I construct a dynamic stochastic general equilibrium (DSGE) model in this paper to study the fluctuations in the U.S. housing markets. The model features a market for newly built houses, a secondary market for old houses, and an endogenous term structure of nominal interest rates. Negative technological progress in the housing sector explains the upward trend in house prices over the past four decades. Housing preference and technology innovations explain about 80% of the volatility of housing investment, real price of new houses, and the old-to-new house price ratio. Monetary factors explain about 15% of the volatility of housing investment, but do not significantly contribute to the price fluctuations of either new or old houses. The preference innovation to old houses is the leading determinant of the run-up in the price of old houses relative to the price of new houses during the 10-year period before the Great Recession. The term structure is endogenous in this paper, and the intertemporal preference innovation makes a non-negligible contribution to the variations in nominal interest rates. Housing market conditions do not contribute much to the fluctuations of interest rates, but significantly affect the shape of the yield curve.

Keywords: DSGE Models, Housing Markets, Term Structure, Monetary Policy.
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1 Introduction

Over the past several decades, the U.S. housing markets have experienced large fluctuations that move together with business cycles, which has stimulated a growing literature on understanding the dynamic interactions between the housing sector and the macroeconomy. In the present work, I construct a dynamic stochastic general equilibrium (DSGE) model that characterizes both a market for newly built houses and a secondary market for old (or existing) houses and endogenously incorporates the term structure of nominal interest rates, motivated by two stylized facts.

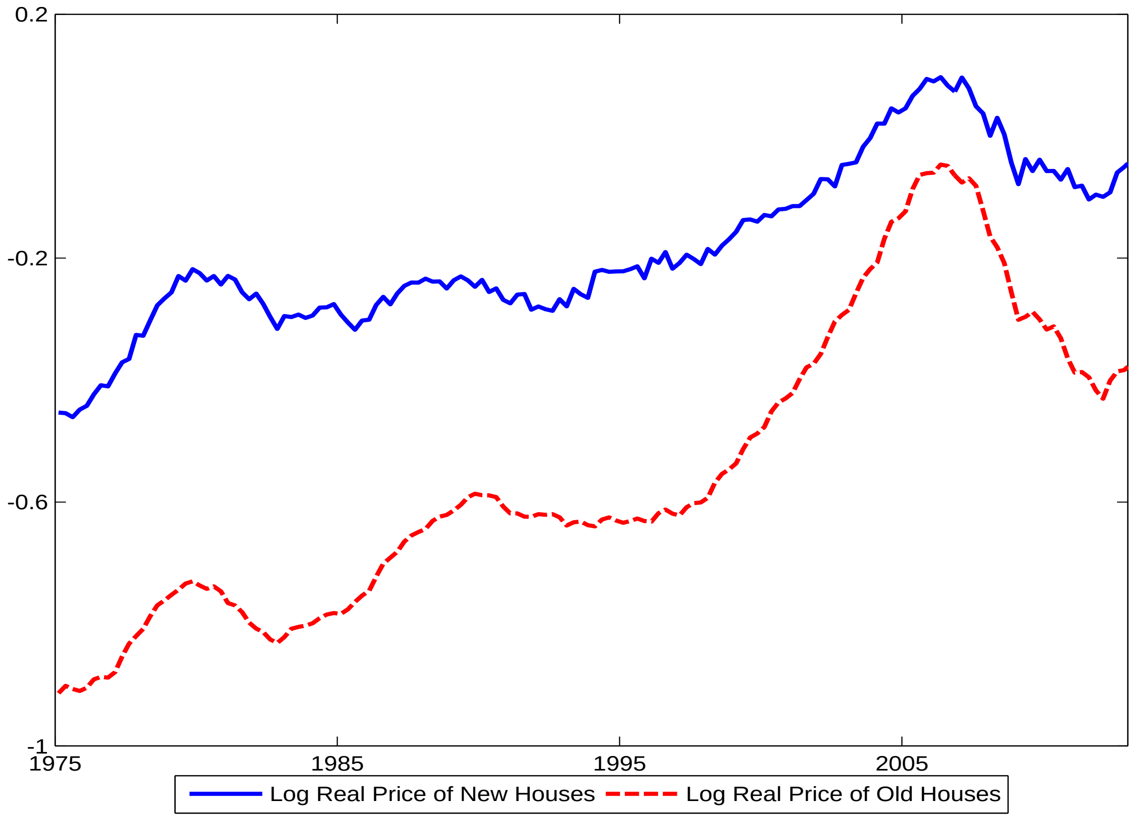


Figure 1: U.S. National House Prices

First, the secondary housing market is more than 10 times larger than the market of new houses in size. In particular, according to the statistics of the U.S. Census Bureau, 306,000, 368,000, and 430,000 units of new houses have been sold in 2011, 2012, and 2013, respectively. During the same years, the sales of existing houses are 3,787,000, 4,128,000, and 4,484,000 units, as indicated by the statistics of National Association of Realtors (NAR). More importantly, as shown in Figure 1, the gap between old house price and new house price shrinks over time during the 10 years prior to the Great Recession.[1] This paper aims at explaining the convergence between prices of old and new houses before the recent recession by explicitly modeling both a new and an old housing market.

Second, households usually borrow to purchase a house and carry the debt over a long period of time, usually 15 years or 30 years. Thus, the relevant borrowing rate is a long-term interest rate or mortgage rate, which ought to be distinguished from the short-term discount rate. In other words, the term structure of nominal interest rates plays a role in households' utility maximization problem and in the dynamics of housing markets. Thus, disturbances in the housing markets might in return affect the term structure. Incorporating the term structure into the DSGE framework enables us to learn more about its bidirectional relations with the housing markets.

Related Literature – Housing is one of the most important sectors over the business cycle. Before Maisel (1968), a substantial literature explores the effects of income and interest rates on the housing sector indicators, such as residential investment, taking the overall business cycle as given (see Ketchum (1954) and Alberts (1962)). However, development in the housing sector is not only a reflection of macroeconomic activities but also one of the driving forces of short-run economic fluctuations. In fact, the investment in the housing sector has been identified as an important channel through which monetary policy affects the economy (see Maisel (1968)) and the best precursor of the business cycle (see Leamer (2007)). More recently, Iacoviello and Neri (2010) apply a dynamic equilibrium model with nominal rigidities (see Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007)) to understanding sources and consequences of fluctuations in the U.S. housing market. Their model features heterogeneity in households' discount factors, a multi-sector structure with consumption and housing goods, nominal rigidities, financing frictions tied to the households, and a rich set of structural shocks. The model attributes most of the volatility of housing investment and house price over the sample period 1965:Q1-2006:Q4 to the housing demand shock and the housing technology shock, and identifies the spillovers from the housing

[1] There have been several indexes constructed for the U.S. house prices, among which the Quarterly Constant-Quality House Price Index by Census Bureau focuses on new houses, and the Quarterly Repeat-Transactions House Price Index by Freddie Mac on existing houses. Both of these two indexes are constructed after adjusting to the quality, including physical size, geographic location, housing type, climate, etc. To make them comparable, I fix the Quarterly Constant-Quality House Price Index for new houses and use the data on new and existing single family house prices of 2012 published by Census Bureau and NAR to normalize the Quarterly Repeat-Transactions House Price Index for existing houses. The comparable indexes (in logs) from the first quarter of 1975 (1975:Q1) to the last quarter of 2013 (2013:Q4), deflated with the implicit price deflator for the nonfarm business sector, are plotted in the figure.

market to the broad economy as a channel of propagation. My contribution to this literature is that I make the term structure of interest rates endogenous in the DSGE model and investigate the bidirectional relations between housing markets and the term structure.

The literature on the term structure of interest rates extends back to Vasicek (1977) and Cox, Ingersoll Jr and Ross (1985). In this literature, it is common to have latent factors driving the dynamics of the term structure of interest rates. These factors are usually interpreted as “level", “slope", and “curvature" factors. For example, Dai and Singleton (2000) construct a three-factor model of the term structure and provide a satisfactory fit of the data. However, the economic forces behind these latent factors are unclear before a number of recent studies focus on the relation between the term structure and macroeconomic dynamics. Ang and Piazzesi (2003) are among the first to incorporate macroeconomic factors in a term structure model. They show that a large fraction of the variations in bond yields can be explained by macro factors. Dewachter and Lyrio (2006) provide a macroeconomic interpretation for the latent factors: the level factor represents the long-run inflation expectation of agents, the slope factor captures business cycle conditions, and the curvature factor expresses an independent monetary policy factor. Hördahl, Tristani and Vestin (2006) and Rudebusch and Wu (2008) append the term structure to a set of standard macroeconomic aggregate relationships for output and inflation, and interpret the dynamics of yields and risk premia in terms of macroeconomic fundamentals. Bekaert, Cho and Moreno (2010) study the term structure in a New Keynesian model framework that features AS (Phillips Curve), IS (derived from utility maximization), and monetary policy equations. The effect of housing market dynamics on the term structure has never been considered, however. This paper contributes to the term structure literature by explicitly modeling housing markets. The transmission mechanism through which monetary policy changes lead to the housing market reactions in the presence of an endogenously adjusted term structure, and the question of whether and how changes in the housing markets affect the term structure are among the issues of interest here.

The second unique feature to the present work is that it explicitly models the secondary housing market; this has never been modeled in a dynamic equilibrium framework. In durable goods markets, however, there is a long-standing literature on the effects of secondary markets. For example, Anderson and Ginsburgh (1994), Hendel and Lizzeri (1999), and Johnson (2011) investigate the allocative effect of secondary markets. More recently, Chen, Esteban and Shum (2013) study different effects – substitution, allocative, and time consistency – of secondary markets in the U.S. automobile industry. They find that the existence of secondary markets is harmful to new car manufacturers and that transaction costs play a key role in secondary markets. This paper models a secondary market of old houses and examines the effects of transaction costs. Raising transaction costs reduces the role of the secondary market. In contrast, reducing transaction costs is able to make transactions in the secondary market frictionless. The optimal level of transaction costs that maintains the overall macroeconomic stability is also investigated. It is found that a reduction in transaction costs in the secondary housing market not only reduces the overall macroeconomic fluctuations but also improves social welfare.

The objective of this paper is to understand the dynamics of housing markets, by studying a DSGE model that features nominal rigidities, the term structure, and the secondary housing market. The rest of this paper proceeds as follows. Section 2 describes the model economy. Section 3 estimates the model using Bayesian methods and performs a variety of counterfactual analyses. Section 4 examines the effects of transaction costs on the secondary housing market and finds the optimal level of transaction costs. Section 5 discusses the sources of housing market fluctuations. Section 6 gives a further discussion on the term structure. Section 7 concludes the paper.

2 The Model Economy

The model with housing spillovers builds on Iacoviello and Neri (2010), augmented with both new and old housing markets, as well as the term structure. On the demand side, there are two types of households: unconstrained (or lenders) and constrained (or borrowers), while constrained households face a borrowing constraint. On the supply side, the consumption sector combines capital and labor to produce consumption goods and business capital. The housing sector produces new houses combining business capital with labor and land. New houses enter the pool of old houses one period later and can be traded again in the secondary market. While the transaction of new houses is frictionless, matching buyers and sellers in the secondary market is costly.

2.1 Households

There is a continuum of measure 1 of agents in each of the unconstrained and constrained groups. The economic size of each group is measured by its wage share, which is determined by the parameter α in the production function. Within each group, a representative household maximizes the lifetime utility:

$$V = E_0 \sum_{t=0}^{\infty} (\beta G_C)^t z_t \left[\Gamma_C \ln(c_t - \epsilon c_{t-1}) + j_{o,t} \ln(h_{o,t}) + j_{n,t} \ln(h_{n,t}) - \frac{\tau_t}{1+\eta} \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{1+\eta}{1+\xi}} \right], \tag{2.1}$$

or

$$V' = E_0 \sum_{t=0}^{\infty} (\beta' G_C)^t z_t \left[\Gamma'_C \ln(c'_t - \epsilon' c'_{t-1}) + j_{o,t} \ln(h'_{o,t}) + j_{n,t} \ln(h'_{n,t}) - \frac{\tau_t}{1+\eta'} \left((n'_{c,t})^{1+\xi'} + (n'_{h,t})^{1+\xi'} \right)^{\frac{1+\eta'}{1+\xi'}} \right], \tag{2.2}$$

where variables without a prime refer to unconstrained households and those with a prime refer to constrained households. Consumption, working hours in the consumption sector, and working hours in the housing sector are denoted by c, n_c, and n_h, respectively. Owner-occupied old houses and new houses are denoted by h_o and h_n, the former of which is a stock variable whereas the later one is a flow variable.[2]

The utility function consists of three separable components: consumption, housing, and labor. First, in the consumption component, G_C is the growth rate of consumption along the balanced growth path; parameters ϵ and ϵ' measure the internal habits in consumption; scaling factors $\Gamma_C = (G_C - \epsilon)/(G_C - \beta\epsilon G_C)$ and $\Gamma'_C = (G_C - \epsilon')/(G_C - \beta'\epsilon' G_C)$ ensure that the marginal utilities of consumption are $1/c$ and $1/c'$ along the balanced growth path. Second, the housing component contains both old houses and new houses, which are perfect substitutes to each other. One can also think of it as a composite housing product of Cobb-Douglas form, $h_{o,t}^{j_{o,t}} h_{h,t}^{j_{n,t}}$, with a weight $j_{o,t}$ on old houses and $j_{n,t}$ on new houses. Preference shocks to two types of houses are measured by $j_{o,t}$ and $j_{n,t}$. Third, the specification of the disutility of labor allows for less than perfect labor mobility between sectors, characterized by parameters ξ, ξ', η, and η'. The term τ_t captures the shock to labor supply. Finally, z_t captures the shock to intertemporal preferences; the discount factors are β and β' which satisfy $\beta' < \beta$.

[2] The rental market is not modeled in the present work. Both old and new houses are owner-occupied.

The shock processes z_t, τ_t, $j_{o,t}$, and $j_{n,t}$ follow:

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_{z,t}, \tag{2.3}$$

$$\ln \tau_t = \rho_\tau \ln \tau_{t-1} + \varepsilon_{\tau,t}, \tag{2.4}$$

$$\ln j_{o,t} = (1-\rho_{j_o}) \ln j_o + \rho_{j_o} \ln j_{o,t-1} + \varepsilon_{j_o,t}, \tag{2.5}$$

$$\ln j_{n,t} = (1-\rho_{j_n}) \ln j_n + \rho_{j_n} \ln j_{n,t-1} + \varepsilon_{j_n,t}. \tag{2.6}$$

where ρ_z, ρ_τ, ρ_{j_o}, and ρ_{j_n} are autoregressive parameters; j_o and j_n are the steady-state values of preference shocks; ε_z, ε_τ, ε_{j_o}, and ε_{j_n} are independently and identically distributed innovations with variances σ_z^2, σ_τ^2, $\sigma_{j_o}^2$, and $\sigma_{j_n}^2$.

Unconstrained households accumulate capital and houses, and make loans to constrained households. They rent capital to firms, choose the capital utilization rates, and sell the remaining undepreciated capital. They maximize their lifetime utility, V, subject to the following budget constraint:

$$\begin{aligned}
&c_t + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + k_{b,t} + q_t[f_t(h_{o,t} - (1-\delta_h)(h_{o,t-1} + h_{n,t-1})) + h_{n,t}] + p_{l,t} l_t + \frac{R_{L,t-1} b_{t-1}}{\pi_t} + TC_t \\
=&\frac{w_{c,t} n_{c,t}}{X_{wc,t}} + \frac{w_{h,t} n_{h,t}}{X_{wh,t}} + DIV_t + \left(R_{c,t} z_{c,t} + \frac{1-\delta_{kc}}{A_{k,t}} \right) k_{c,t-1} + \left(R_{h,t} z_{h,t} + 1 - \delta_{kh} \right) k_{h,t-1} \\
&+ p_{b,t} k_{b,t} + b_t + (p_{l,t} + R_{l,t}) l_{t-1} - \Phi_t - \frac{a(z_{c,t}) k_{c,t-1}}{A_{k,t}} - a(z_{h,t}) k_{h,t-1}.
\end{aligned} \tag{2.7}$$

They choose consumption c_t, capital in the consumption sector $k_{c,t}$, capital $k_{h,t}$ and intermediate inputs $k_{b,t}$ in the housing sector, stock of old houses $h_{o,t}$ and flow of new houses $h_{n,t}$, land stock l_t, working hours $n_{c,t}$ and $n_{h,t}$, capital utilization rates $z_{c,t}$ and $z_{h,t}$, and borrowing b_t (or lending when it is negative) to maximize utility subject to the budget constraint in Equation (2.7). Intermediate inputs in the housing sector and land are priced at $p_{b,t}$ and $p_{l,t}$, respectively. The price of new houses is q_t. Old houses are priced at a fraction f_t of the new house price q_t. Loans are set in nominal terms and yield a riskless long-term nominal return of $R_{L,t}$. The term $A_{k,t}$ denotes the investment-specific technology shock. Real wages are denoted by $w_{c,t}$ and $w_{h,t}$, real rental rates by $R_{c,t}$ and $R_{h,t}$, and depreciation rates by δ_h, δ_{kc} and δ_{kh}. The terms $X_{wc,t}$ and $X_{wh,t}$ characterize the markup between the wage paid by the wholesale firm and the wage paid to the households in the consumption sector and the housing sector, respectively. The money inflation measures the change in the price of consumption goods, i.e., $\pi_t = P_t/P_{t-1}$, where P_t is the price of consumption goods at the retail level. Finally, DIV_t is a lump-sum profit from final good firms and from labor unions; TC_t stands for transaction costs in the secondary housing market; Φ_t is adjustment costs for capital; $z_{c,t}$ and $z_{h,t}$ are capital utilization rates in two production sectors; $a(z)$ is the convex cost of setting the capital utilization to z:

$$DIV_t = \frac{X_t - 1}{X_t} Y_t + \frac{X_{wc,t} - 1}{X_{wc,t}} w_{c,t} n_{c,t} + \frac{X_{wh,t} - 1}{X_{wh,t}} w_{h,t} n_{h,t}, \tag{2.8}$$

$$TC_t = \frac{\phi}{2G_H} \left(\frac{h_{o,t}}{h_{o,t-1}} - G_H \right)^2 h_{o,t-1} q_t f_t, \tag{2.9}$$

$$\Phi_t = \frac{\phi_{kc}}{2G_{KC}} \left(\frac{k_{c,t}}{k_{c,t-1}} - G_{KC} \right)^2 \frac{k_{c,t-1}}{G_{AK}^t} + \frac{\phi_{kh}}{2G_C} \left(\frac{k_{h,t}}{k_{h,t-1}} - G_C \right)^2 k_{h,t-1}, \tag{2.10}$$

$$a(z_{c,t}) = \widetilde{R}_C(\varpi z_{c,t}^2/2 + (1-\varpi) z_{c,t} + (\varpi/2 - 1)), \tag{2.11}$$

$$a(z_{h,t}) = R_h(\varpi z_{h,t}^2/2 + (1-\varpi) z_{h,t} + (\varpi/2 - 1)). \tag{2.12}$$

where G_H, G_Q, G_{KC}, and G_{AK} are growth rates associated with housing investment, house prices, capital in the consumption sector, and the investment-specific technology, respectively; $\widetilde{R}_c$ and R_h are the steady-state values of rental rates of the two types of capital (see more details in the appendix at the end of this paper); Y_t is the production of wholesale goods and X_t is the markup of final goods over wholesale goods. For later convenience, let $\zeta = \varpi/(1+\varpi)$.

Transaction Costs – It is worth a further discussion on the specification of transaction costs TC_t here. In the model, TC_t follows a quadratic form. An alternative specification would be a wedge that is proportional to the transaction of old houses, i.e.,

$$TC_t^w = \phi^w q_t f_t \left| h_{o,t} - (1-\delta_h)(h_{o,t-1} + h_{n,t-1}) \right|, \tag{2.13}$$

where ϕ^w is a parameter which measures the buying or selling cost as in Sommer, Sullivan and Verbrugge (2013). However, the sign of the transaction flow, $h_{o,t} - (1-\delta_h)(h_{o,t-1} + h_{n,t-1})$, is not determinate, since the discount factors β and β' are not different enough for the transaction in the secondary market to be unidirectional. This indeterminacy makes it impracticable to find the steady state in the case of wedge transaction costs.

Notice that the transaction flow can be detrended into $\widetilde{h}_{o,t} - ((1-\delta_h)/G_H)(\widetilde{h}_{o,t-1} + \widetilde{h}_{n,t-1})$, which is equal to $(1-(1-\delta_h)/G_H)\widetilde{h}_o - ((1-\delta_h)/G_H)\widetilde{h}_n$ in the steady state (more details are presented in the appendix). Because $\widetilde{h}_n$ is very small relative to $\widetilde{h}_o$, the transaction flow of old houses between two types of households is tiny around the steady state. The economy cannot get far away from the steady state without having two types of households trade old houses with each other through the secondary market. Off the steady-state path, households participate into the secondary market and incur quadratic transaction costs TC_t, which can be detrended into

$$\widetilde{TC}_t = \frac{\phi}{2}\left(\frac{\widetilde{h}_{o,t}}{\widetilde{h}_{o,t-1}} - 1\right)^2 \widetilde{h}_{o,t-1}\widetilde{q}_t f_t. \tag{2.14}$$

Next, I argue that the quadratic transaction costs are approximately equivalent to the progressive wedge transaction costs. As stated above, the economy stays around the steady state if households do not trade old houses via the secondary market. Suppose they choose to increase (decrease) their stock of old houses by $\Delta \times 100\%$ in period t, they need to buy (sell) about $\Delta \times 100\%$ of their previous period stock, $\widetilde{h}_{o,t-1}$, through the secondary market. The transaction costs in this case are:

$$\widetilde{TC}_t = \frac{\phi}{2}\left(\frac{\widetilde{h}_{o,t}}{\widetilde{h}_{o,t-1}} - 1\right)^2 \widetilde{h}_{o,t-1}\widetilde{q}_t f_t = \frac{\phi}{2}\Delta^2 \widetilde{h}_{o,t-1}\widetilde{q}_t f_t = \frac{\phi\Delta}{2}\left(\Delta\widetilde{h}_{o,t-1}\right)\widetilde{q}_t f_t, \tag{2.15}$$

where the term in parentheses, $\Delta\widetilde{h}_{o,t-1}$, stands for the transaction amount of old houses. As a result, the quadratic transaction costs in the model are equivalent to wedge transaction costs, where the cost rate $\phi\Delta/2$ is increasing in Δ, the magnitude of adjustment.

According to the model specification, both new and old houses depreciate at a constant rate of δ_h which is calibrated to be 0.01 later in Section 3. At this rate, new houses lose more than 75% of their value in 35 years and they can then be ignored or assumed to exit the market. Hence the pool of old houses contains properties at different ages, ranging from 1 quarter up to 35 years. In the steady state, the amount of old houses is equal across ages. It is implicitly assumed that households cannot identify the age of any particular old house. Instead, they purchase or sell an average of the pool, whose quality is constant over time around the steady state.

Constrained households do not accumulate capital and do not own finished good firms or land. Moreover, their maximum borrowing b'_t is given by the expected present value of their houses times the loan-to-value (LTV) ratio m:

$$
\begin{aligned}
& c'_t + q_t[f_t(h'_{o,t} - (1-\delta_h)(h'_{o,t-1} + h'_{n,t-1})) + h'_{n,t}] + \frac{R_{L,t-1} b'_{t-1}}{\pi_t} + TC'_t \\
=& \frac{w'_{c,t} n'_{c,t}}{X_{wc,t}} + \frac{w'_{h,t} n'_{h,t}}{X_{wh,t}} + DIV'_t + b'_t, \qquad (2.16) \\
& b'_t \leq m E_t \left(\frac{f_{t+1} q_{t+1} (1-\delta_h)(h'_{o,t} + h'_{n,t}) \pi_{t+1}}{R_{S,t}} \right), \qquad (2.17)
\end{aligned}
$$

where $R_{S,t}$ is a short-term discount rate and

$$DIV'_t = \frac{X'_{wc,t} - 1}{X'_{wc,t}} w'_{c,t} n'_{c,t} + \frac{X'_{wh,t} - 1}{X'_{wh,t}} w'_{h,t} n'_{h,t}, \qquad (2.18)$$

$$TC'_t = \frac{\phi'}{2G_H} \left(\frac{h'_{o,t}}{h'_{o,t-1}} - G_H \right)^2 h'_{o,t-1} q_t f_t. \qquad (2.19)$$

Given the assumption $\beta' < \beta$, the borrowing constraint (2.17) holds with equality around the steady state for small shocks.

2.2 Production Technologies

Wholesale firms hire capital and labor, and purchase intermediate goods to produce wholesale goods Y_t and new houses IH_t. They seek to maximize their profit:

$$\max \frac{Y_t}{X_t} + q_t IH_t - \left(\sum_{i=c,h} w_{i,t} n_{i,t} + \sum_{i=c,h} w'_{i,t} n'_{i,t} + R_{c,t} z_{c,t} k_{c,t-1} + R_{h,t} z_{h,t} k_{h,t-1} + p_{b,t} k_{b,t} + R_{l,t} l_{t-1} \right),$$

where X_t is the markup of final goods over wholesale goods. The production technologies in two sectors are specified as:

$$Y_t = [A_{c,t}(n^{\alpha}_{c,t} n'^{1-\alpha}_{c,t})]^{1-\mu_c} (z_{c,t} k_{c,t-1})^{\mu_c}, \qquad (2.20)$$

$$IH_t = [A_{h,t}(n^{\alpha}_{h,t} n'^{1-\alpha}_{h,t})]^{1-\mu_h-\mu_b-\mu_l} (z_{h,t} k_{h,t-1})^{\mu_h} k^{\mu_b}_{b,t} l^{\mu_l}_{t-1}. \qquad (2.21)$$

In Equation (2.20), the consumption sector produces output with labor and capital. In Equation (2.21), new houses are produced with labor, capital, intermediate input, and land. The size of land is assumed to be fixed over time and normalized to one. The terms $A_{c,t}$ and $A_{h,t}$ measure productivity in the consumption sector and in the housing sector, respectively. The parameter α measures the labor income share of unconstrained households.

2.3 Price and Wage Rigidities

The model allows for sticky price in the consumption sector and sticky wages in both sectors. Price stickiness in the consumption sector is introduced by assuming monopolistic competition at the retail level, implicit costs of adjusting nominal prices following Calvo-style contracts (see Calvo (1983)), and partial indexation to lagged inflation of those prices that cannot be re-optimized. The resulting Phillips curve is:

$$\ln \pi_t - \iota_\pi \ln \pi_{t-1} = \beta G_C (E_t \ln \pi_{t+1} - \iota_\pi \ln \pi_t) - \frac{(1-\theta_\pi)(1-\beta G_C \theta_\pi)}{\theta_\pi} \ln (X_t / X) + \varepsilon_{p,t}, \quad (2.22)$$

where θ_π is the fraction of retailers that cannot set prices optimally but instead index prices to the previous period inflation.

Wage stickiness is modeled similarly. The Calvo-style pricing with partial indexation to inflation in the previous period yields the following wage Phillips curves:

$$\ln \omega_{i,t} - \iota_{wi} \ln \pi_{t-1} = \beta G_C (E_t \ln \omega_{i,t+1} - \iota_{wi} \ln \pi_t) - \frac{(1-\theta_{wi})(1-\beta G_C \theta_{wi})}{\theta_{wi}} \ln \left(\frac{X_{wi,t}}{X_{wi}} \right), \quad (2.23)$$

$$\ln \omega'_{i,t} - \iota_{wi} \ln \pi_{t-1} = \beta' G_C (E_t \ln \omega'_{i,t+1} - \iota_{wi} \ln \pi_t) - \frac{(1-\theta_{wi})(1-\beta' G_C \theta_{wi})}{\theta_{wi}} \ln \left(\frac{X_{wi,t}}{X_{wi}} \right), \quad (2.24)$$

where $i = c, h$, which denote the consumption sector and the housing sector, respectively. The parameter θ_{wi} characterizes the wage stickiness in sector i; $X_{wi,t}$ is the corresponding wage markup; and $\omega_{i,t}$ is the nominal wage inflation in each sector, i.e., $\omega_{i,t} = \pi_t w_{i,t} / w_{i,t-1}$, where $w_{i,t}$ is the real wage.

2.4 Monetary Policy

Following the term structure literature, I assume that yields of different maturities are driven by two latent factors L_t and S_t:

$$\ln R_{j,t} = \lambda_j + \mathbf{\Lambda}_j \mathbf{F}_t, \tag{2.25}$$

where $\ln R_{j,t}$ is the yield of maturity j periods, $j = 1, ..., J$; λ_j is a constant and $\mathbf{F}_t = (L_t, S_t)'$. The factor loadings on these two yield curve components, $\mathbf{\Lambda}_j$, are modeled as:

$$\mathbf{\Lambda}_j = \left(1, \quad \frac{1 - e^{-\delta j}}{\delta j}\right), \tag{2.26}$$

where δ denotes a decay parameter. The loadings on the first factor, L_t, are constant across the maturity spectrum. A positive shock to this factor induces an essentially parallel shift in the term structure that boosts the level of the whole yield curve, so the L_t factor is often called a "level" factor. The loadings on the second factor, S_t, decrease monotonically with the maturity. A positive shock to this factor increases short-term yields by much more than the long-term yields, thus S_t is usually called the "slope" factor. This setup is similar to the Nelson-Siegel term structure model (see Nelson and Siegel (1987)), without considering the "curvature" factor.[3]

The dynamics of the these latent factors are specified as:

$$L_t = \gamma_L L_{t-1} + (1 - \gamma_L) \ln \pi_t + \varepsilon_{L,t}, \tag{2.27}$$

$$S_t = \gamma_S S_{t-1} + (1 - \gamma_S)\gamma_\pi \ln \pi_t + (1 - \gamma_S)\gamma_Y \ln \left(\frac{GDP_t}{G_C GDP_{t-1}}\right) + \varepsilon_{S,t}, \tag{2.28}$$

where $\varepsilon_{L,t}$ and $\varepsilon_{S,t}$ are independently and identically distributed shocks to the level factor and to the slope factor respectively with variances σ_L^2 and σ_S^2. Here, GDP_t is defined as the sum of the value added of two sectors.

[3]In the Nelson-Siegel model, the loadings on the "curvature" factor follow a hump-shaped pattern and are close to zero for short and long maturities but larger for intermediate maturities. I do not explicitly consider this factor but, later in the estimation, impose measurement errors to the yields of intermediate maturities. Imposing measurement errors partly takes care of this issue.

In Equation (2.27), the level factor L_t is interpreted as the underlying rate of inflation, as in Rudebusch and Wu (2008). This is actually a common interpretation in the recent macro-finance literature, such as Kozicki and Tinsley (2001), Dewachter and Lyrio (2006), and Hördahl, Tristani and Vestin (2006). Equation (2.28) is a classic Taylor rule, indicating that the slope factor S_t reacts to its own lag, inflation rate, and output growth.

2.5 Equilibrium

The consumption sector produces consumption goods, business investment, and intermediate inputs. The housing sector produces new houses. Old houses are traded in the secondary market directly between unconstrained and constrained households. The equilibrium conditions are:

$$C_t + IK_{c,t}/A_{k,t} + IK_{h,t} + k_{b,t} = Y_t - \Phi_t - (TC_t + TC'_t), \tag{2.29}$$

$$h_{n,t} + h'_{n,t} = IH_t, \tag{2.30}$$

$$(h_{o,t} - (1-\delta_h)(h_{o,t-1} + h_{n,t-1})) + \left(h'_{o,t} - (1-\delta_h)(h'_{o,t-1} + h'_{n,t-1})\right) = 0, \tag{2.31}$$

$$b_t + b'_t = 0, \tag{2.32}$$

where $C_t = c_t + c'_t$, $IK_{c,t} = k_{c,t} - (1-\delta_{kc})k_{c,t-1}$, and $IK_{h,t} = k_{h,t} - (1-\delta_{kh})k_{h,t-1}$. Equations (2.29) through (2.32) characterize the clearing conditions of the consumption good market, the new housing market, the old housing market, and the loan market (Walras' law).

2.6 Linear Deterministic Trends

Given the fact that historical data on consumption, residential investment, and house prices all exhibit linear trends, this model allows for linear deterministic trends in the technologies A_c, A_h, and A_k. Let the corresponding gross growth rates be respectively γ_{AC}, γ_{AH}, and γ_{AK}, i.e.,

$$\ln A_{c,t} = t\ln(1+\gamma_{AC}) + \ln Z_{c,t}, \text{ where } \ln Z_{c,t} = \rho_{AC}\ln Z_{c,t-1} + \varepsilon_{c,t}, \tag{2.33}$$

$$\ln A_{h,t} = t\ln(1+\gamma_{AH}) + \ln Z_{h,t}, \text{ where } \ln Z_{h,t} = \rho_{AH}\ln Z_{h,t-1} + \varepsilon_{h,t}, \tag{2.34}$$

$$\ln A_{k,t} = t\ln(1+\gamma_{AK}) + \ln Z_{k,t}, \text{ where } \ln Z_{k,t} = \rho_{AK}\ln Z_{k,t-1} + \varepsilon_{k,t}. \tag{2.35}$$

The terms $\varepsilon_{c,t}$, $\varepsilon_{h,t}$, and $\varepsilon_{k,t}$ are independently and identically innovations with zero mean and variances σ^2_{AC}, σ^2_{AH}, and σ^2_{AK}.

Because of the existence of these trends, the variables $Y_t, c_t, c'_t, k_{c,t}/A_{k,t}, k_{h,t}, q_t IH_t$ all grow at a common rate along the balanced growth path. The production function in the consumption sector infers that this common rate takes the following form:

$$\gamma_Y = \gamma_C = (1-\mu_c)\gamma_{AC} + \mu_c\gamma_{KC}. \tag{2.36}$$

Given $\gamma_Y = \gamma_{KC} - \gamma_{AK}$, it follows that

$$\gamma_Y = \gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK}, \tag{2.37}$$

$$\gamma_{KC} = \gamma_{AC} + \frac{1}{1-\mu_c}\gamma_{AK}, \tag{2.38}$$

$$\gamma_{IH} = (1-\mu_h-\mu_b-\mu_l)\gamma_{AH} + (\mu_h+\mu_b)\left(\gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK}\right), \tag{2.39}$$

$$\gamma_Q = (1-\mu_h-\mu_b)\left(\gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK}\right) - (1-\mu_h-\mu_b-\mu_l)\gamma_{AH}. \tag{2.40}$$

3 Empirical Results

3.1 Data Description

The sample period is 1975:Q1 to 2013:Q4, and 17 observables are included in the analysis: real consumption, real business investment, real residential investment, real price of new houses, the old-to-new house price ratio, inflation rate, working hours and wage inflation in the consumption sector, working hours and wage inflation in the housing sector, nominal 3-month interest rate, and zero-coupon bond yields of maturities 4, 12, 24, 36, 48, and 60 quarters.

Real consumption, real business investment, and real residential investment are obtained from Bureau of Economic Analysis (BEA), expressed in per-capita term; inflation rate is the implicit price deflator for the nonfarm business sector; real price of new houses is the deflated Census Bureau House Price Index whereas the price of old houses is the Freddie Mac Quarterly Repeat-Transactions House Price Index; hours and wage inflations are obtained from Bureau of Labor Statistics (BLS); nominal short-term interest rate is the 3-month Treasury Bill Rate at secondary market from Board of Governors of the Federal Reserve System; zero-coupon bond yields of various maturities are obtained from the Federal Reserve Board (see Gurkaynak, Sack and Wright (2007)).

3.2 Calibration

The discount factors (β and β'), the production function parameters (μ_c, μ_h, μ_b, and μ_l), the depreciation rates (δ_h, δ_{kc}, and δ_{kh}), the LTV ratio (m), the steady state values of preference shocks (j_o and j_n), and the steady state gross price and wage markups (X, X_{wc}, and X_{wh}) are all calibrated. The parameter values are shown in Table 1.

Table 1: Calibrated Parameters

Parameter	Value	Parameter	Value	Parameter	Value
β	0.98285	δ_h	0.01	X	1.15
β'	0.90	δ_{kc}	0.025	X_{wc}	1.15
μ_c	0.35	δ_{kh}	0.03	X_{wh}	1.15
μ_h	0.10	m	0.85		
μ_b	0.10	j_o	0.20		
μ_l	0.10	j_n	0.04		

The model implies a zero inflation rate, i.e., $\pi = 1$, in the steady state, due to the assumption in Equations (2.27) and (2.28) that the central bank targets a zero inflation. The discount factor of unconstrained households is set at $\beta = 0.98285$, implying a steady-state quarterly long-term nominal interest rate of around 1.745 percent, as the sample average of the 60-quarter zero-coupon bond yield. The discount factor of constrained households β' is instead fixed at 0.90 to guarantee an impatience motive for constrained households large enough that they are arbitrarily close to the borrowing limit (see Iacoviello (2005)). The choices $\mu_c = 0.35$, $\mu_h = 0.10$, $\mu_b = 0.10$, and $\mu_l = 0.10$ are same as those in Iacoviello and Neri (2010). The usual markup on prices, 1.15, is chosen for X, X_{wc}, and X_{wh} (see Corsetti, Kuester, Meier and Müller (2013)). The depreciation rates are set at $\delta_h = 0.01$, $\delta_{kc} = 0.025$, and $\delta_{kh} = 0.03$. The LTV ratio is set at $m = 0.85$.[4] The steady-state values of preference shocks are chosen to be $j_o = 0.20$ and $j_n = 0.04$. These choices, together with other estimated parameters, imply a ratio of housing wealth to annual GDP of about 1.4 and an old-to-new house price ratio of around 0.7, as the mean over the sample period.

3.3 Prior and Posterior Distributions

The prior distributions of the structural parameters and shock processes are presented in the left panel of Tables 2 and 3. The labor income share of unconstrained households α has a prior mean of 0.65 and a standard error of 0.05, as implied by Iacoviello (2005). The decay parameter δ has a prior mean of 0.06, as suggested by Diebold and Li (2006), and a relatively large standard error of 0.05. The consumption habit parameters (ϵ and ϵ'), the working disutility parameters (η and η'), the indexation parameters (ι_π, ι_{wc}, and ι_{wh}), the hour substitution parameters (ξ and ξ'), the utilization parameter (ζ), and the Calvo price and wage parameters (θ_π, θ_{wc}, and θ_{wh}) have the same prior distributions as in Iacoviello and Neri (2010). The transaction cost parameters (ϕ and ϕ') are assumed to have a Gamma-distributed prior with a mean of 5 and a relatively large standard error of 2.5. The adjustment cost parameters (ϕ_{kc} and ϕ_{kh}) are assumed to have a prior mean of 10 and a standard error of 2.5. The monetary policy parameters have similar priors to those in Christiano, Eichenbaum and Evans (2005). The technological growth rates (γ_{AC}, γ_{AH}, and γ_{AK}) all have a small prior mean of 0.005. All the autoregressive parameters (ρ_{AC}, ρ_{AH}, ρ_{AK}, ρ_{j_o}, ρ_{j_n}, ρ_τ, and ρ_z) have a Beta-distributed prior with mean 0.8 and standard error 0.1. All the structural shocks (σ_{AC}, σ_{AH}, σ_{AK}, σ_{j_o}, σ_{j_n}, σ_L, σ_S, σ_τ, σ_z, and σ_p) have an Inverse Gamma-distributed standard deviation with mean 0.1 and standard error 2, which corresponds to a rather loose prior. Measurement errors (σ_{n_h}, σ_{w_h}, σ_{R04}, σ_{R12}, σ_{R24}, σ_{R36}, and σ_{R48}) are also specified for the hours and the wage in the housing sector and the bond yields of intermediate maturities.

[4] The LTV ratio is actually varying over time. Data on the LTV ratio is indeed available at the Federal Housing Finance Agency (FHFA). However, this data does not distinguish between unconstrained and constrained households. While most households borrow through the financial markets in reality, the model assumes that a fraction α of the households do not borrow but lend to other households. Here, a sensible number of 0.85 is chosen for constrained households following Iacoviello and Neri (2010).

Table 2: Prior and Posterior Distribution of the Structural Parameters

Parameter	Prior Distribution			Posterior Distribution			
	Distribution	Mean	SD	Mean	5%	Median	95%
α	Beta	0.650	0.050	0.840	0.826	0.841	0.851
δ	Beta	0.060	0.050	0.013	0.011	0.013	0.016
ϵ	Beta	0.500	0.075	0.452	0.404	0.447	0.509
ϵ'	Beta	0.500	0.075	0.628	0.572	0.633	0.677
η	Gamma	0.500	0.100	0.641	0.444	0.606	0.825
η'	Gamma	0.500	0.100	0.512	0.399	0.505	0.613
ϕ	Gamma	5.000	2.500	0.104	0.022	0.092	0.188
ϕ'	Gamma	5.000	2.500	0.210	0.109	0.196	0.343
ϕ_{kc}	Gamma	10.000	2.500	23.131	20.820	22.875	27.503
ϕ_{kh}	Gamma	10.000	2.500	11.073	8.093	10.284	14.830
ι_{π}	Beta	0.500	0.200	0.838	0.747	0.833	0.918
ι_{wc}	Beta	0.500	0.200	0.065	0.024	0.065	0.105
ι_{wh}	Beta	0.500	0.200	0.509	0.293	0.491	0.776
ξ	Normal	1.000	0.100	0.828	0.744	0.827	0.929
ξ'	Normal	1.000	0.100	0.997	0.864	1.002	1.111
r_L	Beta	0.800	0.100	0.596	0.564	0.595	0.622
r_S	Beta	0.200	0.100	0.578	0.536	0.577	0.609
r_{π}	Normal	1.500	0.100	1.315	1.155	1.282	1.510
r_Y	Normal	0.200	0.100	0.800	0.703	0.806	0.901
θ_{π}	Beta	0.667	0.050	0.768	0.739	0.771	0.790
θ_{wc}	Beta	0.667	0.050	0.826	0.801	0.824	0.855
θ_{wh}	Beta	0.667	0.050	0.940	0.930	0.941	0.949
ζ	Beta	0.500	0.200	0.819	0.696	0.803	0.933
$100\gamma_{AC}$	Normal	0.500	1.000	0.312	0.295	0.312	0.326
$100\gamma_{AH}$	Normal	0.500	1.000	-0.042	-0.168	-0.050	0.058
$100\gamma_{AK}$	Normal	0.500	1.000	0.223	0.181	0.225	0.255

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

The right panels report the relevant posterior mean, median, and 90% probability intervals. The estimate of α, the labor income share of credit-unconstrained households, is 0.840, which implies a share of labor income accruing to the credit-constrained households of 16 percent. This value is close to the estimate obtained by Jappelli (1990), using the 1983 Survey of Consumer Finances. The estimate of the decay parameter $\delta = 0.013$ is less than the prior mean, most likely because the "curvature" factor of the term structure has been ignored in the estimation. Both unconstrained and constrained households exhibit a moderate degree of habit formation in consumption ($\epsilon = 0.452$ and $\epsilon' = 0.628$). The estimates of η and η', the labor supply elasticity, are close to their prior mean. The parameters ξ and ξ' are estimated to be 0.828 and 0.997, implying that hours in the two sectors are not perfect substitutes. The estimates of adjustment costs for capital are $\phi_{kc} = 23.131$ and

$\phi_{kh} = 11.073$. The estimate of ζ, the curvature of the capital utilization function, is 0.819, implying that $\varpi = 4.525$.

The transaction costs in the secondary market are characterized by $\phi = 0.104$ and $\phi' = 0.210$, indicating that unconstrained households incur a rate of 2.6% if they choose to change their stock of old houses by 50% while constrained households incur a higher rate of 5.25%. One explanation may be that since the later type of households are constrained, they are willing to adjust their housing stock at higher transaction costs.

Table 3: Prior and Posterior Distribution of the Shock Processes

	Prior Distribution			Posterior Distribution			
Parameter	Distribution	Mean	SD	Mean	5%	Median	95%
ρ_{AC}	Beta	0.800	0.100	0.895	0.867	0.894	0.922
ρ_{AH}	Beta	0.800	0.100	0.985	0.967	0.989	0.998
ρ_{AK}	Beta	0.800	0.100	0.970	0.957	0.970	0.989
ρ_{j_o}	Beta	0.800	0.100	0.971	0.956	0.972	0.982
ρ_{j_n}	Beta	0.800	0.100	0.967	0.956	0.967	0.975
ρ_{τ}	Beta	0.800	0.100	0.901	0.883	0.902	0.915
ρ_{z}	Beta	0.800	0.100	0.835	0.789	0.835	0.866
σ_{AC}	Inv.gamma	0.100	2.000	0.012	0.011	0.011	0.013
σ_{AH}	Inv.gamma	0.100	2.000	0.020	0.019	0.020	0.022
σ_{AK}	Inv.gamma	0.100	2.000	0.019	0.017	0.020	0.022
σ_{j_o}	Inv.gamma	0.100	2.000	0.039	0.030	0.038	0.049
σ_{j_n}	Inv.gamma	0.100	2.000	0.068	0.060	0.069	0.075
σ_{L}	Inv.gamma	0.100	2.000	0.009	0.008	0.009	0.009
σ_{S}	Inv.gamma	0.100	2.000	0.010	0.009	0.010	0.011
σ_{τ}	Inv.gamma	0.100	2.000	0.029	0.023	0.028	0.035
σ_{z}	Inv.gamma	0.100	2.000	0.016	0.014	0.016	0.018
σ_{p}	Inv.gamma	0.100	2.000	0.009	0.008	0.010	0.010
σ_{n_h}	Inv.gamma	0.100	2.000	0.240	0.218	0.241	0.255
σ_{w_h}	Inv.gamma	0.100	2.000	0.010	0.009	0.010	0.011
σ_{R04}	Inv.gamma	0.100	2.000	0.006	0.006	0.006	0.007
σ_{R12}	Inv.gamma	0.100	2.000	0.007	0.006	0.007	0.007
σ_{R24}	Inv.gamma	0.100	2.000	0.006	0.006	0.007	0.007
σ_{R36}	Inv.gamma	0.100	2.000	0.007	0.006	0.007	0.007
σ_{R48}	Inv.gamma	0.100	2.000	0.006	0.006	0.006	0.007

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

According to the estimation results, only 23.2 percent of the retailers are able to set prices optimally. A fraction $\theta_{\pi} = 0.768$ of retailers index prices to the previous period inflation rate with an elasticity equal to $\iota_{\pi} = 0.838$. As for wages, higher wage stickiness and larger wage indexation are found in the housing sector ($\theta_{wh} = 0.940$ versus $\theta_{wc} = 0.826$ and $\iota_{wh} = 0.509$ versus $\iota_{wc} = 0.065$).

Estimates of the monetary policy rule are in line with previous evidence.

It is also found that the consumption sector exhibits the fastest rate of technological progress, followed by that in business investment and by that in the housing sector. The estimates of $\gamma_{AC} = 0.312\%$ and $\gamma_{AK} = 0.223\%$ are close to those in Iacoviello and Neri (2010). However, the estimate of γ_{AH} is negative. This estimate, $\gamma_{AH} = -0.042\%$, is considerably lower than the value obtained by Iacoviello and Neri (2010) but is consistent with the literature.[5] According to the estimation of Corrado, Lengermann, Bartelsman and Beaulieu (2006), the Total Factor Productivity (TFP) in the construction sector has been decreasing at an annual rate of 0.5% over the period 1987-2004; the contribution of labor and purchased inputs more than account for the real output growth of the construction sector; on the contrary, the TFP in the private nonfarm business sector has been increasing at an annual rate of 1.2%. One possible explanation for the negative technological progress is that restrictions and regulations on land use raise expensive barriers to building and lower the efficiency with which housing services are provided to occupants, as suggested by Albouy and Ehrlich (2012).

3.4 Model-Implied Land Price

Given the estimates of γ_{AC}, γ_{AK}, and γ_{AH}, the growth rate of consumption is $\gamma_C = 0.44\%$. The model implies that real land price, $p_{l,t}$, is also growing at this rate. One might argue that the convergence between old and new house prices before the Great Recession is simply a result of the run-up in residential land price, because land accounts for a larger fraction of the value of old houses. In fact, since land is a factor of the housing production, the causality goes the other way from the housing demand to land price.

Though it is an important aspect of the housing markets, land price has been ignored due to lack of data in the literature until recently. Two sets of data on land price index at quarterly frequency are available at the Lincoln Institute of Land Policy. In both datasets, the land price data are derived from data on housing values and estimates of structure costs using price indexes

[5]The negative technological progress in the housing sector confines to residential structures. In nonresidential structures such as highways, airports, bridges, and skyscrapers, an annual technological progress of around 1% is found by Gort, Greenwood and Rupert (1999).

for housing and construction costs. The construction cost data are derived from publicly available data from the Bureau of Economic Analysis (BEA). The house price data are benchmarked to an estimate of the value of the stock of housing based on micro data from the 2000 Decennial Census of Housing and 2001 Residential Finance Survey, and are extrapolated forwards and backwards from the benchmark year using the Macromarkets LLC, formerly Case-Shiller-Weiss (CSW), repeat-sales index for the first data set, and the FHFA repeat-sales index for the second data set (see Davis and Heathcote (2007) for details). Both the CSW-based and the FHFA-based indexes suggest that real land price grows at a rate of 0.75% per quarter over 1975-2013. The model in the present work succeeds explaining about 60% of the land price growth.

Recently, Liu, Wang and Zha (2013) focus on modeling the comovements between land price and business investment. Based on a similar DSGE framework, they do not directly model the housing market but instead the land market, and interpret a shock to households' tastes for land as a housing demand shock. In addition, they assume that firms, instead of households, are credit constrained. Firms finance investment spending by using land as a collateral asset. Even they do not explicitly model the growth rate of land price, an implication of their specification is that land price is growing at the same rate as business investment, which is estimated to be $\gamma_{KC} = 0.68\%$ in the present work. In that case, the model would be able to explain more than 90% of the land price growth. This paper sticks with the current model specification for two reasons. First, land price is not among the primary concerns of the present work. Second, new and old houses cannot be distinguished from each other if households' demand for housing is simply modeled as their demand for land.

The deviation of land price from its trend growth implied by the model is plotted in Figure 2, together with the detrended FHFA-based and CSW-based land price indexes. Even when the estimation does not make use of the data on land price, the model replicates the fluctuations of land price over the sample period very well. This result reflects the fact that movements in land price are a result of house price fluctuations, which are in turn determined by demand and supply conditions. Learning the economic forces behind the fluctuations in the housing markets is among the objectives of this paper.

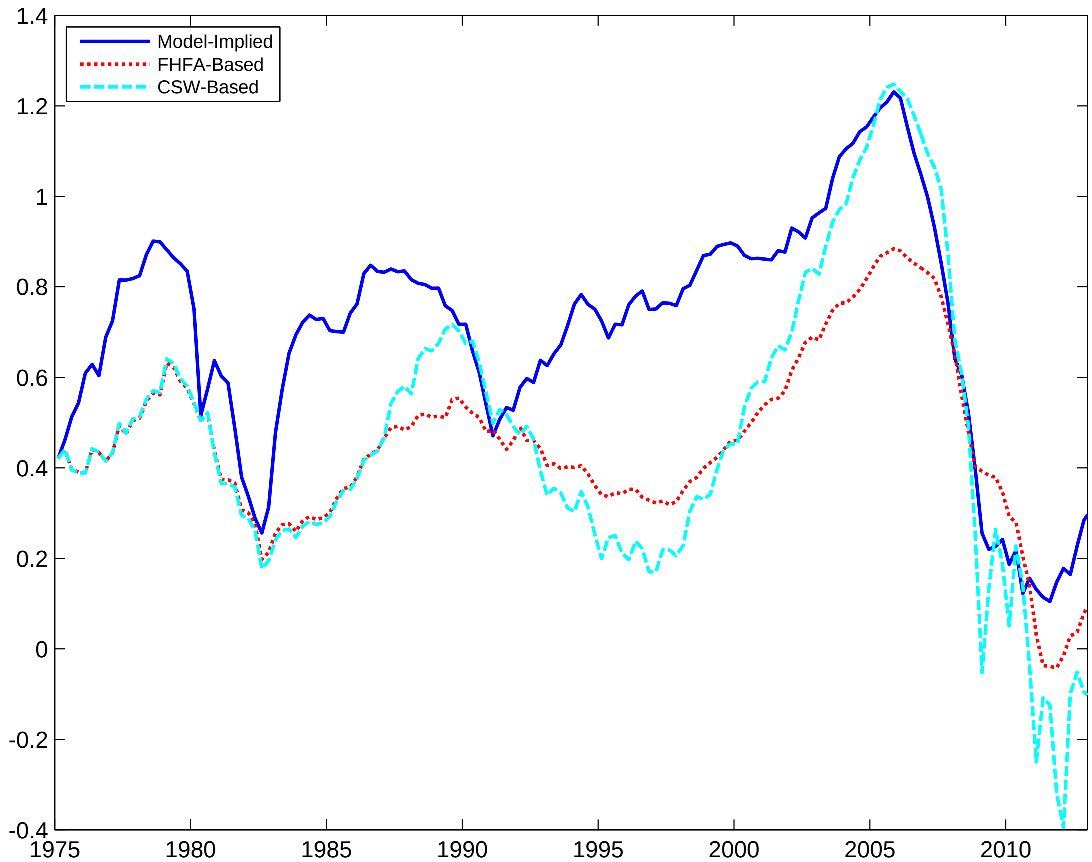


Figure 2: Land Price Fluctuations: A Comparison

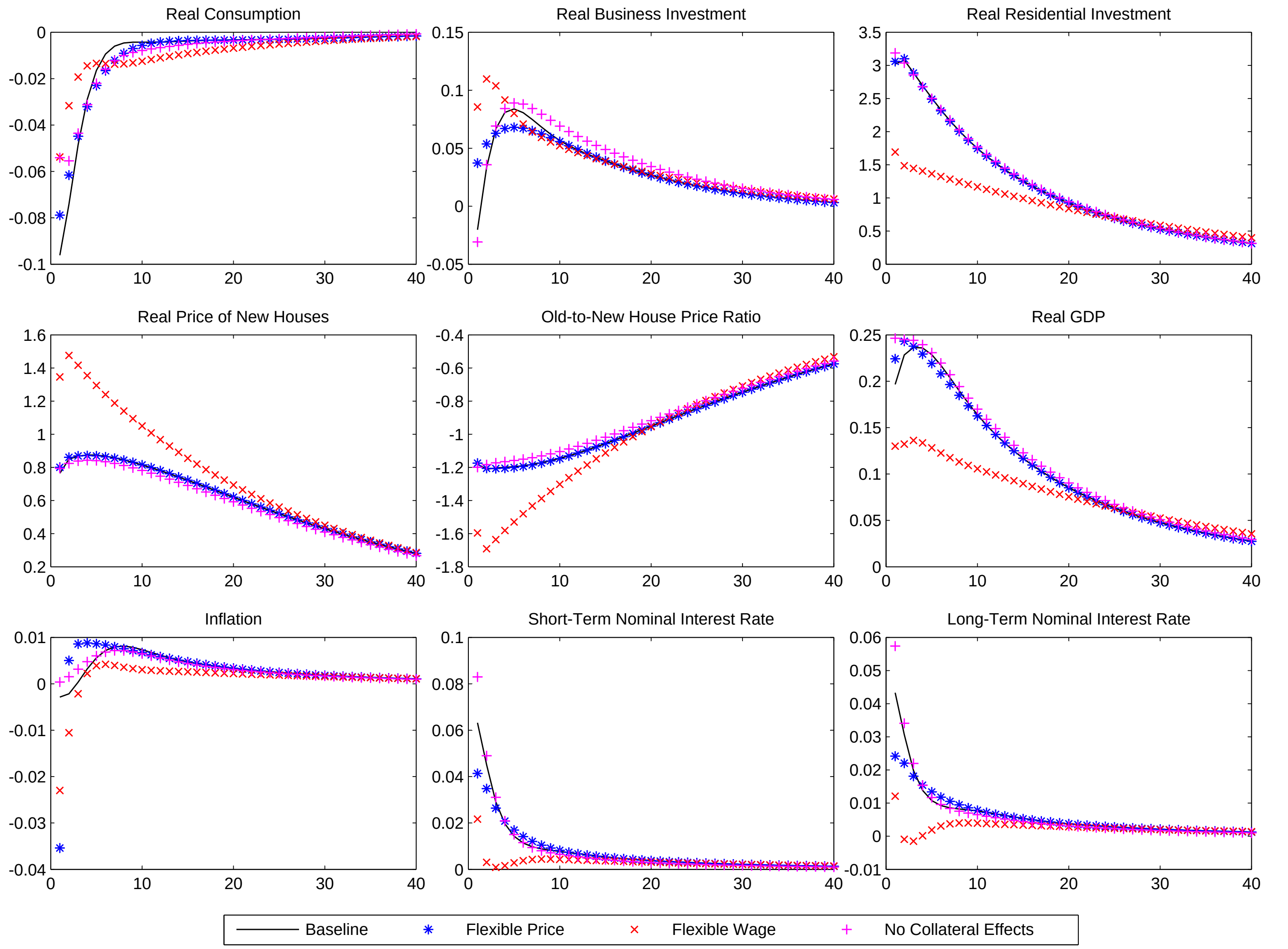
3.5 Impulse Responses

The impulse responses of nine key variables – real consumption, real business investment, real residential investment, real price of new houses, the old-to-new house price ratio, real GDP, inflation, short-term nominal interest rate, and long term nominal interest rate – to preference (or demand) innovations, term-structure (or monetary) innovations, and the housing technology innovation are plotted in Figures 3 to 7.

3.5.1 Preference Innovations

Figure 3 plots impulse responses to the estimated preference innovation in new houses. A positive preference innovation in new houses discourages consumption and encourages business investment in the short run. Due to such a demand innovation, real investment in the housing sector increases by a large magnitude and stays above the steady-state level for a long time period.

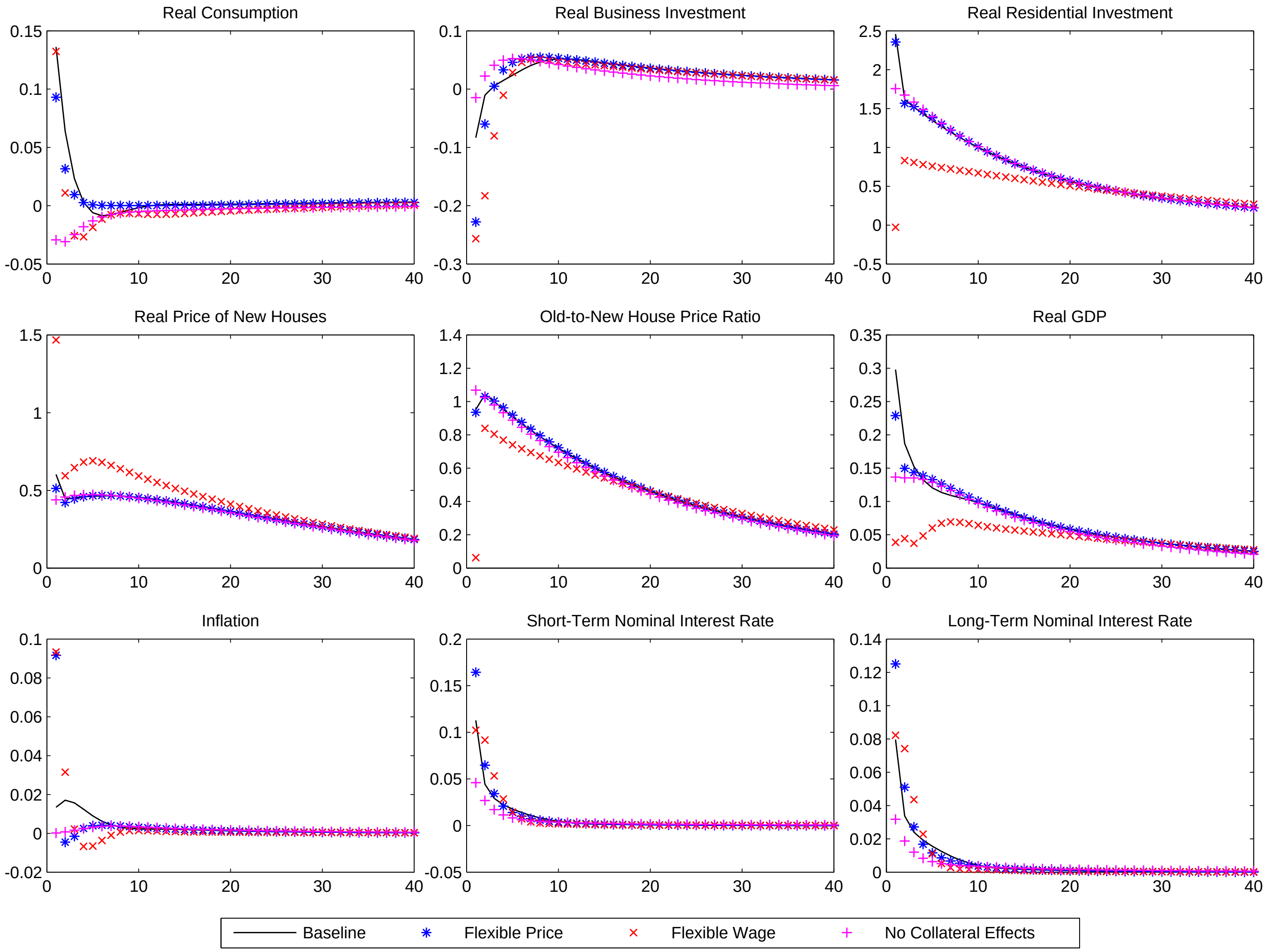
Real price of new houses also increases but old houses become relatively cheaper. The innovation in demand for new houses has a long-lasting positive impact on the real GDP and temporary impacts on inflation and nominal interest rates.



Note: The y-axis measures the percent deviation from the steady state.

Figure 3: Impulse Responses to a Preference Innovation in New Houses

Figure 4 plots impulse responses to the estimated preference innovation in old houses. A positive preference innovation in old houses also encourages residential investment to a large extent but it does not have much impact on business investment. When households face such a disturbance, they switch from new houses to old houses, since new and old houses are perfect substitutes to each other. This switch increases their budget for consumption and stimulates real consumption in the short run. A positive innovation to the demand for old houses not only increases the real price of new houses but also shoots up the relative price of old houses. This innovation is another important driving force of the real GDP. It generates temporary and positive impacts on inflation and nominal interest rates.

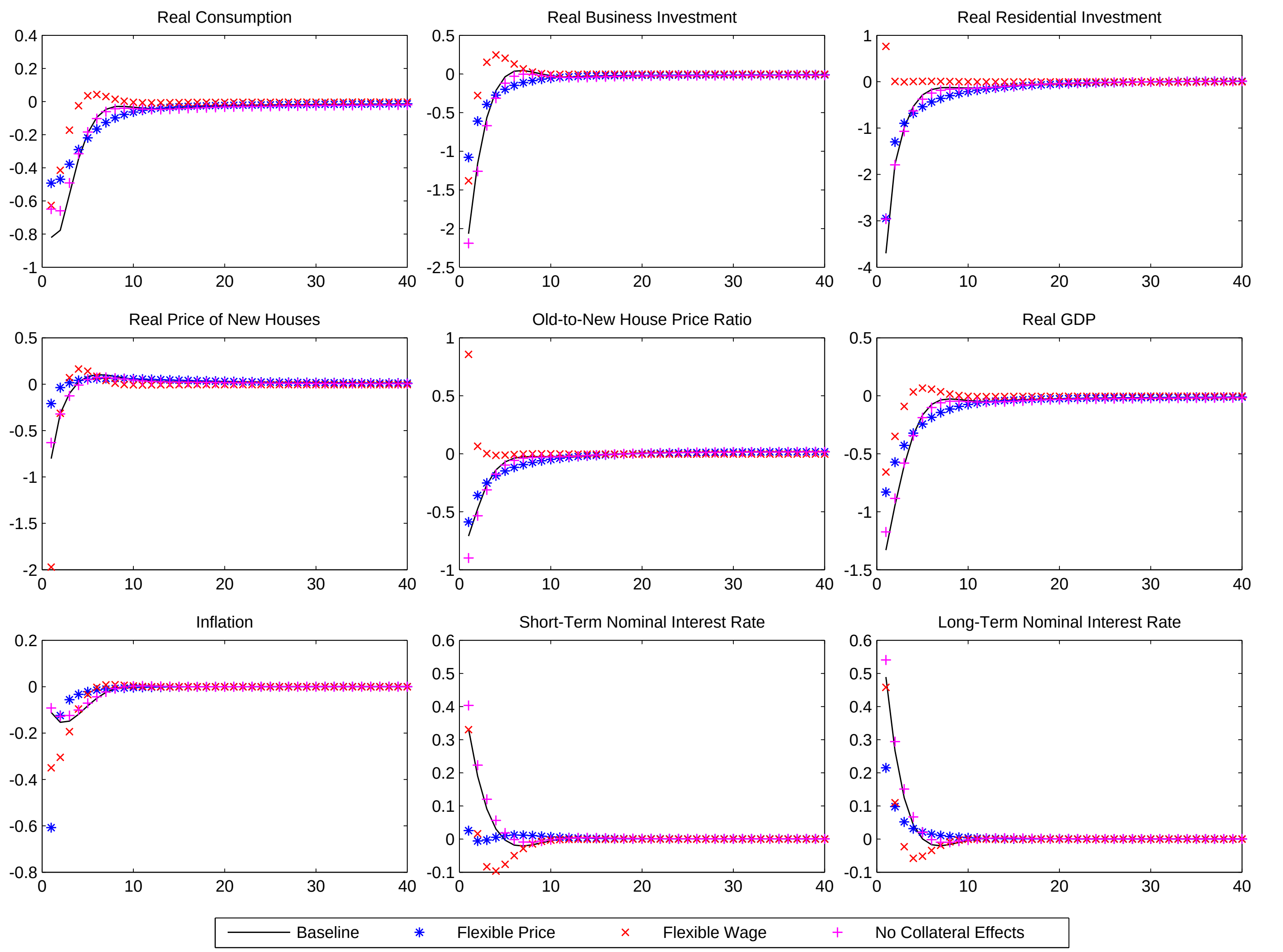


Note: The y-axis measures the percent deviation from the steady state.

Figure 4: Impulse Responses to a Preference Innovation in Old Houses

3.5.2 Term Structure Innovations

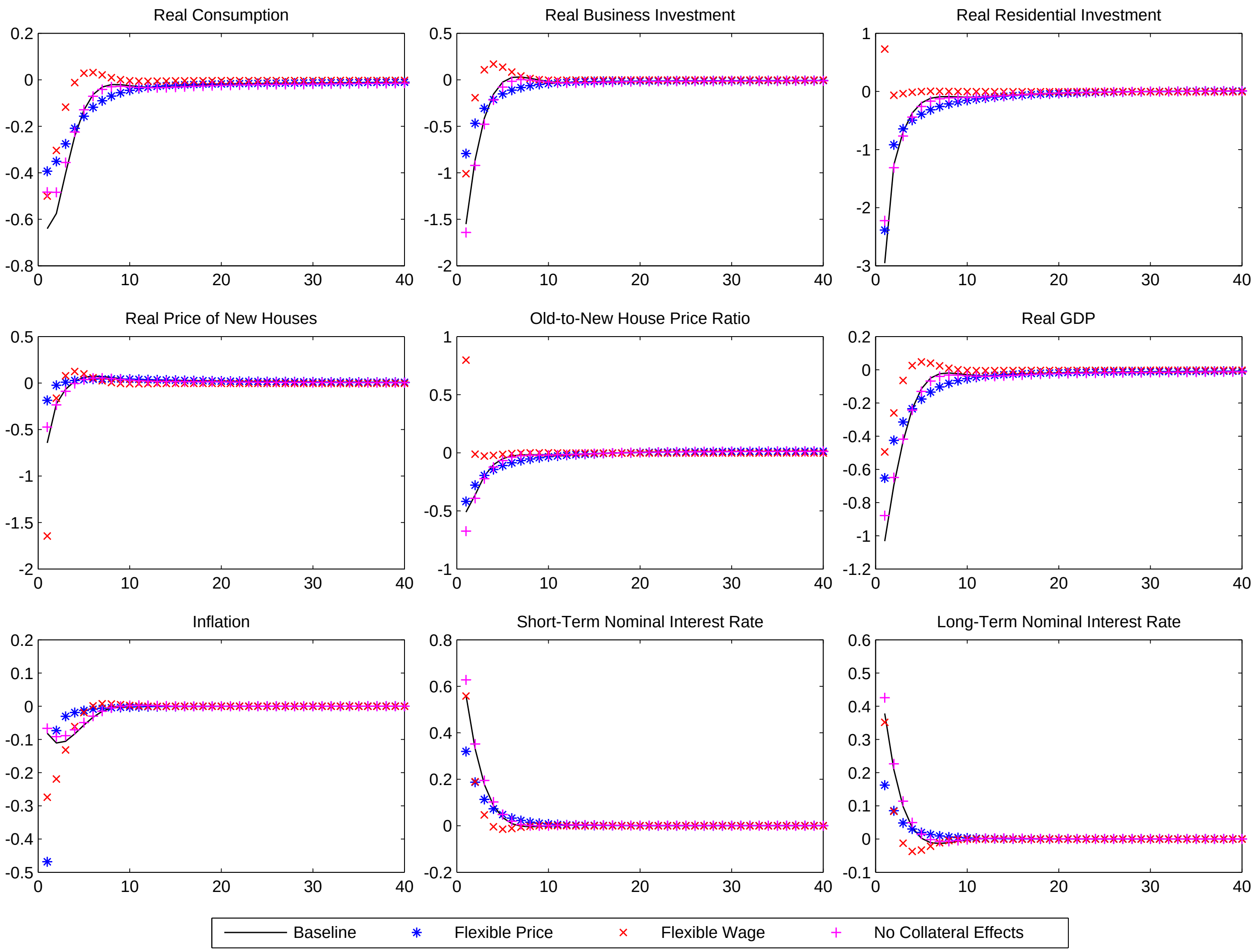
Figures 5 and 6 plot impulse responses to the term structure level innovation and to the term structure slope innovation, respectively. Both level and slope innovations generate temporary (less than 8 quarters) impacts of similar pattern. All components of aggregate demand fall, with the largest drop in residential investment, followed by business investment, and by consumption. Real price of new houses drops and remains below the steady-state value for about 5 quarters. Old houses become cheaper relative to new houses. The slope innovation increases the short-term nominal interest rate by a larger extent, since the loadings on the slope factor decrease monotonically with the maturity.



Note: The y-axis measures the percent deviation from the steady state.

Figure 5: Impulse Responses to a Term Structure Level Innovation

Compared to preference innovations, monetary factors generate larger changes in all components of aggregate demand. The large decline in residential investment is well-documented in the literature. Bernanke and Gertler (1995) find that the earliest and sharpest declines in final demand occur in residential investment in the case of a monetary policy shock.

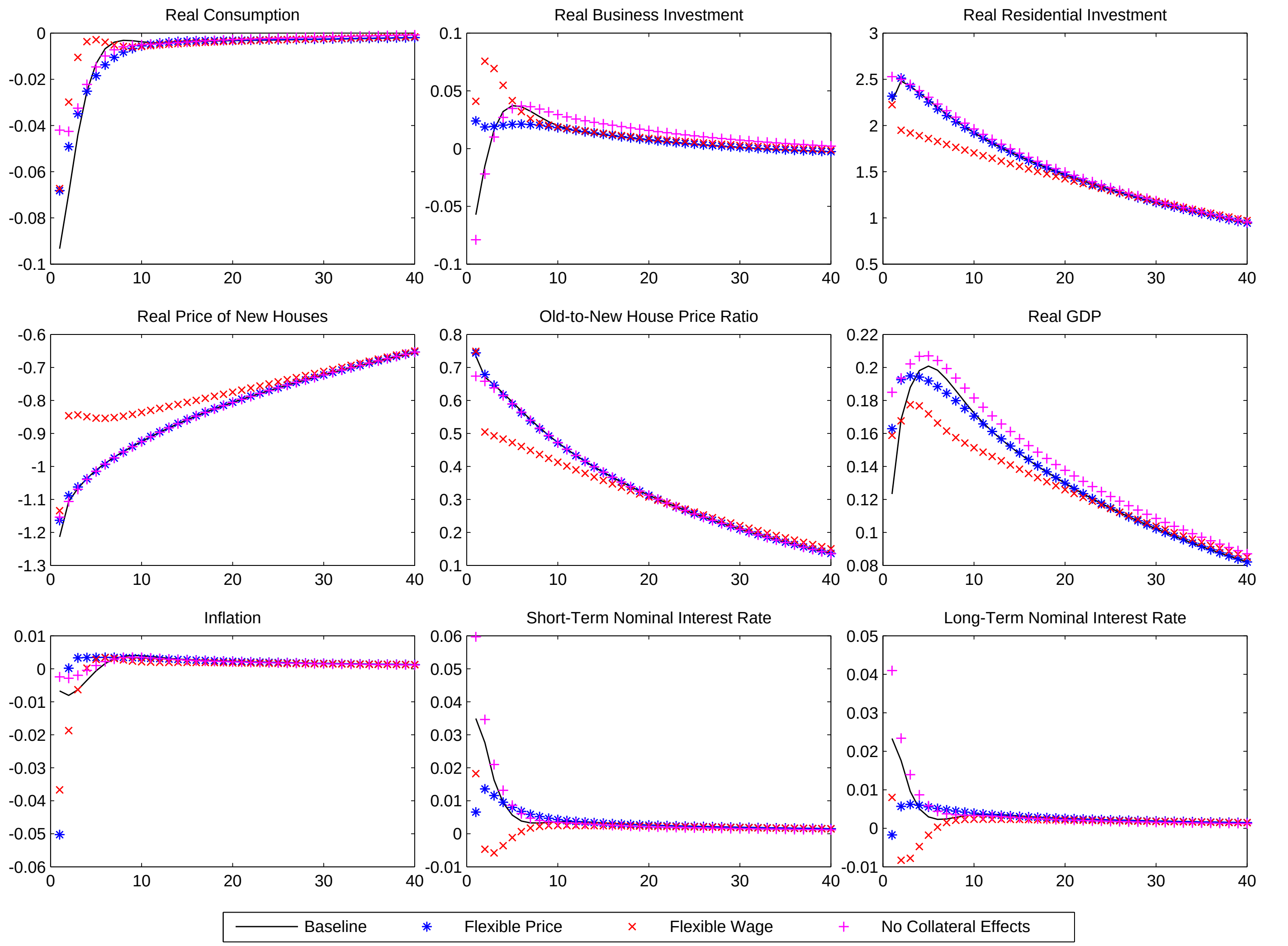


Note: The y-axis measures the percent deviation from the steady state.

Figure 6: Impulse Responses to a Term Structure Slope Innovation

3.5.3 Housing Technology Innovation

A positive technology innovation in the housing sector leads to a large rise in residential investment and a slight drop in business investment. Real consumption also drops in the short run. Due to a technological progress and hence a fall in construction costs, real price of new houses decreases. However, old houses become more expensive relative to new houses. This technology innovation stimulates the economy and has a hump-shaped impact on the real GDP. The innovation generates tiny impacts on inflation and nominal interest rates.



Note: The y-axis measures the percent deviation from the steady state.

Figure 7: Impulse Responses to a Housing Technology Innovation

3.6 Counterfactual Analyses

In this subsection, the scenarios with flexible price, flexible wages, and no collateral effects are examined. In particular, model simulations under $\theta_\pi = 0$ (flexible price), $\theta_{wc} = \theta_{wh} = 0$ (flexible wages), and $\alpha = 1$ (no collateral effects) are in turn conducted, holding all other parameters at their estimated values. The impulse responses for these three versions of the model are displayed in Figures 3 to 7. Three findings are worth mentioning.

First, price stickiness is important for the response of inflation. In the case of flexible price, all innovations generate larger impacts on the gross price inflation, simply because the price of consumption goods can flexibly vary at the retail level.

Second, wage stickiness has important implications for the housing markets. The combination of flexible house prices and sticky wages makes residential investment very sensitive to changes in preference conditions. Absent wage stickiness, the real price of new houses becomes more sensitive to preference innovations. Housing investment is sensitive to the term structure level and slope innovations only when wage stickiness is present. Facing positive term structure innovations, firms are discouraged to invest in the production of new houses in the case of sticky wages, because house price can always flexibly drop but wages remain sticky. The result is a lower level of investment in the housing sector. However, if wages are also flexible, the housing production costs do not change so that residential investment is insensitive to the term structure innovations. Wage stickiness also amplifies the responses of residential investment and new house price to the housing technology innovation.

Third, collateral effects are the key property for a positive response of consumption to a preference innovation in old houses. Without collateral effects, an increase in the demand for old houses would generate a fall in the real consumption. One explanation is that, facing a positive preference innovation in old houses, unconstrained households are willing to spend more on purchasing old houses.[6] In the case of no collateral effects or when α approaches 1, the supply of old houses falls down to zero and the price of old houses shoots up. As a result, a higher spending on old houses crowds out real consumption.

3.7 Variance Decomposition

Table 4 presents results from the conditional variance decomposition of forecast error at business cycle frequency.[7] Housing preference innovations (ε_{j_n} and ε_{j_o}) on the demand side and the housing technology innovation (ε_h) on the supply side explain about 80% of the volatility of residential investment, real price of new houses, and the old-to-new house price ratio. In particular, ε_{j_n} and ε_h each explains $30-40\%$ of the variations in residential investment and new house price. One half

[6] According to the estimation results, there is a transaction flow of old houses from constrained to unconstrained households in the steady state.

[7] The business cycle frequency is usually considered to be 3 to 5 years. Here, the 16-quarter conditional variance decomposition is reported.

of the variation in the old-to-new house price ratio is explained by ε_{j_n}; another 25% by ε_{j_o}. The monetary components ε_l and ε_s explain $10-15\%$ of the variations in residential investment and the price of old houses, but do not contribute much to the variation in new house price. However, the level and slope innovations contribute to 30% of the variation in the growth rate of new house price and 50% of the variation in the growth rate of old house price. The housing demand and supply innovations do not have much explanatory power for consumption and business investment.

Table 4: Variance Decomposition of the Forecast Error

Variable	ε_c C Tech.	ε_h IH Tech.	ε_k IK Tech.	ε_{j_o} Old Pref.	ε_{j_n} New Pref.	ε_l Mon. L	ε_s Mon. S	ε_τ Lab. Sup.	ε_z Inter. Pref.	ε_p Cost Push
Real Consumption	22.90	0.13	4.45	0.19	0.15	14.07	7.87	29.43	7.12	13.69
Real Business Investment	5.98	0.01	66.40	0.04	0.06	6.68	3.74	6.68	3.73	6.68
Real Residential Investment	1.79	31.34	0.12	11.80	34.04	8.56	5.16	4.70	0.33	2.15
Real Price of New Houses	4.03	44.02	0.83	9.75	30.63	2.34	1.44	1.45	1.01	4.51
Growth of New House Price	1.97	29.74	0.21	7.75	11.86	18.93	12.50	0.50	0.35	16.20
Real Price of Old Houses	6.27	7.08	0.84	55.02	4.02	6.78	3.88	5.38	4.59	6.14
Growth of Old House Price	2.83	2.35	0.30	24.75	1.80	31.19	18.28	1.02	2.87	14.61
Old-to-New House Price Ratio	0.79	11.51	0.03	25.94	53.82	2.12	1.14	1.89	2.13	0.63
Real GDP	15.88	2.25	20.22	1.31	2.56	15.08	8.59	21.70	0.46	11.95
Inflation	5.39	0.04	0.23	0.16	0.08	12.59	6.39	1.29	6.18	67.65
Short-Term Interest Rate[1]	4.73	0.19	2.91	1.30	0.63	12.17	36.05	2.84	12.89	26.27
4-Quarter Interest Rate	3.55	0.14	2.15	0.97	0.46	9.88	26.43	2.11	9.68	19.85
12-Quarter Interest Rate	3.39	0.13	1.97	0.89	0.43	11.49	23.75	1.96	9.26	19.33
24-Quarter Interest Rate	3.33	0.12	1.83	0.84	0.39	14.61	21.25	1.85	9.11	19.55
36-Quarter Interest Rate	3.11	0.10	1.60	0.74	0.35	17.04	18.01	1.66	8.51	18.75
48-Quarter Interest Rate	3.13	0.09	1.51	0.71	0.33	20.79	16.39	1.60	8.55	19.33
Long-Term Interest Rate[2]	4.19	0.11	1.90	0.91	0.41	32.88	19.77	2.06	11.39	26.39
Level Factor	1.92	0.01	0.10	0.07	0.04	79.54	2.47	0.58	2.82	12.44
Slope Factor	2.84	0.27	2.84	1.09	0.64	27.53	51.48	2.67	4.44	6.20

Note: [1] 1-Quarter Interest Rate, [2] 60-Quarter Interest Rate. This table reports the 16-quarter conditional forecast error variance decomposition at the posterior mean parameter. All values are in percentage.

As for nominal interest rates, the inflation innovation ε_p and the intertemporal preference innovation ε_z contribute about 20% and 10%, respectively. The term structure innovations ε_l and ε_s together explain $40-50\%$. The rest is attributable to the measurement error or the "curvature" factor which has been ignored in the estimation. Variations in both the level factor and the slope factor of the term structure are mostly explained by their own innovations, while $10-15\%$ are attributable to ε_z and ε_p. Housing demand and technology innovations have a limited effect on the term structure.

4 Application I: The Role of Transaction Costs

According to the specification of transaction costs in Equation (2.9) and (2.19), parameters ϕ and ϕ' do not have any impact on the steady state, where a tiny amount of old houses is transacted from constrained households to unconstrained households. However, these transaction cost parameters do have implications on the dynamics of the secondary housing market. Starting from the steady state, the transaction flows of old houses between two types of households under various combinations of (ϕ, ϕ') are simulated, given the set of estimated structural shocks. The results are presented in Figure 8. The vertical axis measures the transaction flow of old houses from constrained households to unconstrained households.

The graph located at the second row and the second column of the figure corresponds to the baseline model, where ϕ and ϕ' are estimated to be 0.104 and 0.210, respectively. In the baseline model, the transaction flow shows a maximum of 0.29 and a minimum of -0.31 with an average of -0.0022 and a standard error of 0.088. As either parameter increases (decreases), the transaction flow becomes less (more) volatile. In particular, the transaction flow is almost flat in the case of $(\phi, \phi') = (10, 10)$, whereas it shows the largest volatility when transaction costs are absent, i.e., $(\phi, \phi') = (0, 0)$. Further increasing the transaction cost parameters would with certainty shut the secondary market down.

Figure 8 suggests that, even though the transaction costs in the secondary housing market do not affect the steady state of the model, they generally discourage the two types of households from trading old houses with each other.

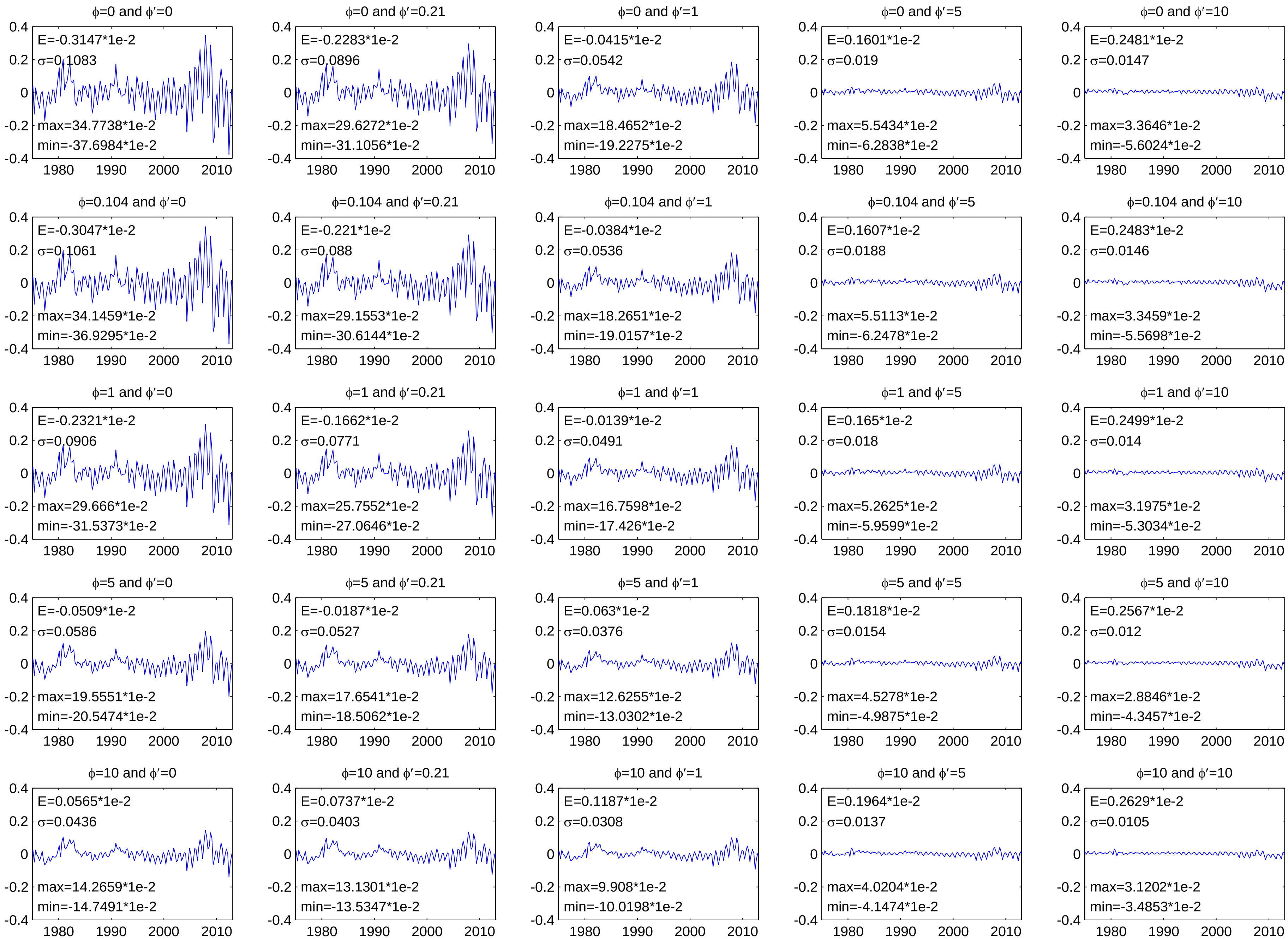


Figure 8: Transaction Costs and the Secondary Housing market

Knowing that the transaction costs stabilize the trade of old houses in the secondary market, it is straightforward to examine whether the existence of transaction costs is harmful to the overall stability of the macroeconomy. It is assumed that the policymaker seeks to minimize an ordinary expected loss criterion as in Giannoni and Woodford (2003):

$$\begin{aligned}\mathfrak{L} = {} & \lambda_\pi var(\ln \pi_t - \iota_\pi \ln \pi_{t-1}) + \lambda_w var(\ln \pi_{wc,t} - \iota_{wc} \ln \pi_{t-1}) \\ & + \lambda_y var(\ln GDP_t - \ln(G_C GDP_{t-1})) + \lambda_r var(\ln R_{L,t} - \ln \overline{rr}),\end{aligned} \tag{4.1}$$

where $\overline{rr}$ is the steady-state value of long-run interest rate; $\pi_{wc,t} = \pi_t(w_{c,t} + w'_{c,t})/((w_{c,t-1} + w'_{c,t-1}))$ is the nominal wage inflation in the consumption sector.[8] Under the objective $\mathfrak{L}$, the policymaker minimizes a weighted variability of money inflation, wage inflation, output growth, and long-run nominal interest rate, with weights $\lambda_\pi = 0.5$, $\lambda_w = 0.5$, $\lambda_y = 0.048$, and $\lambda_r = 0.236$, as calibrated in Woodford (2003).

For later convenience, let $V_\pi = var(\ln \pi_t - \iota_\pi \ln \pi_{t-1})$, $V_w = var(\ln \pi_{wc,t} - \iota_{wc} \ln \pi_{t-1})$, $V_y = var(\ln GDP_t - \ln(G_C GDP_{t-1}))$, and $V_r = var(\ln R_{L,t} - \ln \overline{rr})$ denote the four variance components in the loss function $\mathfrak{L}$. A 2-dimensional optimization over $(\phi, \phi') \in [0, +\infty)^2$ is conducted to identify the combination that minimizes the loss function $\mathfrak{L}$ in Equation (4.1). The results are presented in Table 5. Though the estimated values of transaction cost parameters are not large, the optimal values are zero for both types of households. Such a comparison suggests that reducing transaction costs improves overall macroeconomic stability.

Table 5: Estimated and Optimal Transaction Costs

	Statistics					Parameter Values		Social Welfare	
	V_π	V_w	V_y	V_r	$\mathfrak{L}$	ϕ	ϕ'	V_1^s	V_2^s
Estimated	0.0508	0.0378	0.5234	0.1011	0.0933	0.104	0.210	17.5197	-2.7162
Optimal	0.0511	0.0345	0.4255	0.1089	0.0889	0.000	0.000	17.5252	-2.7005

Note: All statistics are multiplied by 1000. The loss function $\mathfrak{L}$ is a weighted sum of V_π, V_w, V_y, and V_r. Social welfare V_1^s and V_2^s are computed under two different social welfare functions.

Another objective of the policymaker usually considered in the literature is social welfare (see Erceg, Henderson and Levin (2000), Faia and Monacelli (2007), and Schmitt-Grohé and Uribe (2007)

[8] The wage inflation in the housing sector is not considered, since both types of households contribute most of their labor to the production of consumption goods (see Sun and Tsang (2014)).

among others). Social welfare is defined as the weighted average of two types of households' lifetime utilities, i.e.,

$$V_1^s = \alpha V + (1-\alpha)V', \tag{4.2}$$

or

$$V_2^s = (1-\beta G_C)V + (1-\beta' G_C)V', \tag{4.3}$$

where V_1^s and V_2^s denote social welfare; the type-specific welfare V and V' are defined in Equations (2.1) and (2.2), respectively. The first social welfare function weights the welfare of the two types of households according to their economic sizes, α and $1-\alpha$. The second function is specified, following Rubio (2011) and Lambertini, Mendicino and Punzi (2013), such that both types of households achieve the same level of utility given the same constant consumption, housing, and working streams.[9]

The social welfare implied by different transaction costs conditional on the initial state's being the deterministic steady state are computed and presented in the last two columns of Table 5. While reducing overall economic fluctuations, a reduction in transaction costs also improves social welfare. This result is invariant to how these two types of households are weighted.

5 Application II: Sources of Housing Market Fluctuations

There are substantial fluctuations in the housing markets, associated with residential investment, real price of new houses, and the old-to-new house price ratio. Since around the year of 1997, there is an apparent convergence between the price of old houses and the price of new houses up until 2006, when the recent housing market bubble bursts. What factors are driving these changes?

[9] For the same constant consumption, housing, and working streams $(c, h_o, h_n, n_c, n_h) = (c', h_o', h_n', n_c', n_h')$, unconstrained households' lifetime utility is $V = (1/(1-\beta G_C))U(c, h_o, h_n, n_c, n_h)$, whereas constrained households' lifetime utility is $V' = (1/(1-\beta' G_C))U(c', h_o', h_n', n_c', n_h')$. Weighting V by $1-\beta G_C$ and V' by $1-\beta' G_C$ ensures the same level of utility across the two types for given constant consumption, housing, and working streams. Lambertini, Mendicino and Punzi (2013) also consider an alternative social welfare function that weights the welfare of the two types of households equally and obtain consistent results.

5.1 What Moves the Housing Markets?

Figure 9 plots real residential investment (in log, with trend) and real price of new houses (in log, with trend) under different values of γ_{AH}, the housing technological trend. In the baseline model, $\gamma_{AH} = -0.042\%$, indicating that the TFP in the construction sector is *decreasing* at an annual rate of around 0.17%. The negative technological progress significantly explains the upward trend in house prices over the past four decades. Then, a variety of values are specifed for γ_{AH} and the model is simulated for each case. As the figure shows, an increase in γ_{AH} always promotes the investment in the housing sector and hence brings house price down.

As of 2006, residential investment could be 50% higher and the price of new houses could be 25% lower, if the TFP in the housing sector is growing at the same rate as that in the consumption sector, i.e., $\gamma_{AH} = 0.003$.

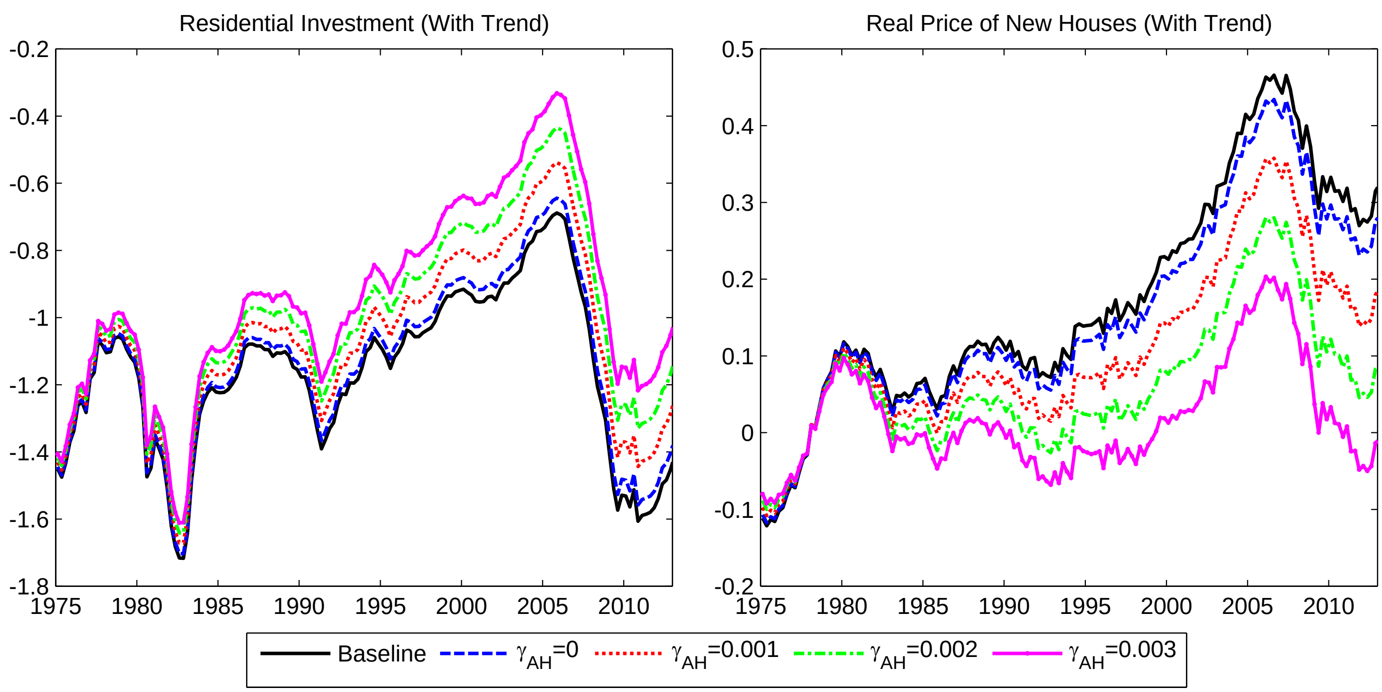


Figure 9: The Effects of the Housing Technological Trend

Given that the upward trend in house prices is a result of the negative technological progress in the housing sector, I turn to exploring the dynamics of the detrended housing market variables. Each panel of Figure 10 plots real residential investment as well as two relevant counterfactual series, shutting down one of the innovations since 1975 and since 1997, respectively. Similar plots are presented in Figure 11 for the real price of new houses and in Figure 12 for the old-to-new house price ratio. Consistent with the variance decompositions, the housing technology innovation, ε_h, and the housing preference innovations, ε_{j_n} and ε_{j_o}, have large impacts on the dynamics of the housing markets.

Since the 1980s, real residential investment and real price of new houses are heavily influenced by the housing technology innovation ε_h, without which residential investment would have been 10% to 20% lower between the mid-1980s and the mid-2000s and the new house price would have been 10% to 20% higher instead. The preference innovation in new houses ε_{j_n} has considerable impacts on residential investment and new house price before the 1990s. It also accelerates the drop in both series during the Great Recession. The preference innovation in old houses ε_{j_o} does not affect residential investment and new house price until 1997. After this time point, it contributes a lot to the run-up in the housing markets. The housing technology innovation ε_h does not have much impact on the ratio of old house price to new house price. Instead, this ratio is mainly affected by the housing preference innovations. Without the preference innovation in new houses ε_{j_n}, the old-to-new price ratio would have been around 10% higher before the 2006 financial crisis and 10% lower after the crisis. The preference innovation in old houses ε_{j_o} increases fluctuations of the house price ratio, especially after 1997. The fact that the old house price increases relative to the price of new houses between the mid-1990s and the mid-2000s is mainly explained by this innovation.

Among other innovations, the consumption sector technology innovation ε_c, the term structure innovations, ε_l and ε_s, and the labor supply innovation, ε_τ, have limited impacts on the housing markets.

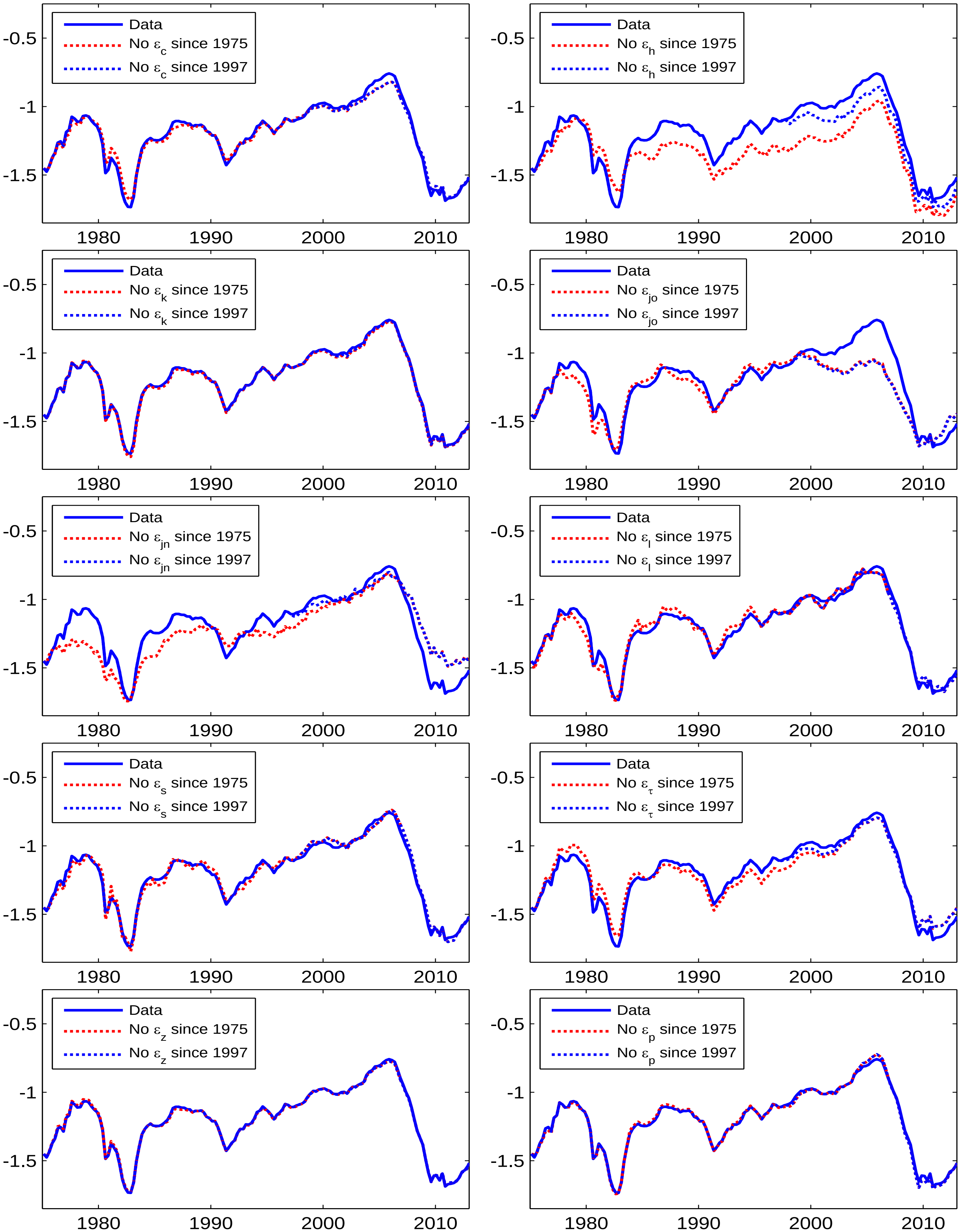


Figure 10: Counterfactuals on Residential Investment (in log)

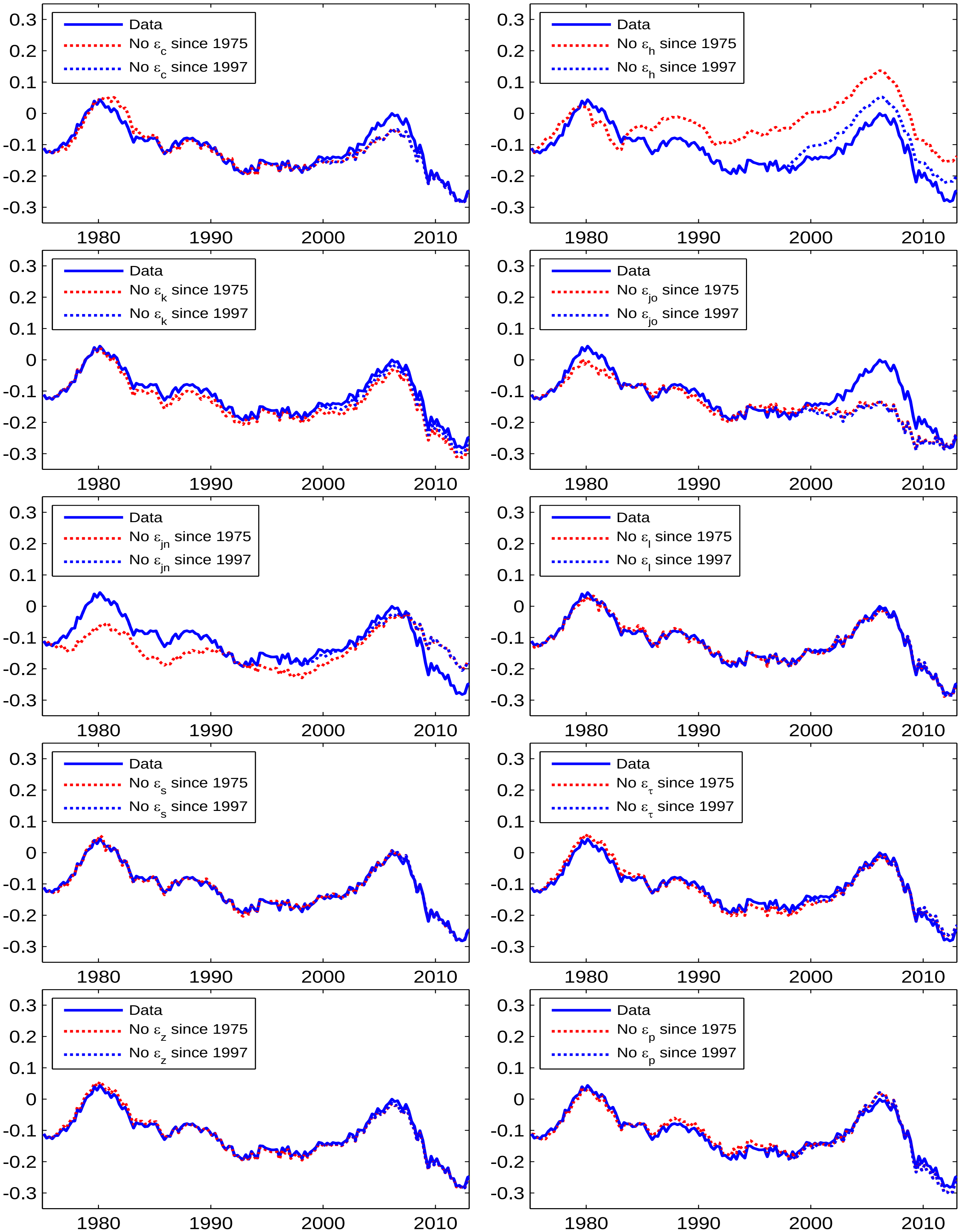


Figure 11: Counterfactuals on Real Price of New Houses (in log)

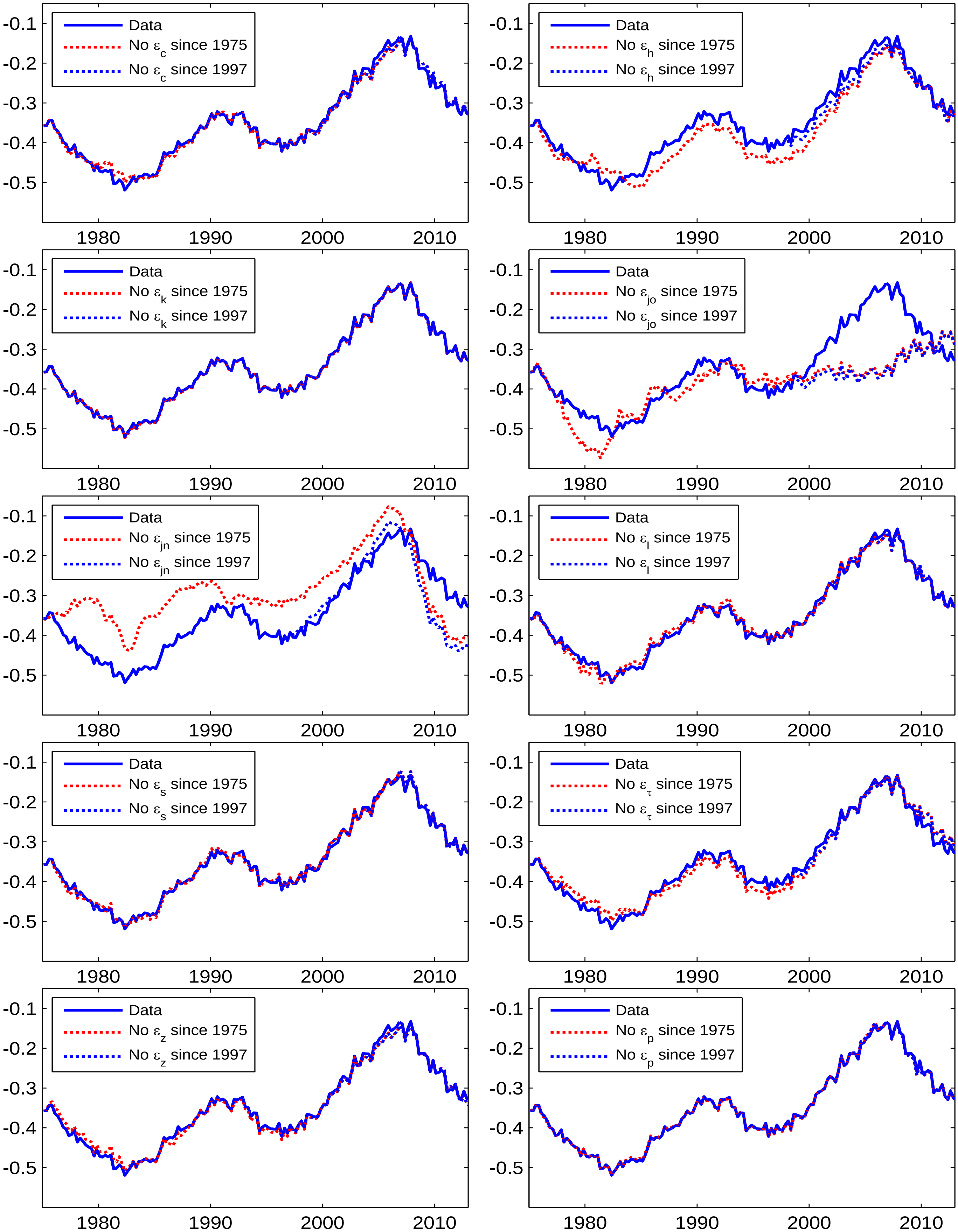


Figure 12: Counterfactuals on Old-to-New House Price Ratio (in log)

5.2 Understanding the Housing Preference Innovations

The previous subsection suggests that the housing preference innovations, ε_{j_n} and ε_{j_o}, account for a sizeable fraction of housing market fluctuations. In the DSGE model, these preference innovations are exogenously introduced, which represent genuine changes in consumer tastes given that the DSGE model "truly" characterizes the economy. However, the concern that these innovations could be nothing but the omitted factors that affect the demand for the two types of houses needs to be addressed. A standard multivariate time series analysis is then conducted for ε_{j_n} and ε_{j_o}, following Evans (1992),

$$\varepsilon_{j_n,t} = B_n \mathbf{x}_t + \nu_{n,t} \tag{5.1}$$

$$\varepsilon_{j_o,t} = B_o \mathbf{x}_t + \nu_{o,t}, \tag{5.2}$$

where B_n and B_o coefficient vectors; $\mathbf{x}$ is a vector of potentially relevant explanatory variables that affect housing preferences; ν_n and ν_o are independently and identically distributed error terms.[10]

The main determinants of the demand for housing are demographic. But other factors, such as income, price of housing, cost and availability of credit, consumer preferences, investor preferences, price of substitutes, and price of complements, all play a role. Some of these determinants have already been included in the model. Several variables that do not appear in the model are chosen as regressors, including the University of Michigan Index of Consumer Sentiment, the Economic Policy Uncertainty Index, the LTV ratio, the employment-to-population ratio, the population share between ages 16 and 19, the population share between ages 20 and 24, the population share above

[10] Iacoviello and Neri (2010) conduct a similar analysis to examine if the preference innovation $u_{j,t}$ can be predicted, using the original framework of Evans (1992):

$$u_{j,t} = A(L)u_{j,t-1} + B(L)\mathbf{x}_{t-1} + \nu_t$$

where ν_t is a mean zero independently and identically distributed random variable, $A(L)$ and $B(L)$ are polynomials in the lag operator L, and $\mathbf{x}$ is a list of potential explanatory variables for housing demand. However, this setup contradicts their model assumption that $u_{j,t}$ is an independently and identically distributed random variable. The regressions (5.1) and (5.2) exclude the lags of each relevant dependent variable from the list of regressors, in order to be consistent with the assumptions in Section 2. By regressing the preference innovation on a variety of variables in their first lags, Iacoviello and Neri (2010) answer the question that whether the preference innovation can possibly be predicted. In the present work, a contemporaneous regression is conducted to answer a different question that how changes in the housing preference shocks can be explained by unmodeled factors.

age 55, and the marriage rate.[11] Among these variables, marriage rate is available from 1979:Q1 to 2009:Q4 only. The trends in data, if exist, are all removed by subtracting a least-squares-fit straight line. Marriage rate is seasonally adjusted. Each variable is in the natural logarithm form. The results are presented in Table 6, in which the first four columns show the results with marriage rate excluded from the regressor list while the last four columns show the results with marriage rated included.

Table 6: Understanding the Housing Preference Innovations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	ε_{j_n} NEW	ε_{j_o} OLD	ε_{j_n} NEW	ε_{j_o} OLD	ε_{j_n} NEW	ε_{j_o} OLD	ε_{j_n} NEW	ε_{j_o} OLD
Consumer	0.162***	0.144***	0.146***	0.143***	0.149***	0.134***	0.135***	0.143***
Sentiment	(0.0436)	(0.0225)	(0.0381)	(0.0213)	(0.0466)	(0.0234)	(0.0414)	(0.0206)
Uncertainty	0.0219	-0.00867			-0.00187	-0.00363		
Index	(0.0341)	(0.0176)			(0.0389)	(0.0195)		
LTV	0.0155	-0.442***		-0.480***	-0.278	-0.677***		-0.695***
Ratio	(0.238)	(0.123)		(0.117)	(0.256)	(0.128)		(0.120)
Employment-to-	-1.156***	0.332**	-1.185***	0.210**	-1.466***	0.282	-1.474***	
Population	(0.286)	(0.148)	(0.281)	(0.0981)	(0.424)	(0.212)	(0.345)	
Population Share	0.0396	0.144***		0.166***	0.0250	0.115*		0.153***
b/w 16 and 19	(0.0959)	(0.0494)		(0.0438)	(0.123)	(0.0616)		(0.0489)
Population Share	0.130	0.104			-0.0199	0.127		
b/w 20 and 24	(0.187)	(0.0963)			(0.238)	(0.119)		
Population Share	-0.647**	0.0254	-0.484**		-0.774***	-0.238	-0.702***	-0.238**
above 55	(0.273)	(0.141)	(0.198)		(0.294)	(0.147)	(0.215)	(0.0921)
Marriage					0.0897**	-0.0108	0.0786**	
Rate					(0.0443)	(0.0222)	(0.0376)	
N	156	156	156	156	124	124	124	124
adj. R^2	0.108	0.305	0.126	0.308	0.153	0.362	0.172	0.372

Note: Standard errors in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$; constant terms are not reported.

Columns (1) and (2) indicate that changes in the selected regressors explain only 10% of the variation in the preference innovation to new houses and 30% of the variation in the preference

[11] Data sources are as follows. Consumer Sentiment: Thomson Reuters/University of Michigan Surveys of Consumers; Uncertainty Index: Economic Policy Uncertainty; LTV Ratio: Federal Housing Finance Agency; the employment-to-population ratio and population shares: Federal Reserve Bank of St. Louis; marriage rate: Centers for Disease Control and Prevention.

innovation to old houses. Consumer sentiment is statistically significant and has the expected sign in both regressions. As households become more optimistic about the economic prospects, they are willing to own more of both types of houses. The economic policy uncertainty does not significantly affect either of the preference innovations. A higher LTV ratio lowers the demand for old houses but does not affect the demand for the other type. One explanation is perhaps that an increase in LTV ratio is usually implemented during hard times when households attempt to save more for the future. The employment-to-population ratio affects the demand for new houses negatively and the demand for old houses positively. Age structure has important implications for the housing demand. As the share of population between ages 16 and 19 increases, the demand for old houses rises; the share of population above age 55 instead affects the demand for new houses negatively. This suggests that, firstly, households with more teenagers have higher demand for housing but they are likely to own more of old instead of new houses, and secondly, the elderly population tend not to own new houses. The backward-stepwise selection is then used to keep those variables that have significant impacts on the preference innovations. The main results presented in columns (3) and (4) stay the same. When marriage rate is included in the regressor list, similar results are obtained in columns (5)-(8). An increase in marriage rate significantly promotes the households' demand for new houses. One explanation would be that as more people, usually young people, get married and leave their families, higher housing demand is created and newly married people prefer owning a new house. The results with backward-stepwise selection are similar.

The results provide some evidence that part of the preference innovations can be captured by some omitted variables. However, the low goodness-of-fit in the multivariate regression analysis suggests that most of the movements in housing demand represent shifts in households' tastes for housing.

6 Application III: The Term Structure of Nominal Interest Rates

Modeling the term structure of nominal interest rates in a structural framework is a recent exercise. Before Rudebusch and Wu (2008), studies on the relationship between the term structure and macroeconomic factors focus particularly on statistical descriptions (see for example Ang and

Piazzesi (2003)). Rudebusch and Wu (2008) develop a macro-finance model that combines the term structure with standard macroeconomic aggregate relationships for output and inflation. They find that up to 95% of the variations in interest rates are driven by the term structure level and factor shocks. However, their model specification is far from complete. In addition, the housing markets have never been considered among the determinants of the term structure. The present paper incorporates the term structure in a DSGE model that features a variety of structural innovations, including those related to the housing markets. As the variance decomposition in Table 4 shows, the term structure level and slope factors explain 40-50% of the variations in nominal interest rates. Among other innovations, the cost-push innovation and the intertemporal preference innovation also significantly contribute to the interest rate fluctuations.

How each of the structural innovations affects the shape of the yield curve? Each panel of Figure 13 shows the comparison between the data and the counterfactual scenario, with one of the innovations being shut down. Circles denote the average of the observed nominal interest rates over the whole sample period 1975:Q1-2013:Q4 at different maturities, whereas plus signs denote the average of the counterfactual nominal interest rates. A line of best fit using the method of least squares is added for each series. The shape of the yield curve is characterized by the intercept and the slope of the best-fit line, the later of which is also reported in each panel of the figure.

Over the sample period, even the term structure innovations, ε_l and ε_s, and the cost-push innovation ε_p significantly contribute to the variations in nominal interest rates, they do not change the average level of interest rates at any maturity, so that the shape of the yield curve is insensitive. Some of the structural innovations, such as the technology innovation in the consumption sector ε_c, the labor supply innovation ε_τ, and the intertemporal preference innovation ε_z, affect interest rates negatively. Others, including the housing productivity innovation ε_h, the investment-specific technology innovation ε_k, the housing preference innovations ε_{j_o} and ε_{j_n}, have positive impacts on interest rates. More importantly, the existence of these innovations significantly changes the shape of the yield curve. The shocks negatively affecting interest rates tend to make the yield curve steeper and those positively affecting interest rates make the yield curve flatter.[12]

[12]This is simply because structural innovations usually have larger impacts on shorter-term interest rates which exhibit much more volatility.

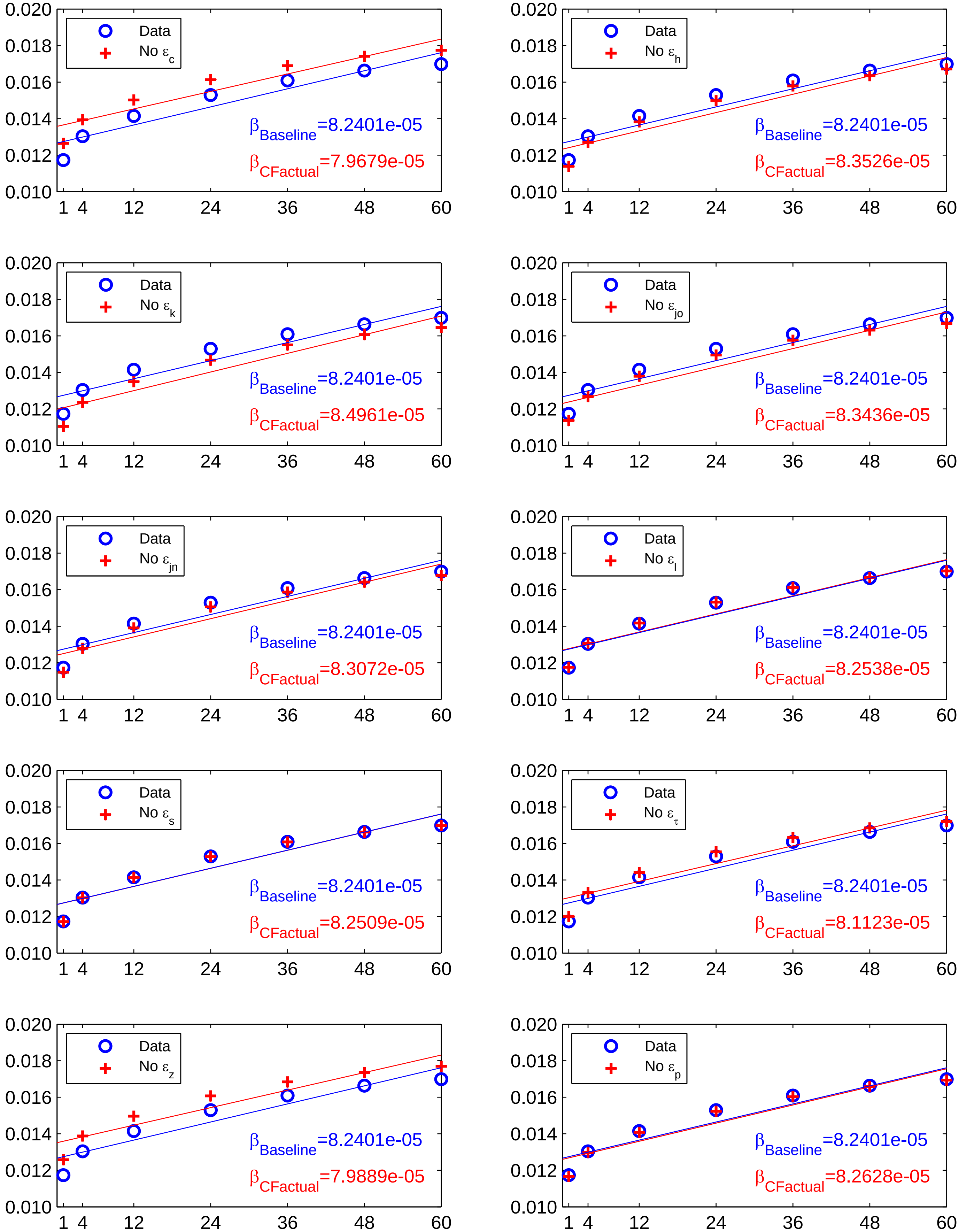


Figure 13: Structural Innovations and the Yield Curve

It is worth emphasizing that all of the housing market innovations, i.e., ε_h, ε_{j_o}, and ε_{j_n}, have been positively affecting nominal interest rates at different maturities and making the yield curve flatter in the past four decades. To my knowledge, this is the first-ever finding on the relationship between the housing market conditions and the yield curve.

7 Conclusion

In this paper, I construct a DSGE model that features a market of newly built houses and a secondary market of old houses. I incorporate the term structure of nominal interest rates into the DSGE model in order to investigate the bidirectional relations between housing markets and the term structure. The negative TFP progress in the housing sector significantly explains the upward trend in house prices over the past four decades. The model attributes about 80% of the volatility of residential investment, real price of new houses, and the old-to-new house price ratio to housing preference and housing technology innovations. Monetary factors explain about 15% of the volatility of housing investment, but do not significantly contribute to the price fluctuations of either new or old houses. The preference innovation to old houses mainly explains the run-up in the old house price relative to the price of new houses between the mid-1990s and the mid-2000s. The intertemporal preference innovation has a non-negligible contribution to the dynamics of nominal interest rates. Housing market conditions do not contribute much to the fluctuations of interest rates, but significantly affect the shape of the yield curve.

The exogenously-specified shifts to housing preferences are related to several variables. Multivariate regressions suggest that consumer sentiment, the loan-to-value ratio, the employment-to-population ratio, and age structure, as well as marriage rate, are all potential variables that have explanatory power for the estimated housing preference innovations. Further research might improve such a DSGE model by taking these factors, such as age structure, into consideration. However, most of the movements in housing demand represent shifts in households' tastes.

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A Derivations for the Model Economy

A.1 Unconstrained Households

The lifetime utility of unconstrained households is given by:

$$V_t = E_0 \sum_{t=0}^{\infty} (\beta G_C)^t z_t \left[\frac{G_C - \epsilon}{G_C - \beta \epsilon G_C} \ln(c_t - \epsilon c_{t-1}) + j_{o,t} \ln(h_{o,t}) + j_{n,t} \ln(h_{n,t}) - \frac{\tau_t}{1+\eta} \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{1+\eta}{1+\xi}} \right]. \quad \text{(A.1)}$$

The marginal utility of consumption is:

$$u_{c,t} = \left(\frac{G_C - \epsilon}{G_C - \beta \epsilon G_C} \right) \left(\frac{z_t}{c_t - \epsilon c_{t-1}} - \frac{\beta \epsilon G_C z_{t+1}}{c_{t+1} - \epsilon c_t} \right). \quad \text{(A.2)}$$

The marginal utility of old house is:

$$u_{h_o,t} = \frac{z_t j_{o,t}}{h_{o,t}}, \quad \text{(A.3)}$$

and the marginal utility of new house is:

$$u_{h_n,t} = \frac{z_t j_{n,t}}{h_{n,t}}. \quad \text{(A.4)}$$

The marginal disutilities of working in the goods sector and in the housing sector are:

$$u_{n_c,t} = z_t \tau_t \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{\eta-\xi}{1+\xi}} n_{c,t}^{\xi}, \quad \text{(A.5)}$$

$$u_{n_h,t} = z_t \tau_t \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{\eta-\xi}{1+\xi}} n_{h,t}^{\xi}. \quad \text{(A.6)}$$

Along the balanced growth path, consumption grows at the rate G_C every quarter, the marginal utility of consumption falls at this rate. Hence the transformed marginal utility $\widetilde{u}_{c,t} = u_{c,t} G_C^t$ is stationary around the steady state and is equal to:

$$\widetilde{u}_{c,t} = \left(\frac{G_C - \epsilon}{G_C - \beta \epsilon G_C} \right) \left(\frac{z_t}{\widetilde{c}_t - \frac{\epsilon}{G_C} \widetilde{c}_{t-1}} - \frac{\beta \epsilon z_{t+1}}{\widetilde{c}_{t+1} - \frac{\epsilon}{G_C} \widetilde{c}_t} \right). \quad \text{(A.7)}$$

Transformed consumption, $\widetilde{c}_t = c_t / G_C^t$, and the scaled marginal utility of consumption $\widetilde{u}_{c,t}$:

$$\widetilde{u}_c = z \left(\frac{G_C - \epsilon}{G_C - \beta \epsilon G_C} \right) \left(\frac{1}{\widetilde{c} - \frac{\epsilon}{G_C} \widetilde{c}} - \frac{\beta \epsilon}{\widetilde{c} - \frac{\epsilon}{G_C} \widetilde{c}} \right) = \frac{1}{\widetilde{c}}, \quad \text{(A.8)}$$

are both constant in the steady state, since z is equal to one.

The marginal utilities of housing $u_{h_o,t} = z_t j_{o,t} / h_{o,t}$ and $u_{h_n,t} = z_t j_{n,t} / h_{n,t}$ both decline at the rate G_H. Therefore the transformed marginal utilities $\widetilde{u}_{h_o,t} = u_{h_o,t} G_H^t$ and $\widetilde{u}_{h_n,t} = u_{h_n,t} G_H^t$ are stationary around the steady state and are equal to:

$$\widetilde{u}_{h_o,t} = \frac{z_t j_{o,t}}{\widetilde{h}_{o,t}}, \quad \widetilde{u}_{h_n,t} = \frac{z_t j_{n,t}}{\widetilde{h}_{n,t}}. \quad \text{(A.9)}$$

In the steady state, $\widetilde{u}_{h_o} = j_o / \widetilde{h}_o$ and $\widetilde{u}_{h_n} = j_n / \widetilde{h}_n$.

Notice that hours worked in the two sectors, $n_{c,t}$ and $n_{h,t}$, are stationary already in the level economy. Unconstrained households' budget constraint is given by:

$$
\begin{aligned}
& c_t + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + k_{b,t} + q_t[f_t(h_{o,t} - (1-\delta_h)h_{o,t-1} - (1-\delta_h)h_{n,t-1}) + h_{n,t}] + p_{l,t}l_t + \frac{R_{L,t-1}b_{t-1}}{\pi_t} + TC_t \\
= & \frac{w_{c,t}n_{c,t}}{X_{wc,t}} + \frac{w_{h,t}n_{h,t}}{X_{wh,t}} + DIV_t + \left(R_{c,t}z_{c,t} + \frac{1-\delta_{kc}}{A_{k,t}}\right)k_{c,t-1} + (R_{h,t}z_{h,t} + 1 - \delta_{kh})\,k_{h,t-1} \\
& + p_{b,t}k_{b,t} + b_t + (p_{l,t} + R_{l,t})l_{t-1} - \Phi_t - \frac{a(z_{c,t})k_{c,t-1}}{A_{k,t}} - a(z_{h,t})k_{h,t-1}, \qquad \text{(A.10)}
\end{aligned}
$$

which can be transformed as follows:

$$
\begin{aligned}
& \widetilde{c}_t + \frac{\widetilde{k}_{c,t}}{a_{k,t}} + \widetilde{k}_{h,t} + \widetilde{k}_{b,t} + \widetilde{q}_t\left[f_t\left(\widetilde{h}_{o,t} - \frac{(1-\delta_h)\widetilde{h}_{o,t-1}}{G_H} - \frac{(1-\delta_h)\widetilde{h}_{n,t-1}}{G_H}\right) + \widetilde{h}_{n,t}\right] + \widetilde{p}_{l,t}l_t + \frac{R_{L,t-1}\widetilde{b}_{t-1}}{\pi_t G_C} + \widetilde{TC}_t \\
= & \widetilde{w}_{c,t}n_{c,t} + \widetilde{w}_{h,t}n_{h,t} + \left(1 - \frac{1}{X_t}\right)\widetilde{Y}_t + \left(\widetilde{R}_{c,t}z_{c,t} + \frac{1-\delta_{kc}}{a_{k,t}}\right)\frac{\widetilde{k}_{c,t-1}}{G_{KC}} + (R_{h,t}z_{h,t} + 1 - \delta_{kh})\,\frac{\widetilde{k}_{h,t-1}}{G_C} \\
& + p_{b,t}\widetilde{k}_{b,t} + \widetilde{b}_t + (\widetilde{p}_{l,t} + \widetilde{R}_{l,t})l_{t-1} - \widetilde{\Phi}_t - \frac{a(z_{c,t})\widetilde{k}_{c,t-1}}{a_{k,t}G_{KC}} - a(z_{h,t})\frac{\widetilde{k}_{h,t-1}}{G_C}, \qquad \text{(A.11)}
\end{aligned}
$$

where $\widetilde{c}_t = c_t/G_C^t$, $\widetilde{k}_{c,t} = k_{c,t}/G_{KC}^t$, $a_{k,t} = A_{k,t}/G_{AK}^t$, $\widetilde{k}_{h,t} = k_{h,t}/G_C^t$, $\widetilde{k}_{b,t} = k_{b,t}/G_C^t$, $\widetilde{q}_t = q_t/G_Q^t$, $\widetilde{h}_{o,t} = h_{o,t}/G_H^t$, $\widetilde{h}_{n,t} = h_{n,t}/G_H^t$, $\widetilde{p}_{l,t} = p_{l,t}/G_C^t$, $\widetilde{R}_{l,t} = R_{l,t}/G_C^t$, $\widetilde{b}_t = b_t/G_C^t$, $\widetilde{TC}_t = TC_t/G_C^t$, $\widetilde{w}_{c,t} = w_{c,t}/G_C^t$, $\widetilde{w}_{h,t} = w_{h,t}/G_C^t$, $\widetilde{R}_{c,t} = R_{c,t}G_{AK}^t$.

Note that

$$
\begin{aligned}
DIV_t &= \frac{X_t - 1}{X_t}Y_t + \frac{X_{wc,t} - 1}{X_{wc,t}}w_{c,t}n_{c,t} + \frac{X_{wh,t} - 1}{X_{wh,t}}w_{h,t}n_{h,t}, && \text{(A.12)} \\
TC_t &= \frac{\phi}{2G_H}\left(\frac{h_{o,t}}{h_{o,t-1}} - G_H\right)^2 h_{o,t-1}q_t f_t, && \text{(A.13)} \\
\Phi_t &= \frac{\phi_{kc}}{2G_{KC}}\left(\frac{k_{c,t}}{k_{c,t-1}} - G_{KC}\right)^2 \frac{k_{c,t-1}}{G_{AK}^t} + \frac{\phi_{kh}}{2G_C}\left(\frac{k_{h,t}}{k_{h,t-1}} - G_C\right)^2 k_{h,t-1}, && \text{(A.14)} \\
a(z_{c,t}) &= \widetilde{R}_c(\varpi z_{c,t}^2/2 + (1-\varpi)z_{c,t} + (\varpi/2 - 1)), && \text{(A.15)} \\
a(z_{h,t}) &= R_h(\varpi z_{h,t}^2/2 + (1-\varpi)z_{h,t} + (\varpi/2 - 1)), && \text{(A.16)}
\end{aligned}
$$

and the detrended counterparts of TC_t and Φ_t take the following form:

$$
\begin{aligned}
\widetilde{TC}_t &= \frac{\phi}{2}\left(\frac{\widetilde{h}_{o,t}}{\widetilde{h}_{o,t-1}} - 1\right)^2 \widetilde{h}_{o,t-1}\widetilde{q}_t f_t, && \text{(A.17)} \\
\widetilde{\Phi}_t &= \frac{\phi_{kc}}{2}\left(\frac{\widetilde{k}_{c,t}}{\widetilde{k}_{c,t-1}} - 1\right)^2 \widetilde{k}_{c,t-1} + \frac{\phi_{kh}}{2}\left(\frac{\widetilde{k}_{h,t}}{\widetilde{k}_{h,t-1}} - 1\right)^2 \widetilde{k}_{h,t-1}. && \text{(A.18)}
\end{aligned}
$$

The choice variables for unconstrained households are the following: $b_t, h_{o,t}, h_{n,t}, k_{c,t}, k_{h,t}, n_{c,t}, n_{h,t}, l_t$. The first-order conditions of unconstrained households' maximization problem are:

$$
b_t: \quad u_{c,t} = \beta G_C E_t\left(u_{c,t+1}\frac{R_{L,t}}{\pi_{t+1}}\right), \qquad \text{(A.19)}
$$

$$h_{o,t}: \quad u_{c,t}q_t f_t \left[1+\frac{\phi}{G_H}\left(\frac{h_{o,t}}{h_{o,t-1}}-G_H\right)\right]$$

$$= u_{h_o,t} + \beta G_C(1-\delta_h)E_t(u_{c,t+1}q_{t+1}f_{t+1}) + \beta G_C E_t\left[u_{c,t+1}q_{t+1}f_{t+1}\frac{\phi}{2G_H}\left(\frac{h^2_{o,t+1}}{h^2_{o,t}}-G_H^2\right)\right], \tag{A.20}$$

$$h_{n,t}: \quad u_{c,t}q_t = u_{h_n,t} + \beta G_C(1-\delta_h)E_t(u_{c,t+1}q_{t+1}f_{t+1}), \tag{A.21}$$

$$n_{c,t}: \quad u_{n_c,t} = u_{c,t}w_{c,t}/X_{wc,t}, \tag{A.22}$$

$$n_{h,t}: \quad u_{n_h,t} = u_{c,t}w_{h,t}/X_{wh,t}, \tag{A.23}$$

$$k_{c,t}: \quad u_{c,t}\left[\frac{1}{A_{k,t}}+\frac{\phi_{kc}}{G_{KC}}\left(\frac{k_{c,t}}{k_{c,t-1}}-G_{KC}\right)\frac{1}{G^t_{AK}}\right]$$

$$= \beta G_C E_t\left[u_{c,t+1}\left(R_{c,t+1}z_{c,t+1}+\frac{1-\delta_{kc}}{A_{k,t+1}}+\frac{\phi_{kc}}{2G_{KC}}\left(\frac{k^2_{c,t+1}}{k^2_{c,t}}-G^2_{KC}\right)\frac{1}{G^{t+1}_{AK}}-\frac{a(z_{c,t+1})}{A_{k,t+1}}\right)\right], \tag{A.24}$$

$$k_{h,t}: \quad u_{c,t}\left[1+\frac{\phi_{kh}}{G_C}\left(\frac{k_{h,t}}{k_{h,t-1}}-G_C\right)\right]$$

$$= \beta G_C E_t\left[u_{c,t+1}\left(R_{h,t+1}z_{h,t+1}+1-\delta_{kh}+\frac{\phi_{kh}}{2G_C}\left(\frac{k^2_{h,t+1}}{k^2_{h,t}}-G^2_C\right)-a(z_{h,t+1})\right)\right], \tag{A.25}$$

$$k_{b,t}: \quad u_{c,t}(p_{b,t}-1)=0, \tag{A.26}$$

$$l_t: \quad u_{c,t}p_{l,t} = \beta G_C E_t\left[u_{c,t+1}(p_{l,t+1}+R_{l,t+1})\right], \tag{A.27}$$

which can be transformed in the following way:

$$\widetilde{u}_{c,t} = \beta E_t\left(\widetilde{u}_{c,t+1}\frac{R_{L,t}}{\pi_{t+1}}\right), \tag{A.28}$$

$$\widetilde{u}_{c,t}\widetilde{q}_t f_t\left[1+\phi\left(\frac{\widetilde{h}_{o,t}}{\widetilde{h}_{o,t-1}}-1\right)\right]$$

$$=\widetilde{u}_{h_o,t}+\beta G_Q(1-\delta_h)E_t(\widetilde{u}_{c,t+1}\widetilde{q}_{t+1}f_{t+1})+\beta G_Q E_t\left[\widetilde{u}_{c,t+1}\widetilde{q}_{t+1}f_{t+1}\frac{\phi}{2}G_H\left(\frac{\widetilde{h}^2_{o,t+1}}{\widetilde{h}^2_{o,t}}-1\right)\right], \tag{A.29}$$

$$\widetilde{u}_{c,t}\widetilde{q}_t = \widetilde{u}_{h_n,t}+\beta G_Q(1-\delta_h)E_t(\widetilde{u}_{c,t+1}\widetilde{q}_{t+1}f_{t+1}), \tag{A.30}$$

$$u_{n_c,t} = \widetilde{u}_{c,t}\widetilde{w}_{c,t}/X_{wc,t}, \tag{A.31}$$

$$u_{n_h,t} = \widetilde{u}_{c,t}\widetilde{w}_{h,t}/X_{wh,t}, \tag{A.32}$$

$$\widetilde{u}_{c,t}\left[\frac{1}{a_{k,t}}+\phi_{kc}\left(\frac{\widetilde{k}_{c,t}}{\widetilde{k}_{c,t-1}}-1\right)\right]$$

$$=\frac{\beta}{G_{AK}}E_t\left[\widetilde{u}_{c,t+1}\left(\widetilde{R}_{c,t+1}z_{c,t+1}+\frac{1-\delta_{kc}}{a_{k,t+1}}+\frac{\phi_{kc}G_{KC}}{2}\left(\frac{\widetilde{k}^2_{c,t+1}}{\widetilde{k}^2_{c,t}}-1\right)-\frac{a(z_{c,t+1})}{a_{k,t+1}}\right)\right], \tag{A.33}$$

$$\widetilde{u}_{c,t}\left[1+\phi_{kh}\left(\frac{\widetilde{k}_{h,t}}{\widetilde{k}_{h,t-1}}-1\right)\right]$$

$$=\beta E_t\left[\widetilde{u}_{c,t+1}\left(R_{h,t+1}z_{h,t+1}+1-\delta_{kh}+\frac{\phi_{kh}G_C}{2}\left(\frac{\widetilde{k}^2_{h,t+1}}{\widetilde{k}^2_{h,t}}-1\right)-a(z_{h,t+1})\right)\right], \tag{A.34}$$

$$p_{b,t}=1, \tag{A.35}$$

$$\widetilde{u}_{c,t}\widetilde{p}_{l,t} = \beta G_C E_t\left[\widetilde{u}_{c,t+1}(\widetilde{p}_{l,t+1}+\widetilde{R}_{l,t+1})\right]. \tag{A.36}$$

The first-order conditions w.r.t. $z_{c,t}$ and $z_{h,t}$ are as follows:

$$R_{c,t} = \frac{a'(z_{c,t})}{A_{k,t}},$$
$$R_{h,t} = a'(z_{h,t}),$$

which can be transformed as:

$$\widetilde{R}_{c,t} = \frac{a'(z_{c,t})}{a_{k,t}} = \widetilde{R}_c(\varpi z_{c,t} + (1-\varpi)),$$
$$R_{h,t} = a'(z_{h,t}) = R_h(\varpi z_{h,t} + (1-\varpi)).$$

where $\widetilde{R}_c$ and R_h are the steady state values of the rental rates of the two types of capital.

A.2 Constrained Households

The lifetime utility of constrained households is given by:

$$V'_t = E_0 \sum_{t=0}^{\infty} (\beta' G_C)^t z_t \left[\frac{G_C - \epsilon'}{G_C - \beta'\epsilon' G_C} \ln(c'_t - \epsilon' c'_{t-1}) + j_{o,t} \ln(h'_{o,t}) + j_{n,t} \ln(h'_{n,t}) - \frac{\tau_t}{1+\eta'} \left((n'_{c,t})^{1+\xi'} + (n'_{h,t})^{1+\xi'} \right)^{\frac{1+\eta'}{1+\xi'}} \right]. \tag{A.37}$$

The marginal utility of consumption is:

$$u'_{c,t} = \left(\frac{G_C - \epsilon'}{G_C - \beta'\epsilon' G_C} \right) \left(\frac{z_t}{c'_t - \epsilon' c'_{t-1}} - \frac{\beta'\epsilon' G_C z_{t+1}}{c'_{t+1} - \epsilon' c'_t} \right). \tag{A.38}$$

The marginal utility of old house is:

$$u'_{h_o,t} = \frac{z_t j_{o,t}}{h'_{o,t}}, \tag{A.39}$$

and the marginal utility of new house is:

$$u'_{h_n,t} = \frac{z_t j_{n,t}}{h'_{n,t}}. \tag{A.40}$$

The marginal disutilities of working in the goods and the housing sectors are:

$$u'_{n_c,t} = z_t \tau_t \left((n'_{c,t})^{1+\xi'} + (n'_{h,t})^{1+\xi'} \right)^{\frac{\eta'-\xi'}{1+\xi'}} (n'_{c,t})^{\xi'}, \tag{A.41}$$
$$u'_{n_h,t} = z_t \tau_t \left((n'_{c,t})^{1+\xi'} + (n'_{h,t})^{1+\xi'} \right)^{\frac{\eta'-\xi'}{1+\xi'}} (n'_{h,t})^{\xi'}. \tag{A.42}$$

The transformed marginal utility of consumption $\widetilde{u}'_{c,t} = u'_{c,t} G_C^t$ is stationary around the steady state and is equal to:

$$\widetilde{u}'_{c,t} = \left(\frac{G_C - \epsilon'}{G_C - \beta'\epsilon' G_C} \right) \left(\frac{z_t}{\widetilde{c}'_t - \frac{\epsilon'}{G_C} \widetilde{c}'_{t-1}} - \frac{\beta'\epsilon' z_{t+1}}{\widetilde{c}'_{t+1} - \frac{\epsilon'}{G_C} \widetilde{c}'_t} \right). \tag{A.43}$$

Transformed consumption, $\widetilde{c}'_t = c'_t / G^t_C$, and the scaled marginal utility of consumption $\widetilde{u}'_{c,t}$:

$$\widetilde{u}'_c = z\left(\frac{G_C - \epsilon'}{G_C - \beta'\epsilon' G_C}\right)\left(\frac{1}{1-\frac{\epsilon'}{G_C}} - \frac{\beta'\epsilon'}{1-\frac{\epsilon'}{G_C}}\right)\frac{1}{\widetilde{c}'} = \frac{1}{\widetilde{c}'}, \tag{A.44}$$

are both stationary in the steady state.

The marginal utilities of housing $u'_{h_o,t} = z_t j_{o,t}/h'_{o,t}$ and $u'_{h_n,t} = z_t j_{n,t}/h'_{n,t}$ both decline at the rate G_H. Therefore the transformed marginal utilities $\widetilde{u}'_{h_o,t} = u'_{h_o,t} G^t_H$ and $\widetilde{u}'_{h_n,t} = u'_{h_n,t} G^t_H$ are stationary around the steady state and are equal to:

$$\widetilde{u}'_{h_o,t} = \frac{z_t j_{o,t}}{\widetilde{h}_{o,t}}, \quad \widetilde{u}'_{h_n,t} = \frac{z_t j_{n,t}}{\widetilde{h}_{n,t}}. \tag{A.45}$$

In the steady state, $\widetilde{u}'_{h_o} = j_o / \widetilde{h}'_o$ and $\widetilde{u}'_{h_n} = j_n / \widetilde{h}'_n$.

Notice that hours worked in the two sectors, $n'_{c,t}$ and $n'_{h,t}$, are stationary already in the level economy.

Constrained households' budget constraint is given by:

$$\begin{aligned} &c'_t + q_t[f_t(h'_{o,t} - (1-\delta_h)h'_{o,t-1} - (1-\delta_h)h'_{n,t-1}) + h'_{n,t}] + \frac{R_{L,t-1}b'_{t-1}}{\pi_t} + TC'_t \\ =&\frac{w'_{c,t}n'_{c,t}}{X_{wc,t}} + \frac{w'_{h,t}n'_{h,t}}{X_{wh,t}} + DIV'_t + b'_t, \end{aligned} \tag{A.46}$$

which can be transformed as follows:

$$\widetilde{c}'_t + \widetilde{q}_t\left[f_t\left(\widetilde{h}'_{o,t} - \frac{(1-\delta_h)\widetilde{h}'_{o,t-1}}{G_H} - \frac{(1-\delta_h)\widetilde{h}'_{n,t-1}}{G_H}\right) + \widetilde{h}'_{n,t}\right] + \frac{R_{L,t-1}}{\pi_t}\frac{\widetilde{b}'_{t-1}}{G_C} + \widetilde{TC}'_t = \widetilde{w}'_{c,t}n'_{c,t} + \widetilde{w}'_{h,t}n'_{h,t} + \widetilde{b}'_t, \tag{A.47}$$

where

$$TC'_t = \frac{\phi'}{2G_H}\left(\frac{h'_{o,t}}{h'_{o,t-1}} - G_H\right)^2 h'_{o,t-1}q_t f_t, \tag{A.48}$$

$$DIV'_t = \frac{X_{wc,t}-1}{X_{wc,t}}w'_{c,t}n'_{c,t} + \frac{X_{wh,t}-1}{X_{wh,t}}w'_{h,t}n'_{h,t}. \tag{A.49}$$

The borrowing constraint is given by:

$$b'_t = mE_t\left(\frac{f_{t+1}q_{t+1}(1-\delta_h)(h'_{o,t}+h'_{n,t})\pi_{t+1}}{R_{S,t}}\right), \tag{A.50}$$

which can be transformed as:

$$\widetilde{b}'_t = mE_t\left(\frac{G_Q f_{t+1}\widetilde{q}_{t+1}(1-\delta_h)(\widetilde{h}'_{o,t}+\widetilde{h}'_{n,t})\pi_{t+1}}{R_{S,t}}\right). \tag{A.51}$$

The choice variables for constrained households are the following: $b'_t, h'_{o,t}, h'_{n,t}, n'_{c,t}, n'_{h,t}$. The first-order conditions of constrained households' maximization problem are:

$$
\begin{aligned}
b'_t: \quad & u'_{c,t} = \beta' G_C E_t \left(\frac{u'_{c,t+1} R_{L,t}}{\pi_{t+1}} \right) + \lambda_t, && \text{(A.52)} \\
h'_{o,t}: \quad & u'_{c,t} q_t f_t \left[1 + \frac{\phi'}{G_H} \left(\frac{h'_{o,t}}{h'_{o,t-1}} - G_H \right) \right] \\
& = u'_{h_o,t} + \beta' G_C (1-\delta_h) E_t (u'_{c,t+1} q_{t+1} f_{t+1}) + \beta' G_C E_t \left[u_{c,t+1} q_{t+1} f_{t+1} \frac{\phi'}{2G_H} \left(\frac{h'^2_{o,t-1}}{h'^2_{o,t}} - G_H^2 \right) \right] \\
& \quad + E_t \left(\lambda_t \frac{m(1-\delta_h) q_{t+1} f_{t+1} \pi_{t+1}}{R_{S,t}} \right), && \text{(A.53)} \\
h'_{n,t}: \quad & u'_{c,t} q_t = u'_{h_n,t} + \beta' G_C (1-\delta_h) E_t (u'_{c,t+1} q_{t+1} f_{t+1}) + E_t \left(\lambda_t \frac{m(1-\delta_h) q_{t+1} f_{t+1} \pi_{t+1}}{R_{S,t}} \right), && \text{(A.54)} \\
n'_{c,t}: \quad & u'_{n_c,t} = u'_{c,t} w'_{c,t} / X_{wc,t}, && \text{(A.55)} \\
n'_{h,t}: \quad & u'_{n_h,t} = u'_{c,t} w'_{h,t} / X_{wh,t}. && \text{(A.56)}
\end{aligned}
$$

which can be transformed in the following way:

$$
\begin{aligned}
& \widetilde{u}'_{c,t} = \beta' E_t \left(\frac{\widetilde{u}'_{c,t+1} R_{L,t}}{\pi_{t+1}} \right) + \widetilde{\lambda}_t, && \text{(A.57)} \\
& \widetilde{u}'_{c,t} \widetilde{q}_t f_t \left[1 + \phi' \left(\frac{\widetilde{h}'_{o,t}}{\widetilde{h}'_{o,t-1}} - 1 \right) \right] \\
& = \widetilde{u}'_{h_o,t} + \beta' (1-\delta_h) E_t (\widetilde{u}'_{c,t+1} \widetilde{q}_{t+1} f_{t+1} G_Q) + \beta' G_Q E_t \left[\widetilde{u}'_{c,t+1} \widetilde{q}_{t+1} f_{t+1} \frac{\phi'}{2} G_H \left(\frac{\widetilde{h}'^2_{o,t+1}}{\widetilde{h}'^2_{o,t}} - 1 \right) \right] \\
& \quad + E_t \left(\widetilde{\lambda}_t \frac{m(1-\delta_h) \widetilde{q}_{t+1} f_{t+1} \pi_{t+1} G_Q}{R_{S,t}} \right), && \text{(A.58)} \\
& \widetilde{u}'_{c,t} \widetilde{q}_t = \widetilde{u}'_{h_n,t} + \beta' (1-\delta_h) E_t (\widetilde{u}'_{c,t+1} \widetilde{q}_{t+1} f_{t+1} G_Q) + E_t \left(\widetilde{\lambda}_t \frac{m(1-\delta_h) \widetilde{q}_{t+1} f_{t+1} \pi_{t+1} G_Q}{R_{S,t}} \right), && \text{(A.59)} \\
& u'_{n_c,t} = \widetilde{u}'_{c,t} \widetilde{w}'_{c,t} / X_{wc,t}, && \text{(A.60)} \\
& u'_{n_h,t} = \widetilde{u}'_{c,t} \widetilde{w}'_{h,t} / X_{wh,t}. && \text{(A.61)}
\end{aligned}
$$

A.3 Firms

Wholesale firms solve the following maximization problem:

$$
\max \frac{Y_t}{X_t} + q_t IH_t - \left(\sum_{i=c,h} w_{i,t} n_{i,t} + \sum_{i=c,h} w'_{i,t} n'_{i,t} + R_{c,t} z_{c,t} k_{c,t-1} + R_{h,t} z_{h,t} k_{h,t-1} + p_{b,t} k_{b,t} + R_{l,t} l_{t-1} \right).
$$

The two production technologies are:

$$
\begin{aligned}
Y_t &= [A_{c,t} (n^{\alpha}_{c,t} n'^{1-\alpha}_{c,t})]^{1-\mu_c} (z_{c,t} k_{c,t-1})^{\mu_c}, && \text{(A.62)} \\
IH_t &= [A_{h,t} (n^{\alpha}_{h,t} n'^{1-\alpha}_{h,t})]^{1-\mu_h-\mu_b-\mu_l} (z_{h,t} k_{h,t-1})^{\mu_h} k^{\mu_b}_{b,t} l^{\mu_l}_{t-1}, && \text{(A.63)}
\end{aligned}
$$

which can be transformed as:

$$\widetilde{Y}_t = [a_{c,t}(n_{c,t}^{\alpha} n_{c,t}^{\prime 1-\alpha})]^{1-\mu_c} \left(\frac{z_{c,t}\widetilde{k}_{c,t-1}}{G_{KC}} \right)^{\mu_c}, \tag{A.64}$$

$$\widetilde{IH}_t = [a_{h,t}(n_{h,t}^{\alpha} n_{h,t}^{\prime 1-\alpha})]^{1-\mu_h-\mu_b-\mu_l} \left(\frac{z_{h,t}k_{h,t-1}}{G_C} \right)^{\mu_h} \widetilde{k}_{b,t}^{\mu_b}. \tag{A.65}$$

The first-order conditions are:

$$n_{c,t}: \quad (1-\mu_c)\alpha \frac{Y_t}{X_t n_{c,t}} = w_{c,t}, \tag{A.66}$$

$$n'_{c,t}: \quad (1-\mu_c)(1-\alpha) \frac{Y_t}{X_t n'_{c,t}} = w'_{c,t}, \tag{A.67}$$

$$n_{h,t}: \quad (1-\mu_h-\mu_b-\mu_l)\alpha \frac{q_t IH_t}{n_{h,t}} = w_{h,t}, \tag{A.68}$$

$$n'_{h,t}: \quad (1-\mu_h-\mu_b-\mu_l)(1-\alpha) \frac{q_t IH_t}{n'_{h,t}} = w'_{h,t}, \tag{A.69}$$

$$k_{c,t-1}: \quad \mu_c \frac{Y_t}{X_t k_{c,t-1}} = R_{c,t} z_{c,t}, \tag{A.70}$$

$$k_{h,t-1}: \quad \mu_h \frac{q_t IH_t}{k_{h,t-1}} = R_{h,t} z_{h,t}, \tag{A.71}$$

$$k_{b,t}: \quad \mu_b q_t IH_t / k_{b,t} = p_{b,t} = 1, \tag{A.72}$$

$$l_{t-1}: \quad \mu_l q_t IH_t = R_{l,t}, \tag{A.73}$$

which can be transformed as:

$$(1-\mu_c)\alpha \frac{\widetilde{Y}_t}{X_t n_{c,t}} = \widetilde{w}_{c,t}, \tag{A.74}$$

$$(1-\mu_c)(1-\alpha) \frac{\widetilde{Y}_t}{X_t n'_{c,t}} = \widetilde{w}'_{c,t}, \tag{A.75}$$

$$(1-\mu_h-\mu_b-\mu_l)\alpha \frac{\widetilde{q}_t \widetilde{IH}_t}{n_{h,t}} = \widetilde{w}_{h,t}, \tag{A.76}$$

$$(1-\mu_h-\mu_b-\mu_l)(1-\alpha) \frac{\widetilde{q}_t \widetilde{IH}_t}{n'_{h,t}} = \widetilde{w}'_{h,t}, \tag{A.77}$$

$$\frac{\mu_c G_{KC}}{X_t} \frac{\widetilde{Y}_t}{\widetilde{k}_{c,t-1}} = \widetilde{R}_{c,t} z_{c,t}, \tag{A.78}$$

$$\mu_h G_C \frac{\widetilde{q}_t \widetilde{IH}_t}{\widetilde{k}_{h,t-1}} = R_{h,t} z_{h,t}, \tag{A.79}$$

$$\mu_b \widetilde{q}_t \widetilde{IH}_t / \widetilde{k}_{b,t} = 1, \tag{A.80}$$

$$\mu_l \widetilde{q}_t \widetilde{IH}_t = \widetilde{R}_{l,t}. \tag{A.81}$$

A.4 Wage Stickiness

As in Iacoviello and Neri (2010), two types of households supply their homogenous labor services to labor unions. There are two unions for each sector, each one acting in the interest of either type of households. The unions differentiate labor services, set nominal wages subject to a Calvo scheme and offer labor services to intermediate labor packers who assemble the differentiated labor services into the homogenous labor composites n_c, n'_c, n_h, n'_h and provide wholesale firms with labor services. Partial indexation of nominal wages to past inflation yields the following four wage Phillips curves:

$$\ln \omega_{c,t} - \iota_{wc} \ln \pi_{t-1} = \beta G_C (E_t \ln \omega_{c,t+1} - \iota_{wc} \ln \pi_t) - \epsilon_{wc} \ln(X_{wc,t}/X_{wc}), \tag{A.82}$$
$$\ln \omega'_{c,t} - \iota_{wc} \ln \pi_{t-1} = \beta' G_C (E_t \ln \omega'_{c,t+1} - \iota_{wc} \ln \pi_t) - \epsilon'_{wc} \ln(X'_{wc,t}/X_{wc}), \tag{A.83}$$
$$\ln \omega_{h,t} - \iota_{wh} \ln \pi_{t-1} = \beta G_C (E_t \ln \omega_{h,t+1} - \iota_{wh} \ln \pi_t) - \epsilon_{wh} \ln(X_{wh,t}/X_{wh}), \tag{A.84}$$
$$\ln \omega'_{h,t} - \iota_{wh} \ln \pi_{t-1} = \beta' G_C (E_t \ln \omega'_{h,t+1} - \iota_{wh} \ln \pi_t) - \epsilon'_{wh} \ln(X'_{wh,t}/X_{wh}), \tag{A.85}$$

with $\omega_{i,t}$ nominal wage inflation, that is $\omega_{i,t} = \pi_t w_{i,t}/w_{i,t-1}$ for each sector/household pair, and

$$\epsilon_{wc} = (1-\theta_{wc})(1-\beta G_C \theta_{wc})/\theta_{wc}, \tag{A.86}$$
$$\epsilon'_{wc} = (1-\theta_{wc})(1-\beta' G_C \theta_{wc})/\theta_{wc}, \tag{A.87}$$
$$\epsilon_{wh} = (1-\theta_{wh})(1-\beta G_C \theta_{wh})/\theta_{wh}, \tag{A.88}$$
$$\epsilon'_{wh} = (1-\theta_{wh})(1-\beta' G_C \theta_{wh})/\theta_{wh}. \tag{A.89}$$

A.5 Price Stickiness

Price stickiness in the consumption-business investment sector is introduced by assuming monopolistic competition at the retail level, implicit costs of adjusting nominal prices following Calvo-style contracts and partial indexation to lagged inflation of those prices that cannot be reoptimized. The resulting inflation equation is:

$$\ln \pi_t - \iota_\pi \ln \pi_{t-1} = \beta G_C (E_t \ln \pi_{t+1} - \iota_\pi \ln \pi_t) - \epsilon_\pi \ln (X_t/X) + u_{p,t}, \tag{A.90}$$

where $\epsilon_\pi = (1-\theta_\pi)(1-\beta G_C \theta_\pi)/\theta_\pi$.

A.6 Monetary Policy

It is assumed that yields of different maturities are driven by two zero-mean latent factors L_t and S_t:

$$\ln R_{j,t} = \lambda_j + \mathbf{\Lambda}_j \mathbf{F}_t, \tag{A.91}$$

where λ_j is a constant and $\mathbf{F}_t = (L_t, S_t)'$. The factor loadings on these two yield curve components, $\mathbf{\Lambda}_j$, are modeled as:

$$\mathbf{\Lambda}_j = \left(1, \quad \frac{1-e^{-\delta j}}{\delta j}\right), \tag{A.92}$$

where δ denotes a decay parameter and j maturity. The loadings on the first factor, L_t, are constant across the maturity spectrum. A positive shock to this factor induces an essentially parallel shift in the term structure that boosts the level of the whole yield curve, so the L_t factor is often called a "level" factor. The loadings on the second factor, S_t, decrease monotonically with the maturity. A positive shock to this factor increases short-term yields by much more than the long-term yields, thus S_t is termed the "slope" factor.

The dynamics of the these latent factors are specified as:

$$L_t = \gamma_L L_{t-1} + (1-\gamma_L) \ln \pi_t + \varepsilon_{L,t}, \tag{A.93}$$

$$S_t = \gamma_S S_{t-1} + (1-\gamma_S)\gamma_\pi \ln \pi_t + (1-\gamma_S)\gamma_Y \ln\left(\frac{GDP_t}{G_C GDP_{t-1}}\right) + u_{S,t}, \tag{A.94}$$

where $u_{S,t} = \rho_S u_{S,t-1} + \varepsilon_{S,t}$; $\varepsilon_{L,t}$ and $\varepsilon_{S,t}$ are independently and identically distributed shocks to the level factor and to the slope factor respectively with variances σ_L^2 and σ_S^2. GDP_t is defined as the sum of the value added of two sector, i.e., $GDP_t = Y_t - k_{b,t} + \overline{q} IH_t$.

The interest rate data used in this study are the nominal 3-month interest rate and zero-coupon yields of maturities 4, 12, 24, 36, 48, and 60 quarters. In the setup, the short-term nominal rate $R_{S,t}$ refers to the 1-quarter rate $R_{1,t}$ and the long-term nominal rate $R_{L,t}$ refers to the 60-quarter rate $R_{60,t}$.

A.7 Market Clearing

The market clearing conditions are:

$$C_t + IK_{c,t}/A_{k,t} + IK_{h,t} + k_{b,t} = Y_t - \Phi_t, \tag{A.95}$$

$$h_{n,t} + h'_{n,t} = IH_t, \tag{A.96}$$

$$(h_{o,t} - (1-\delta_h)h_{o,t-1} - (1-\delta_h)h_{n,t-1}) + (h'_{o,t} - (1-\delta_h)h'_{o,t-1} - (1-\delta_h)h'_{n,t-1}) = 0, \tag{A.97}$$

$$b_t + b'_t = 0, \tag{A.98}$$

which can be transformed as:

$$\widetilde{C}_t + \widetilde{IK}_{c,t}/a_{k,t} + \widetilde{IK}_{h,t} + \widetilde{k}_{b,t} = \widetilde{Y}_t - \widetilde{\Phi}_t, \tag{A.99}$$

$$\widetilde{h}_{n,t} + \widetilde{h}'_{n,t} = \widetilde{IH}_t, \tag{A.100}$$

$$\left(\widetilde{h}_{o,t} - \frac{(1-\delta_h)\widetilde{h}_{o,t-1}}{G_H} - \frac{(1-\delta_h)\widetilde{h}_{n,t-1}}{G_H}\right) + \left(\widetilde{h}'_{o,t} - \frac{(1-\delta_h)\widetilde{h}'_{o,t-1}}{G_H} - \frac{(1-\delta_h)\widetilde{h}'_{n,t-1}}{G_H}\right) = 0, \tag{A.101}$$

$$\widetilde{b}_t + \widetilde{b}'_t = 0, \tag{A.102}$$

B Linear Deterministic Trends

Suppose there are linear deterministic trends in the technologies A_c, A_h, and A_k. Let the corresponding gross growth rates be respectively:

$$\gamma_{AC}, \gamma_{AH}, \gamma_{AK}.$$

Because of these trends, the variables:

$$Y, c, c', \frac{k_c}{A_k}, k_h, qIH$$

all grow at a common rate along the balanced growth path.

The following is obtained from the production function,

$$\gamma_Y = \gamma_C = (1 - \mu_c)\gamma_{AC} + \mu_c \gamma_{KC}.$$

Since $\gamma_Y = \gamma_{KC} - \gamma_{AK}$, it follows that

$$\begin{aligned}
\gamma_Y &= \gamma_{AC} + \frac{\mu_c}{1 - \mu_c}\gamma_{AK}, \\
\gamma_{KC} &= \gamma_{AC} + \frac{1}{1 - \mu_c}\gamma_{AK}, \\
\gamma_{IH} &= (1 - \mu_h - \mu_b - \mu_l)\gamma_{AH} + (\mu_h + \mu_b)\gamma_Y, \\
&= (1 - \mu_h - \mu_b - \mu_l)\gamma_{AH} + (\mu_h + \mu_b)\left(\gamma_{AC} + \frac{\mu_c}{1 - \mu_c}\gamma_{AK}\right), \\
\gamma_Q &= \gamma_Y - \gamma_{IH} = (1 - \mu_h - \mu_b)\left(\gamma_{AC} + \frac{\mu_c}{1 - \mu_c}\gamma_{AK}\right) - (1 - \mu_h - \mu_b - \mu_l)\gamma_{AH}.
\end{aligned}$$

C Steady State of the Model

C.1 Calculations

Marginal utilities of consumption, old house, and new house are equal, respectively, to $1/\widetilde{c}$ (or $1/\widetilde{c}'$), $j_o/\widetilde{h}_o$ (or $j_o/\widetilde{h}'_o$), and $j_n/\widetilde{h}_n$ (or $j_n/\widetilde{h}'_n$). From the transformed consumption Euler equation, the steady state level of the real interest rate can be derived once $\overline{\pi} = 1$ has been imposed:

$$R_L = \frac{1}{\beta}. \tag{C.1}$$

From the Euler equations for the two capital stocks, the steady state values for the rental rates can be derived:

$$\widetilde{R}_c = \frac{G_{AK}}{\beta} - (1 - \delta_{kc}), \tag{C.2}$$

$$R_h = \frac{1}{\beta} - (1 - \delta_{kh}), \tag{C.3}$$

$$r \equiv \frac{R_L}{G_C} - 1. \tag{C.4}$$

Combining the Euler equation for k_c from the optimal demand for capital by firms in the good sector and the expression for $\widetilde{R}_c$, the following ratio is obtained:

$$\zeta_0 = \frac{\widetilde{k}_c}{\widetilde{Y}} = \frac{\mu_c G_{KC}}{\widetilde{R}_c X} = \frac{\beta \mu_c G_{KC}}{(G_{AK} - \beta(1 - \delta_{kc}))X}. \tag{C.5}$$

Combing the Euler equation for k_h from the optimal demand for capital by firms in the good sector and the expression for R_h the following ratio is obtained:

$$\zeta_1 = \frac{\widetilde{k}_h}{\widetilde{q}\widetilde{IH}} = \frac{\mu_h G_C}{R_h} = \frac{\beta \mu_h G_C}{1 - \beta(1 - \delta_{kh})}. \tag{C.6}$$

From the Euler equations for h_o and h_n:

$$\zeta_2 = \frac{f\widetilde{q}\widetilde{h}_o}{\widetilde{c}} = \frac{j_o}{1 - \beta G_Q(1 - \delta_h)}, \tag{C.7}$$

$$\zeta_3 = \frac{\widetilde{q}\widetilde{h}_n}{\widetilde{c}} = \frac{j_n}{1 - f\beta G_Q(1 - \delta_h)}. \tag{C.8}$$

From the Euler equations for h'_o, h'_n and b':

$$\zeta_4 = \frac{f\widetilde{q}\widetilde{h}'_o}{\widetilde{c}'} = \frac{j_o}{1 - \beta' G_Q(1 - \delta_h) - m(1 - \beta'/\beta)G_Q(1 - \delta_h)/R_S}, \tag{C.9}$$

$$\zeta_5 = \frac{\widetilde{q}\widetilde{h}'_n}{\widetilde{c}'} = \frac{j_n}{1 - f\beta' G_Q(1 - \delta_h) - fm(1 - \beta'/\beta)G_Q(1 - \delta_h)/R_S}, \tag{C.10}$$

$$\widetilde{\lambda} = \frac{1}{\widetilde{c}'}\left(1 - \frac{\beta'}{\beta}\right). \tag{C.11}$$

Using the Walras' law $\widetilde{b}+\widetilde{b}'=0$ where $\widetilde{b}'=mG_Q(1-\delta_h)f\widetilde{q}(\widetilde{h}'_o+\widetilde{h}'_n)/R_S$, steady state repayment is $r\widetilde{b}'$:

$$r\widetilde{b}'=\frac{rmG_Q(1-\delta_h)}{R_S}f\widetilde{q}(\widetilde{h}'_o+\widetilde{h}'_n).$$

Define the adjusted depreciation rates:

$$\widetilde{\delta}_h=1-\frac{1-\delta_h}{G_H},\quad \widetilde{\delta}_{kc}=1-\frac{1-\delta_{kc}}{G_{KC}},\quad \widetilde{\delta}_{kh}=1-\frac{1-\delta_{kh}}{G_{KH}}. \tag{C.12}$$

From above ratios and the budget constraints of the two types of households, the following is obtained:

$$\widetilde{k}_c=\zeta_0\widetilde{Y}, \tag{C.13}$$
$$\widetilde{k}_h=\zeta_1\widetilde{q}\widetilde{IH}, \tag{C.14}$$
$$f\widetilde{q}\widetilde{h}_o=\zeta_2\widetilde{c}, \tag{C.15}$$
$$\widetilde{q}\widetilde{h}_n=\zeta_3\widetilde{c}, \tag{C.16}$$
$$f\widetilde{q}\widetilde{h}'_o=\zeta_4\widetilde{c}', \tag{C.17}$$
$$\widetilde{q}\widetilde{h}'_n=\zeta_5\widetilde{c}', \tag{C.18}$$
$$\widetilde{h}_n+\widetilde{h}'_n=\widetilde{IH}, \tag{C.19}$$
$$\widetilde{\delta}_h\widetilde{h}_o-(1-\widetilde{\delta}_h)\widetilde{h}_n+\widetilde{\delta}_h\widetilde{h}'_o-(1-\widetilde{\delta}_h)\widetilde{h}'_n=0, \tag{C.20}$$
$$\widetilde{c}+\widetilde{c}'+\widetilde{\delta}_{kc}\widetilde{k}_c+\widetilde{\delta}_{kh}\widetilde{k}_h+\widetilde{k}_b=\widetilde{Y}, \tag{C.21}$$
$$\begin{aligned}\widetilde{c}+\widetilde{q}\left[f\left(\widetilde{\delta}_h\widetilde{h}_o-(1-\widetilde{\delta}_h)\widetilde{h}_n\right)+\widetilde{h}_n\right]&=\frac{X-1}{X}\widetilde{Y}+r\widetilde{k}_c+r\widetilde{k}_h+\mu_l\widetilde{q}\widetilde{IH}+\widetilde{w}_cn_c+\widetilde{w}_hn_h\\&\quad+\frac{rmG_Q(1-\delta_h)}{R_S}f\widetilde{q}(\widetilde{h}'_o+\widetilde{h}'_n),\end{aligned} \tag{C.22}$$
$$\widetilde{c}'+\widetilde{q}\left[f\left(\widetilde{\delta}_h\widetilde{h}'_o-(1-\widetilde{\delta}_h)\widetilde{h}'_n\right)+\widetilde{h}'_n\right]=\widetilde{w}'_cn'_c+\widetilde{w}'_hn'_h-\frac{rmG_Q(1-\delta_h)}{R_S}f\widetilde{q}(\widetilde{h}'_o+\widetilde{h}'_n). \tag{C.23}$$

The equations in the labor market satisfy from the demand side:

$$(1-\mu_c)\alpha\frac{\widetilde{Y}}{Xn_c}=\widetilde{w}_c, \tag{C.24}$$
$$(1-\mu_c)(1-\alpha)\frac{\widetilde{Y}}{Xn'_c}=\widetilde{w}'_c, \tag{C.25}$$
$$(1-\mu_h-\mu_b-\mu_l)\alpha\frac{\widetilde{q}\widetilde{IH}}{n_h}=\widetilde{w}_h, \tag{C.26}$$
$$(1-\mu_h-\mu_b-\mu_l)(1-\alpha)\frac{\widetilde{q}\widetilde{IH}}{n'_h}=\widetilde{w}'_h. \tag{C.27}$$

The total wage bill plus union dividends earned by each group follows:

$$\widetilde{w}_cn_c+\widetilde{w}_hn_h=\alpha\left((1-\mu_c)\frac{\widetilde{Y}}{X}+(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH}\right), \tag{C.28}$$
$$\widetilde{w}'_cn'_c+\widetilde{w}'_hn'_h=(1-\alpha)\left((1-\mu_c)\frac{\widetilde{Y}}{X}+(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH}\right). \tag{C.29}$$

Then,

$$\zeta_3\widetilde{c}+\zeta_5\widetilde{c}' = \widetilde{q}\widetilde{IH}, \tag{C.30}$$

$$\widetilde{c}+\widetilde{c}'+\widetilde{\delta}_{kc}\zeta_0\widetilde{Y}+\widetilde{\delta}_{kh}\zeta_1\widetilde{q}\widetilde{IH}+\mu_b\widetilde{q}\widetilde{IH} = \widetilde{Y}, \tag{C.31}$$

$$\begin{aligned}\widetilde{c}+\widetilde{\delta}_h\zeta_2\widetilde{c}+(1-f(1-\widetilde{\delta}_h))\zeta_3\widetilde{c} = {} & \frac{X-1}{X}\widetilde{Y}+r\zeta_o\widetilde{Y}+r\zeta_1\widetilde{q}\widetilde{IH}+\mu_l\widetilde{q}\widetilde{IH}\\ & +\alpha\left((1-\mu_c)\frac{\widetilde{Y}}{X}+(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH}\right)\\ & +\frac{rmG_Q(1-\delta_h)}{R_S}(\zeta_4+f\zeta_5)\widetilde{c}',\end{aligned} \tag{C.32}$$

$$\begin{aligned}\widetilde{c}'+\widetilde{\delta}_h\zeta_4\widetilde{c}'+(1-f(1-\widetilde{\delta}_h))\zeta_5\widetilde{c}' = {} & (1-\alpha)\left((1-\mu_c)\frac{\widetilde{Y}}{X}+(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH}\right)\\ & -\frac{rmG_Q(1-\delta_h)}{R_S}(\zeta_4+f\zeta_5)\widetilde{c}'.\end{aligned} \tag{C.33}$$

Using the formula for $\widetilde{q}\widetilde{IH}$:

$$\begin{aligned}&\left(1+\widetilde{\delta}_h\zeta_2+(1-f(1-\widetilde{\delta}_h))\zeta_3\right)\widetilde{c}\\ =&\left(\frac{X-1}{X}+r\zeta_0\right)\widetilde{Y}+(r\zeta_1+\mu_l)(\zeta_3\widetilde{c}+\zeta_5\widetilde{c}')+\alpha\left((1-\mu_c)\frac{\widetilde{Y}}{X}+(1-\mu_h-\mu_b-\mu_l)(\zeta_3\widetilde{c}+\zeta_5\widetilde{c}')\right)\\ &+\frac{rmG_Q(1-\delta_h)}{R_S}(\zeta_4+f\zeta_5)\widetilde{c}',\end{aligned} \tag{C.34}$$

$$\begin{aligned}&\left(1+\widetilde{\delta}_h\zeta_4+(1-f(1-\widetilde{\delta}_h))\zeta_5\right)\widetilde{c}'\\ =&(1-\alpha)\left((1-\mu_c)\frac{\widetilde{Y}}{X}+(1-\mu_h-\mu_b-\mu_l)(\zeta_3\widetilde{c}+\zeta_5\widetilde{c}')\right)-\frac{rmG_Q(1-\delta_h)}{R_S}(\zeta_4+f\zeta_5)\widetilde{c}'.\end{aligned} \tag{C.35}$$

or equivalently,

$$\begin{aligned}&\left(1+\widetilde{\delta}_h\zeta_2+(1-f(1-\widetilde{\delta}_h))\zeta_3-(r\zeta_1+\mu_l)\zeta_3-\alpha(1-\mu_h-\mu_b-\mu_l)\zeta_3\right)\widetilde{c}\\ &-((r\zeta_1+\mu_l)\zeta_5+\alpha(1-\mu_h-\mu_b-\mu_l)\zeta_5+rmG_Q(1-\delta_h)(\zeta_4+f\zeta_5)/R_S)\,\widetilde{c}' = \left(\frac{X-1}{X}+r\zeta_0+\frac{\alpha(1-\mu_c)}{X}\right)\widetilde{Y},\end{aligned} \tag{C.36}$$

and

$$\begin{aligned}&\left(1+\widetilde{\delta}_h\zeta_4+(1-f(1-\widetilde{\delta}_h))\zeta_5-(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\zeta_5+rmG_Q(1-\delta_h)(\zeta_4+f\zeta_5)/R_S\right)\widetilde{c}'\\ &-((1-\alpha)(1-\mu_h-\mu_b-\mu_l)\zeta_3)\,\widetilde{c} = \frac{(1-\alpha)(1-\mu_c)}{X}\widetilde{Y}.\end{aligned} \tag{C.37}$$

Redefining

$$\Xi_1\frac{\widetilde{c}}{\widetilde{Y}}-\Xi_2\frac{\widetilde{c}'}{\widetilde{Y}} = \Xi_3, \tag{C.38}$$

$$\Xi_4\frac{\widetilde{c}'}{\widetilde{Y}}-\Xi_5\frac{\widetilde{c}}{\widetilde{Y}} = \Xi_6, \tag{C.39}$$

delivers the following solution:

$$\frac{\widetilde{c}}{\widetilde{Y}} = \frac{\Xi_3\Xi_4 + \Xi_2\Xi_6}{\Xi_1\Xi_4 - \Xi_2\Xi_5}, \tag{C.40}$$

$$\frac{\widetilde{c}'}{\widetilde{Y}} = \frac{\Xi_1\Xi_6 + \Xi_3\Xi_5}{\Xi_1\Xi_4 - \Xi_2\Xi_5}, \tag{C.41}$$

$$\frac{\widetilde{q}\widetilde{IH}}{\widetilde{Y}} = \zeta_3\frac{\widetilde{c}}{\widetilde{Y}} + \zeta_5\frac{\widetilde{c}'}{\widetilde{Y}}. \tag{C.42}$$

Plug back to the rest equation,

$$(1 + (\widetilde{\delta}_{kh}\zeta_1 + \mu_b)\zeta_3)\frac{\Xi_3\Xi_4 + \Xi_2\Xi_6}{\Xi_1\Xi_4 - \Xi_2\Xi_5} + (1 + (\widetilde{\delta}_{kh}\zeta_1 + \mu_b)\zeta_5)\frac{\Xi_1\Xi_6 + \Xi_3\Xi_5}{\Xi_1\Xi_4 - \Xi_2\Xi_5} = 1 - \widetilde{\delta}_{kc}\zeta_0. \tag{C.43}$$

Rearrange the above equation and solve for f,

$$(1 + (\widetilde{\delta}_{kh}\zeta_1 + \mu_b)\zeta_3)(\Xi_3\Xi_4 + \Xi_2\Xi_6) + (1 + (\widetilde{\delta}_{kh}\zeta_1 + \mu_b)\zeta_5)(\Xi_1\Xi_6 + \Xi_3\Xi_5) = (1 - \widetilde{\delta}_{kc}\zeta_0)(\Xi_1\Xi_4 - \Xi_2\Xi_5). \tag{C.44}$$

C.2 Levels

In order to compute the levels of the variables in steady state, the value of hours worked is needed. The labor market equilibrium is of the kind:

$$(1-\mu_c)\alpha\frac{\widetilde{Y}}{X} = \widetilde{c}X_{wc}\left(n_c^{1+\xi} + n_h^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}} n_c^{1+\xi}, \tag{C.45}$$

$$(1-\mu_h-\mu_b-\mu_l)\alpha\widetilde{q}\widetilde{IH} = \widetilde{c}X_{wh}\left(n_c^{1+\xi} + n_h^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}} n_h^{1+\xi}, \tag{C.46}$$

so that the ratio of hours worked is:

$$\frac{n_h}{n_c} = \left(\frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH}X}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{1}{1+\xi}}, \tag{C.47}$$

where $X_{wc} = X_{wh} \equiv X_w$.

Plug back to get:

$$(1-\mu_c)\alpha\frac{\widetilde{Y}}{X\widetilde{c}} = X_w\left(1 + \frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH}X}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{\eta-\xi}{1+\xi}} n_c^{1+\eta}, \tag{C.48}$$

knowing $\widetilde{c}/\widetilde{Y}$ and $\widetilde{q}\widetilde{IH}/\widetilde{Y}$, this can be solved for n_c,

$$n_c = \left(\frac{(1-\mu_c)\alpha\frac{\widetilde{Y}}{X\widetilde{c}}}{X_w\left(1 + \frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IH}X}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{\eta-\xi}{1+\xi}}}\right)^{\frac{1}{1+\eta}}, \tag{C.49}$$

$$n_h = \left(\frac{(1-\mu_c)\alpha \frac{\widetilde{Y}}{X\widetilde{c}}}{X_w \left(1 + \frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IHX}}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{\eta-\xi}{1+\xi}}} \right)^{\frac{1}{1+\eta}} \left(\frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IHX}}{(1-\mu_c)\widetilde{Y}} \right)^{\frac{1}{1+\xi}}. \tag{C.50}$$

Similarly,

$$n'_c = \left(\frac{(1-\mu_c)(1-\alpha) \frac{\widetilde{Y}}{X\widetilde{c}'}}{X_w \left(1 + \frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IHX}}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{\eta'-\xi'}{1+\xi'}}} \right)^{\frac{1}{1+\eta'}}, \tag{C.51}$$

$$n'_h = \left(\frac{(1-\mu_c)(1-\alpha) \frac{\widetilde{Y}}{X\widetilde{c}'}}{X_w \left(1 + \frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IHX}}{(1-\mu_c)\widetilde{Y}}\right)^{\frac{\eta'-\xi'}{1+\xi'}}} \right)^{\frac{1}{1+\eta'}} \left(\frac{(1-\mu_h-\mu_b-\mu_l)\widetilde{q}\widetilde{IHX}}{(1-\mu_c)\widetilde{Y}} \right)^{\frac{1}{1+\xi'}}. \tag{C.52}$$

Once the levels of hours worked by the two households in the two sectors are obtained, $\widetilde{Y}$, $\widetilde{k}_c$, and $\widetilde{k}_h$ can be computed,

$$\widetilde{Y} = \left[a_c \left(n_c^{\alpha} n_c'^{1-\alpha}\right)\right]^{1-\mu_c} \left(\frac{z_c \widetilde{k}_c}{G_{KC}}\right)^{\mu_c} = \left(n_c^{\alpha} n_c'^{1-\alpha}\right)^{1-\mu_c} \left(\frac{\widetilde{k}_c}{G_{KC}}\right)^{\mu_c}, \tag{C.53}$$

where $\widetilde{k}_c = \zeta_0 \widetilde{Y}$. As a result,

$$\widetilde{Y} = \left(n_c^{\alpha} n_c'^{1-\alpha}\right) \left(\frac{\zeta_0}{G_{KC}}\right)^{\frac{\mu_c}{1-\mu_c}}, \tag{C.54}$$

$$\widetilde{k}_c = \zeta_0 \left(n_c^{\alpha} n_c'^{1-\alpha}\right) \left(\frac{\zeta_0}{G_{KC}}\right)^{\frac{\mu_c}{1-\mu_c}}, \tag{C.55}$$

$$\widetilde{k}_h = \frac{\zeta_1 \widetilde{q}\widetilde{IH}}{\widetilde{Y}} \widetilde{Y} = \zeta_1 \left(\zeta_3 \frac{\Xi_3\Xi_4 + \Xi_2\Xi_6}{\Xi_1\Xi_4 - \Xi_2\Xi_5} + \zeta_5 \frac{\Xi_1\Xi_6 + \Xi_3\Xi_5}{\Xi_1\Xi_4 - \Xi_2\Xi_5} \right) \left(n_c^{\alpha} n_c'^{1-\alpha}\right) \left(\frac{\zeta_0}{G_{KC}}\right)^{\frac{\mu_c}{1-\mu_c}}, \tag{C.56}$$

$$\widetilde{k}_b = \mu_b \widetilde{q}\widetilde{IH} = \mu_b \left(\zeta_3 \frac{\Xi_3\Xi_4 + \Xi_2\Xi_6}{\Xi_1\Xi_4 - \Xi_2\Xi_5} + \zeta_5 \frac{\Xi_1\Xi_6 + \Xi_3\Xi_5}{\Xi_1\Xi_4 - \Xi_2\Xi_5} \right) \left(n_c^{\alpha} n_c'^{1-\alpha}\right) \left(\frac{\zeta_0}{G_{KC}}\right)^{\frac{\mu_c}{1-\mu_c}}. \tag{C.57}$$

$\widetilde{c}$ and $\widetilde{c}'$ can also be computed,

$$\widetilde{c} = \frac{\Xi_3\Xi_4 + \Xi_2\Xi_6}{\Xi_1\Xi_4 - \Xi_2\Xi_5} \widetilde{Y}, \tag{C.58}$$

$$\widetilde{c}' = \frac{\Xi_1\Xi_6 + \Xi_3\Xi_5}{\Xi_1\Xi_4 - \Xi_2\Xi_5} \widetilde{Y}, \tag{C.59}$$

$$\widetilde{q}\widetilde{IH} = \left(\zeta_3 \frac{\Xi_3\Xi_4 + \Xi_2\Xi_6}{\Xi_1\Xi_4 - \Xi_2\Xi_5} + \zeta_5 \frac{\Xi_1\Xi_6 + \Xi_3\Xi_5}{\Xi_1\Xi_4 - \Xi_2\Xi_5} \right) \widetilde{Y}. \tag{C.60}$$

Using the market clearing conditions for two types of houses,

$$0 = \widetilde{\delta}_h \widetilde{h}_o - (1-\widetilde{\delta}_h)\widetilde{h}_n + \widetilde{\delta}_h \widetilde{h}'_o - (1-\widetilde{\delta}_h)\widetilde{h}'_n, \tag{C.61}$$

$$\widetilde{IH} = \widetilde{h}_n + \widetilde{h}'_n, \tag{C.62}$$

$$\frac{\widetilde{h}_o}{\widetilde{h}_n} = \frac{\zeta_2}{\zeta_3}\frac{1}{f}, \tag{C.63}$$

$$\frac{\widetilde{h}'_o}{\widetilde{h}'_n} = \frac{\zeta_4}{\zeta_5}\frac{1}{f}, \tag{C.64}$$

the following can be obtained,

$$0 = \left(\widetilde{\delta}_h \frac{\zeta_2}{\zeta_3}\frac{1}{f} - (1-\widetilde{\delta}_h)\right)\widetilde{h}_n + \left(\widetilde{\delta}_h \frac{\zeta_4}{\zeta_5}\frac{1}{f} - (1-\widetilde{\delta}_h)\right)\widetilde{h}'_n, \tag{C.65}$$

$$\widetilde{IH} = \left(1 - \frac{\widetilde{\delta}_h \frac{\zeta_2}{\zeta_3}\frac{1}{f} - (1-\widetilde{\delta}_h)}{\widetilde{\delta}_h \frac{\zeta_4}{\zeta_5}\frac{1}{f} - (1-\widetilde{\delta}_h)}\right)\widetilde{h}_n, \tag{C.66}$$

$$\widetilde{h}_n = \frac{\widetilde{\delta}_h \frac{\zeta_4}{\zeta_5}\frac{1}{f} - (1-\widetilde{\delta}_h)}{\left(\widetilde{\delta}_h \frac{\zeta_4}{\zeta_5}\frac{1}{f} - (1-\widetilde{\delta}_h)\right) - \left(\widetilde{\delta}_h \frac{\zeta_2}{\zeta_3}\frac{1}{f} - (1-\widetilde{\delta}_h)\right)}\widetilde{IH}, \tag{C.67}$$

$$\widetilde{h}'_n = \widetilde{IH} - \widetilde{h}_n, \tag{C.68}$$

$$\widetilde{h}_o = \frac{\zeta_2}{\zeta_3}\frac{1}{f}\widetilde{h}_n, \tag{C.69}$$

$$\widetilde{h}'_o = \frac{\zeta_4}{\zeta_5}\frac{1}{f}\widetilde{h}'_n. \tag{C.70}$$

To find $\widetilde{q}$ and $\widetilde{IH}$ separately,

$$\widetilde{IH} = \left(n_h^{\alpha} n_h'^{1-\alpha}\right)^{1-\mu_h-\mu_b-\mu_l}\left(\frac{\widetilde{k}_h}{G_C}\right)^{\mu_h}\widetilde{k}_b^{\mu_b}, \tag{C.71}$$

and

$$\widetilde{q} = \widetilde{q}\widetilde{IH}/\widetilde{IH}. \tag{C.72}$$

GDP_t is the sum of the value added of the two sectors, that is $GDP_t = Y_t - k_{b,t} + q_t IH_t = Y_t + (1 - \mu_b)q_t IH_t$. In the steady state, $\widetilde{GDP} = \widetilde{Y} + (1-\mu_b)\widetilde{q}\widetilde{IH}$.