Crime and Vigilance[*]

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Abstract

This paper develops a novel equilibrium theory of property crime. A population of potential victims elects how much costly vigilance to exert to guard their property, while a population of potential criminals chooses whether to engage in crime, and if so, how much, and what caliber of offenses to attempt. Crimes arise from random encounters of criminals and potential victims. The deterrence rate — i.e., the failure chance of an attempted crime — rises in vigilance and falls in caliber. It acts as a market-clearing price in the unique equilibrium, equilibrating actions of all agents.

Our model predicts how changes in the values of goods to criminal or owner, legal punishment, the technology of theft, or vigilance, or policing affect seven observables: the crime rate, attempted crime rate, deterrence rate, criminal entry rates, offenses per criminal, criminal caliber, and victims' vigilance expenses. Most predictions are new, and make sense of a wide array of empirical work. Many are also contrary to intuitive decision theory predictions of criminal or victim behavior, due to equilibrium feedback effects. For instance, we contradict two key theoretical predictions in Becker (1968) — e.g., he claimed that more severe punishment lowers the criminal offenses, but we find instead that it crowds out vigilance, and thereby raises offenses for all criminals.

Aside from positive predictions, our paper offers the first analysis of the social costs of crime in a graphical framework. We find that crime is a classic case of the "Tullock Paradox" — total social costs are strictly less than the potential transfers. The reason is the randomness of crime, diminishing returns to vigilance, and criminal heterogeneity.

Finally, a gentle twist on our model yields the first theoretical equilibrium analysis of the positive spillovers of unobservable vigilance, found in Ayres and Levitt (1998).

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1 Introduction

Motivated by its massive social costs, the path-breaking paper by Becker (1968) gave the first economic analysis of crime. Accounting for the optimal criminal response to changes in punishment and the probability of capture, he explored the socially optimal law enforcement.

This paper develops a new strategic model of crime, centered on the competition between optimizing criminals and potential victims. Our model gives a new vehicle for thinking about crime and its determinants, showing exactly how the different forces interact, and affording unambiguous predictions. In our model, criminals choose whether to commit thefts, and if so, the number and caliber of offenses, whereas their innocent rivals decide how vigilant to be. While formulated as a game, we show how this framework generates a supply *and* demand for crime in an implicit market that essentially realizes Becker's partial equilibrium vision for crime. Our theory explains a wide range of equilibrium variables: the crime rate, the attempted crime rate, the deterrence rate, victim's vigilance, the level of offenses per criminal, and the criminal caliber of every offense. We also create a graphical analysis for understanding not only comparative statics but also welfare gains and losses.

Chance is a crucial aspect of crime; few of us are victims in any month. According to the Bureau of Justice Statistics, in 2013, only 9% of 11.5 million households experienced one or more property victimizations. Our model captures the stochastic nature of crime firstly by assuming random encounters of criminals and victims. Randomness also plays a key role since not all attempted crimes succeed. One often fails to break into a house or steal a car. To explain variation in the attempted crime data, we introduce another decision margin that we call criminal *caliber*. Not all crimes are alike: Some are criminal masterpieces and most are petty. For any vigilance level, a higher caliber crime succeeds with greater chance. So contrary to Becker (1968), we divorce the events that a crime fails and the criminal is caught. The failure chance, or the *deterrence rate*, is endogenous; it rises in the vigilance, falls in the caliber, and holds constant when vigilance and caliber each double. This homogeneity ensures diminishing returns to vigilance by potential victims and to caliber by criminals. Since deterrence is positive, the attempted crime rate exceeds the crime rate. Because our focus is on vigilance, unlike in Becker's paper, we model law enforcement as an automaton: We simply assume that it enhances individual deterrence with diminishing marginal efficacy.

As in Becker, we assume that potential criminals have an extensive margin — whether to engage in crime. Doing so risks one's outside option, say one's career. A greater punishment makes the life of crime less appealing, and so formally raises the outside option. We assume that criminals vary by their outside options. As a result, the supply of criminals is increasing in how lucrative it is, and decreasing in the value of noncriminal life endeavors.

But unlike Becker, intensive margins play a central role in our individual decision-making. Consider the synopsis of Becker's paper in Lazear (2015): "The potential criminal will engage in the crime if the expected benefit is greater than the expected fine. . .[1] This simple inequality is rich in implications." We instead assume that criminals engage in marginal analysis and not just cost-benefit analysis. Criminals do not simply choose whether to commit a single crime, but instead explore a richer lifetime decision. For intuitively, if a criminal considers a one-shot crime as worthwhile, he would repeatedly do so. He would inevitably be caught, but his probability of capture would translate into an expected *number of offenses* λ before arrest. Since this variable is a centerpiece of the criminal career literature (Piquero et al., 2003), we assume that criminals have two intensive margins: caliber and offenses λ. Naturally, both margins are costly. For a more prolific criminal must expend greater resources in planning and executing every crime to ensure he is not quickly caught. We assume that the legal punishment is constant in the expected number of offenses.[2,3]

All told, we explore the steady-state of a dynamic model in which a continuum of potential criminals each makes three choices: whether to engage in crime, and if so, the caliber of crime and number of offenses. Meanwhile, a continuum of potential victims each elects how vigilant to be. The number of criminals and per-criminal offenses then fix the attempted crime rate, while the caliber and vigilance dictate the deterrence. Potential victims are hurt by a greater attempted crime rate and criminal caliber, while greater vigilance frustrates criminals. Our *crime equilibrium* is a Nash equilibrium, in which everyone optimizes, taking other actions as given. The crime rate is then a derived quantity, reflecting the undeterred attempted crimes.

We re-interpret our crime equilibrium as a competitive equilibrium, partially depicted in a supply and demand framework. For we can think of this game as an "implicit market", in which deterrence — namely, a probability — plays the role of a price, and the attempted crime rate is the quantity. For a fixed caliber, optimal criminal behavior induces a *downward-sloping supply of crime*, as fewer crimes are committed with greater deterrence. Equally well, for a fixed caliber, optimal vigilance induces an *upward-sloping derived demand for crime*, since a greater victimization chance elicits a more vigilant response. For every caliber, the supply-demand crossing yields a unique *market-clearing deterrence*; we prove that this locus slopes down in caliber, reflecting how criminal costs rise in caliber while the efficacy of

[1] In Becker (1968), a risk neutral criminal commits a crime if $m - pf \geq \bar{u}$, where m is the gain, p the chance of getting caught, f the fine, and $\bar{u}$ the outside option.

[2] Repeating Becker's single crime logic, the expected number of crimes committed until capture is $\lambda = 1/(1-p)$, so that his incentive condition is $m\lambda - fp\lambda \geq \bar{u}\lambda$. So *his expected legal penalty* $\ell = \lambda f p$ *is linear in the expected number of crimes* λ. Also, Becker's criminal would seek to commit as many crimes as possible, and so only be constrained by capture. We instead secure a finite solution instead by assuming cost convexity — indeed, evasion from capture, planning and carrying out crimes are increasingly costly at the margin.

[3] This is a model simplification: "The punishment for two murders is the same as for one." (*Spellbound*)

vigilance falls in it. Finally, we introduce another graphical apparatus, an *optimal caliber locus*, that relates deterrence to caliber. Crossing these two loci then fully determines the one extensive and three intensive margins of the crime equilibrium. We prove that this is unique, rendering unambiguous all comparative statics analyses we perform.

Our crime equilibrium is a long run steady-state. But not all margins are equally flexible, and our model yields different predictions for the medium and short runs. Arguably, the extensive margin is the most inflexible, and so is assumed fixed in these runs. For entry into crime, or exit from it, is a major career choice. The next most sticky choice is the caliber, since this surely entails a learning curve. We assume it is also fixed in the short run. As predicted by Le Châtelier, the supply of attempted crime is more elastic as the run lengthens.

An important property of any competitive equilibrium is its robustness to changes in the environment. We show that our crime equilibrium is stable under an economically motivated tâtonnement process in which each intensive margin adjusts in proportion to the marginal gains, and the criminal entry or exit speed is proportionate to criminal profits or losses. We prove that the crime equilibrium is stable in the short and medium runs, and also in the long run when entry is slow enough, or the level of offenses adjusts sufficiently fast.

We focus on property crime, with victims losing more than criminals gain from crimes. This excludes victimless crimes, like prostitution or drug dealing, but may subsume violent crime, if one imagines that victims lose much more than perpetrators gain from violence. For such situations, our equilibrium offers predictions for changes in all equilibrium variables as the payoff stakes change for victims or criminals, as policing improves, legal penalties change, new or cheaper vigilance technologies are introduced, or criminal innovations occur. As with standard partial equilibrium analysis, each change affects supply or demand or both.

Predictions Overview. To see how we diverge from Becker, consider his most basic claim that aggregate and individual criminal activity should fall in legal penalties. By contrast, since we assume heterogeneous criminals, a higher expected legal penalty deters entry. The supply curve shifts down, but demand is unaffected. So greater punishment discourages private vigilance, raising the marginal profitability of every offense. On balance, deterrence falls. At the end of the feedback loop captured by our other two loci, Proposition 1 finds that the attempted crime rate falls, but unlike in Becker (1968), *the offenses per criminal* λ *rises with increased punishment*. The attempted crime rate reflects the criminal intensive and extensive margins, and these margins move in opposition. *There are fewer criminals, but each commits more crimes* and is more poorly deterred to make up for the greater punishment.

Becker (1968) argues (on p. 188) that improved law enforcement has an ambiguous effect on individual and total offenses. No such ambiguity arises here. The derived demand for crime increases while supply falls, and both reduce deterrence. Becker also presumes that this would be partially offset by a drop in private expenditures. In fact, Proposition 2 asserts that better policing so much displaces private vigilance, leading to fewer criminals, each committing fewer and lower caliber crimes, and thus a lower attempted crime rate.[4] While lacking our notion of deterrence, Becker would surely be stunned at our finding that the *vigilance is crowded out so much that improved policing leads to reduced deterrence.*

Since our centerpiece is the endogenous vigilance by potential victims, we explore how vigilance technology innovations impact the crime equilibrium. Unlike policing, cheaper vigilance lowers demand, but leaves supply unaffected: Proposition 3 uncovers a *vigilance magnification effect* that reflects the competitive nature of crime: *When the marginal costs of vigilance fall, potential victims grow less vigilant* — the opposite response in producer theory, when one's marginal cost fall. The reason is that *criminals optimally reduced their caliber so much more that deterrence rises.* This key feedback effect underscores the importance of analyzing crime as a strategic game. In the end, we find fewer criminals, each committing fewer offenses, and each crime deterred with a higher chance, so that the crime rate drops.

In §7, we provide the first theoretical model for the Lojack vigilance technology improvement for cars, explored in Ayres and Levitt (1998). When the feasible vigilance levels jump over the optimal one, a *mixed strategy equilibrium* emerges: Some opt for high vigilance and buy Lojack, and others do not. We find Lojack users confer a positive spillover on those who do not, by lowering their attempted crime rate. We use our graphical framework to depict the external benefit, finding that this might attenuate in the long run once caliber readjusts.

A major source of exogenous variation across crimes is the value of property to victims or criminals. At one extreme is money, worth the same to victims and criminals, and at the opposite are very personal goods, or violent crime. But arguably most property crimes entail a positive but partial *criminal markdown* — since owners invariably prefer their property more than others do, and because laundering criminal spoils is by no means easy. The markdown on automobiles might be around 80%, for instance. Assume that criminal stakes rise. Decision-theoretic logic argues that the equilibrium theft chance should rise. In fact, Proposition 4(a) predicts that *vigilance rises so much than caliber that the deterrence rises*; the attempted crime rate and offenses per criminal rise, and crime rate ultimately rises too.

By the same token, a rise in the property loss boosts victims' vigilance. This inflates the deterrence rate for any attempted crime rate, and — thinking inversely — lowers the derived demand for crime for any deterrence. But the supply curve is unaffected. So Proposition 4(b) finds that vigilance and deterrence unambiguously rise, while the offenses per criminal, the

[4] The first finding is documented in Vollaard and Koning (2009). The second facts echo Philipson and Posner (1996); Levitt (1997); McCrary (2002); Di Tella and Schargrodsky (2004); Nagin (2013).

attempted crime rate, and crime rate fall. When vigilance is not too high, the optimal caliber also rises as potential property losses climb. Among the competing forces, most important is that heightened vigilance raises the marginal profits of caliber, because caliber and vigilance are strategic complements in our model. This result offers insights into a stereotypical distinction — that petty thieves steal low value property, while criminal masterminds, who opt for fewer but higher caliber crime, abscond with the highest value property.

Next, Proposition 4(c) fixes the criminal markdown, and explores a simultaneous increase in the stakes of the game for everyone: property losses and criminal gains. This naturally applies to a fixed class of goods, as the value of the good rises for owner and criminal alike. This change impacts both sides of the market: Facing a greater loss, potential victims grow more vigilant, while criminals respond to the increased stakes with more offenses. So the victims prevail more in each crime, but more crimes are attempted: the derived demand for crime falls and the supply rises, thereby raising deterrence and the attempted crime rate.

Since the attempted crime rate and deterrence co-move, the crime rate shift is unclear — for more crimes are initiated, but fewer succeed. This conflict is fundamental, and depends on whether the crime rate rises in deterrence along the demand curve (★). We show that it does when expected property losses are more than twice vigilance costs. In this case, if legal penalties rise, then supply shifts left along the demand curve, reducing both the attempted crime *and* crime rate. But otherwise, greater legal penalties can raise the crime rate. In general, given (★), the attempted crime rate rises faster than deterrence with worse policing, greater criminal gains, and — as we next see — lower criminal costs, raising the crime rate.

In Proposition 5, we explore how falling criminal costs impact crime. *Surprisingly, a fixed cost reduction reduces the offenses per criminal.* In producer theory, this should have no effect on the intensive margins of any producing firm. But here it leads to more criminal entry, eliciting more vigilance, and this depresses λ. For the criminal supply rises while the derived demand is unchanged. Generally marginal costs fall too, and in this case, criminal caliber, vigilance, offenses, the attempted crime rate, and crime rate all rise. Proposition 5 then distinguishes two forms of innovation, since we find that deterrence falls with *caliber-augmenting* technological innovation and rises with *offenses-augmenting* technological change.

The anonymity in cities naturally reduces the costs of each offense, since criminals can more easily evade notice and capture. Proposition 5 suggests that rural areas enjoy lower crime rates and vigilance than cities, but the causation is instructive. For since people lock cars and houses in cities and not rural areas, one might ask why criminals do not relocate. But this reverses the causation. The derived demand for crime works in reverse: Vigilance is higher in cities precisely because the attempted crime rate is higher. Likewise, residence burglary is lower at night than during the day (according to the FBI). Criminals enjoy a lower cost for each criminal attempt because fewer residences are occupied then. So daytime captures an offense-augmenting innovation. Proposition 5 explains this crime rate difference.

The supply and demand graphical framework is prized in economics not only for its easy comparative statics analysis, but also for its transparent depiction of welfare gains and losses. With no police, deterrence is the chance that a theft fails. In this special case, in our space of deterrence and attempted crime rates, the mass of successful crimes is an area. Since criminals and victims differently value stolen goods, we must scale supply and demand curves separately. When we do so in §6, we find that the area under the falling supply curve and above the equilibrium deterrence represents the producer surplus of crime — namely, the criminal profits of the inframarginal criminals. More subtly, the consumer surplus of potential victims is not the area over the rising demand curve below the market deterrence, but instead the area under this curve. This twist arises because "trade" here is not win-win: Rather, criminal offenses constitute an economic good for criminals, but a bad for potential victims. Towards a smarter foundation for measuring the true costs of crime, we prove that social costs of crime do not depend explicitly on criminal costs, the markdown, or the caliber.

In the same vein, we offer a contribution to rent-seeking (Tullock, 1967). For criminals expend resources to procure victims' property, and victims invest resources to forestall this. Over and above property losses, Anderson (2012) estimates $480 billion spent on private vigilance efforts, in addition to public expenditures. Crime can thus be viewed as a set of rent-seeking contests. We have discovered that this offers a clear example of the famous Tullock Paradox — namely, that social costs may be less than the magnitude of potential transfers (the stolen property). This owes to the random nature of crime. We give a precise measure of the Tullock gap in terms of primitives. Only in the case of no criminal markdown, no diminishing returns to vigilance, and no criminal heterogeneity does the paradox disappear.

Literature Review. Aside from first treating criminals as maximizing agents, Becker (1968) introduced an aggregate supply of crime with intuitive derivatives in fundamentals. Ehrlich (1981) later posited an intuitive demand for crime, where the net payoff per offense acts as the price.[5] So his supply curve slopes up, and demand slopes down. He might be the first to think of the number of crimes as an equilibrium object. By contrast, we formulate the criminal game, specifying all payoff functions. Thus, our supply explicitly arises from the criminal maximization, and our demand is derived from the victim's optimization problem. Moreover, Ehrlich's verbal story suggests that victims' vigilance costs directly reduce criminal

[5]He offers a verbal reduced form logic: "the demand schedule for offenses represents the average potential payoff per offense at alternative frequencies of offenses". Ehrlich (2010) is recent survey. Hotte et al. (2003) develop a model of criminals and victims. While their supply follows from criminals entry, their demand reflects the criminals' optimization. So their market equilibrium takes vigilance as given.

payoffs.[6] By contrast, we assume that the main effect of vigilance, like burglar systems, is to deter crimes (maybe with a delay, as with Lojack). In so doing, we take seriously the randomness of crime by modeling both probabilistic encounters and stochastic deterrence.[7]

The easy demand side of our crime model is inspired by the counterfeiting model of Quercioli and Smith (2015), analogizing their verification chance for deterrence, and their counterfeiting rate for an attempted crime rate. They focus on a money passing game, trivializing the criminal side, assuming homogenous criminals who counterfeit an exogenously fixed amount of money.[8] The richness of our model is in the criminal analysis. Our falling supply of crime owes to the endogenous level of offenses and the heterogeneous outside options of potential criminals. These twin aspects allow us to differentiate individual and total market offenses. They yield many subtleties, e.g., our optimal caliber locus is nonmonotone because criminals choose to reduce their offenses to zero when deterrence approaches perfect.

We owe a conceptual heritage to Knowles et al. (2001) who may be the first to study a population game of crime. Like Becker, they focus on the interaction between police and potential criminals, and seek to identify whether police stops for exhibit racial or statistical discrimination. Their potential criminals are heterogeneous and decide whether to carry contraband. The probability of a police search of their vehicle incentivizes their decision to carry drugs. The models differ in most respects but share the feature that the criminal extensive margin, and police search decision (like our vigilance) are mutual equilibrium best replies.[9]

The empirical literature on crime has raced far ahead of the theory, and we cannot do justice to it. There are many great surveys and analyses, like Levitt (2004).[10] But our full equilibrium should allows us to speak to many of them, and we cite them in the text, as our theorems permit. Our framework captures the incentive effects of all agents — from the effect of more police (Levitt, 1997; McCrary, 2002) or greater legal penalties (Levitt, 2004), the spillover effects of vigilance (Ayres and Levitt, 1998), the investments in private security (Clotfelter, 1978), and the spatial variation in crime rates (Glaeser and Sacerdote, 1999).

In §2, we set up the model, and illustrate our solution concept in §3. Next, in §4, we construct the graphical apparatus, and characterize the unique equilibrium. In §5, we develop theoretical predictions, and compare them to the empirical literature on crime. In §6, we study the social costs of crime, and finally, in §7, we explore the positive spillovers of discrete unobservable vigilance. Section §8 concludes. Omitted proofs are in the Appendix.

[6] He writes: "Burglar alarm systems, guards, . . . all serve the similar purpose of decreasing the gross loot per offense, or increasing the cost and effort to the offender of committing the offense."

[7] Crime is not a simultaneous move game if vigilance raises criminal costs, and cannot be solved by Nash.

[8] So there is no markdown in Quercioli and Smith (2015). They also do not pursue a welfare analysis.

[9] In the same spirit, Persico (2002) performs a theoretical analysis of the effects of police fairness on crime.

[10] The normative approach to crime and public enforcement is surveyed in Polinsky and Shavell (2000).

2 The Model

A. Overview. A competition occurs at times 1,2,3... between a unit mass continuum of homogeneous risk-neutral innocent *potential victims*, and a heterogeneous continuum of risk-neutral potential criminals; once they enter, they become *criminals*. All actions are per period, and all payoffs and outside options flows. We analyze a steady-state of this game.[11]

Potential criminals vary by their *outside options* ℓ. The variable ℓ subsumes the expected legal costs from crime, which we assume does not vary in the number of crimes. Criminals treat ℓ as the fixed cost of their illicit activities. The *outside option mass distribution* F has a density $f(\ell) \equiv F'(\ell) > 0$ on $[0, \infty)$.[12] We say that criminals have *higher outside options* with F than G if F is higher than G in the sense of first order stochastic dominance.

Criminals specialize in the theft of a specific good. Goods are described by pairs (m, M), where M is the *property loss*,[13] and $m \leq M$ the *criminal's gain*. This permits a comparative static moving from, say, auto theft with a large *markdown* $1 - m/M \gg 0$ to jewel theft (intermediate markdown $1-m/M \in (0,1)$) to bank robberies with no markdown $(m = M)$.[14]

In every period, any potential criminal decides whether to enter. If so, he chooses an expected level of *offenses* $\lambda > 0$ to attempt (per period) before capture. Criminals and potential victims are randomly matched every period, with all potential victims equally likely targets; thus, each is chosen with the same probability α, the *attempted crime rate* (per capita).[15] The *crime rate* $\kappa \in [0, 1]$ is the chance that they are successfully victimized.

B. Endogenous Deterrence and the Police. Chance plays two roles in our model. First, whether one is a victim of a *potential crime* is random. Second, whether a crime succeeds or is deterred is random. Victims choose levels of costly *vigilance* $v \geq 0$ to guard against crime, and criminals choose costly criminal *caliber* $q \geq 0$. The resulting *deterrence rate* $\delta \in [0, 1]$ is the chance that a criminal attempt fails, and depends on vigilance and caliber via $\delta = \Delta(v, q)$. An attempted crime succeeds more often the higher the criminal caliber, and the lower the vigilance. Naturally, victims wish to raise their deterrence and criminals wish to lower it. The *deterrence function* $\Delta(v, q)$ is twice smooth and obeys $\Delta_v(v, q) > 0 > \Delta_q(v, q)$.[16] To capture the competitive nature of the game, we assume that the same proportionate rise in vigilance and caliber exactly offset each other: Deterrence

[11] We thus avoid discussions of discounting, and not need time subscripts.

[12] To capture the vast criminal pool given the right conditions, F need not be a probability distribution.

[13] Given a competitive insurance market (or actuarially fair insurance prices) and risk neutral victims, our analysis works provided the property M is the net loss after any insurance payments.

[14] Despite our property crime focus, one could think that a violent crime has a high markdown $1-m/M \approx 1$.

[15] For example, if there is a mass ν of criminals each attempting λ crimes, then $\alpha = \nu\lambda$. Also, α denotes the tightness of the "criminal market" — namely, the ratio of attempted crimes to potential victims.

[16] For any smooth function $x \mapsto f(x)$ and $x \in \mathbb{R}^n$, we denote partial derivatives by $f_{x_i}(x) \equiv \partial f(x)/\partial x_i$.

$\Delta(v,q)$ is homogeneous of degree zero. As result, deterrence obeys the implicit relation $v \equiv q\chi(\Delta(v,q))$, for some twice smooth increasing *vigilance function* $\chi(\delta)$. For any positive criminal caliber, zero vigilance deters no crimes, or $\Delta(0,q) = 0$ for $q > 0$, and so $\chi(0) = 0$. Observe that $\delta = \Delta(v,q) \equiv \chi^{-1}(v/q)$ for $v \leq q\chi(1)$, and $\Delta(v,q) = 1$ for all $v > q\chi(1)$, thus any criminal caliber can be defeated with enough vigilance. As a result, $\chi(1) < \infty$. So, for any vigilance $v > 0$, deterrence would be perfect for all criminal calibers $q \leq v/\chi(1)$.

Next, suppose that $\delta = \Delta(v,q) < 1$. Differentiating the identity $v \equiv q\chi(\Delta(v,q))$ yields:

$$\Delta_v(v,q) = 1/q\chi'(\delta) \qquad \text{and} \qquad \Delta_q(v,q) = -\chi(\delta)/q\chi'(\delta) \tag{1}$$

To ensure a positive optimal vigilance, we need the Inada condition $\Delta_v(v,q) \uparrow \infty$ as $v \downarrow 0$ if $q > 0$. We equivalently assume that the marginal cost of zero deterrence vanishes: $\chi'(0) = 0$.

Given deterrence $\delta = \Delta(v,q)$, a crime succeeds with *theft function* $\Theta(v,q) \equiv \theta(\Delta(v,q)) = \theta(\delta) \in [0, 1-\delta]$, where θ is twice differentiable, decreasing and convex: $\theta'' \geq 0 > \theta'$.

C. Victim and Criminal Optimizations. Potential victims minimize their expected total costs from crime — i.e., expected property losses and vigilance costs. A victim loses his property given an undeterred attempted crime. Thus, he chooses vigilance v to minimize his *expected losses*[17],[18]

$$\mathcal{L}(v,q,\alpha) \equiv \alpha\Theta(v,q)M + v \tag{2}$$

Assume an interior solution with $\Delta(v,q) < 1$, so that we may twice differentiate $\Theta(v,q)$. As optimization demands, the expected losses exhibit diminishing returns in vigilance, and thus:

$$q^2\Theta_{vv}(v,q) = \theta' \frac{1}{(\chi')^2}\left(\frac{\theta''}{\theta'} - \frac{\chi''}{\chi'}\right) \geq 0 \tag{3}$$

In the *no police* case ($\theta(\delta) \equiv 1-\delta$), this is equivalent to the convexity of χ which we assume.

Given entry, we ignore discounting, and assume that criminals maximize expected flow profits — namely, expected the flow arrival of revenues $\Theta(v,q)\lambda m$ minus criminal flow costs $c(\lambda,q)$. The cost function $c(\lambda,q)$ is twice smooth, increasing and strictly convex, with $c(0,0) = c_\lambda(0,0) = c_q(0,0) = 0$. Also, to ensure a finite solution for the maximization, we posit the Inada conditions $\lim_{q\uparrow\infty} c_q(\lambda,q) = \lim_{\lambda\uparrow\infty} c_\lambda(\lambda,q) = \infty$. We assume that $\mathcal{E}_\lambda(c_\lambda) \geq 1$

[17]With risk averse potential victims, and actuarially fair insurance prices, the problem is slightly more complex — for a victim chooses $v \geq 0$ to *maximize* $u(w - \alpha\Theta(v,q)M) - v$, for some increasing and strictly concave $u(\cdot)$, and large enough income $w > 0$. This is similar to Ehrlich and Becker (1972) who study the relation between (competitive) market insurance and self protection. Nevertheless, their results do not extend to our case, since their "loss chance" depends only on a victim's own effort.

[18]We can think of vigilance costs either as ongoing self-protection and awareness costs, with (2) solved daily, or the annuity value of initial one-shot costs incurred by an individual who cares about the future.

and $\mathcal{E}_q(c_q) \geq 1$,[19] which holds if the cost function is sufficiently convex. A quadratic and separable cost function like $c(\lambda, q) = \lambda^2 + q^2$ obeys these assumptions.

All told, a criminal with outside option ℓ maximizes his expected flow *profits*, taking vigilance as given:

$$\Pi(\lambda, q, v|\ell) \equiv \Theta(v, q)\lambda m - c(\lambda, q) - \ell \tag{4}$$

entering when $\Pi(\lambda, q, v|\ell) \geq 0$. Criminals clearly choose the same (λ, q) for any outside option ℓ. Given convex costs, profits have diminishing returns in λ ($\Pi_{\lambda\lambda} < 0$). To ensure that $\Pi_{qq} < 0$ for any strictly convex cost function $c(\lambda, q)$, we assume diminishing returns to caliber:

$$q^2\Theta_{qq}(v, q) = \theta' \left(\frac{\chi}{\chi'}\right)^2 \left(\frac{\theta''}{\theta'} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\chi'}{\chi}\right) \leq 0 \tag{5}$$

For an instructive special case, assume a convex but log-concave vigilance function (so $\log \chi$ is concave), as is true for any vigilance function $\chi(\delta) = \delta^\gamma$ with $\gamma \geq 2$. Then (5) holds for with no police (so that $\theta'' = 0$), and more generally, when the police response to deterrence is not excessively elastic ($|\theta''/\theta'|$ not too large). For instance, with our geometric vigilance function and quadratic theft function $\theta(\delta) = (1 - \sigma\delta)(1 - \delta)$, (5) holds when $\sigma \leq (1 + \gamma)/(3 + \gamma)$.[20]

A critical distinction is how caliber and offenses interact. Intuitively, higher caliber crimes are more carefully executed, and so each is more costly. Formally, greater caliber q raises the marginal cost of offenses λ, or $c_{\lambda q} \geq 0$. The key question is whether it raises the marginal benefits of offenses more or less. We will see in §4 that equilibrium requires that $\mathcal{E}_\lambda(c_q) < 1$, namely, the marginal benefits of offenses rise more than marginal costs.

Property crime is a competitive game, and so logically cannot exhibit complementarities for both sides. We venture that vigilance raises the marginal efficacy of caliber, while caliber undermines the marginal fruits of vigilance.[21] More formally, vigilance and caliber should be *strategic complements* for criminals (profits satisfy $\Pi_{vq} \geq 0$) and *strategic substitutes* for victims (losses obey $\mathcal{L}_{vq} \geq 0$). To ensure this, we posit a *theft complementarity inequality*:

$$q^2\Theta_{vq} = -\theta' \frac{\chi}{(\chi')^2} \left(\frac{\theta''}{\theta'} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'}\right) > 0 \tag{6}$$

This inequality forces a log-concave vigilance function (i.e. $(\log \chi)'' \leq 0$), whose extreme case is a geometric function, like $\chi(\delta) = \delta^\gamma$. In this case, (6) holds for $\theta(\delta) = (1 - \sigma\delta)(1 - \delta)$ when $\sigma \leq 1/3$. So (6) can be understood as a limit on the diminishing returns to police.

[19] We define the *elasticity of a real-valued function f with respect to $x \in \mathbb{R}$* by $f \mapsto \mathcal{E}_x(f) \equiv xf_x/f$.

[20] This restriction is only needed for high deterrence $\delta > (1+\gamma)/(2+\gamma)$. Otherwise, (5) holds for all $\sigma \leq 1$.

[21] As a mutual matching story, one might wonder what sorting emerges. But potential victims have no say in who steals from them. We will generally find that vigilance and caliber co-move in our comparative statics.

Standard economic logic demands that own effects dominate cross effects for the rival choice variables (q, v). To ensure this conclusion holds, our theft function $\Theta(v, q)$ must obey $|\mathcal{E}_q(\Theta_q)| \geq \mathcal{E}_v(\Theta_q)$ and $|\mathcal{E}_v(\Theta_v)| \geq |\mathcal{E}_q(\Theta_v)|$. Comparing (5) and (6) yields $|\mathcal{E}_q(\Theta_q)| \geq \mathcal{E}_v(\Theta_q)$. But the second inequality is restrictive. We want that it holds absent police, when $\Theta(v, q) \equiv 1 - \Delta(v, q)$, and in fact this suffices.[22] Seeing inequalities (3) and (6), this inequality reduces to the assumption:

$$2\chi''/\chi' \geq \chi'/\chi \tag{7}$$

Precisely, the vigilance function χ is sufficiently convex so that $\sqrt{\chi}$ is convex. This holds, in particular, for any geometric vigilance function $\chi(\delta) = \delta^\gamma$ with $\gamma \geq 2$.

D. Equilibrium. We explore a stationary Nash equilibrium in which both potential victims and criminals independently optimize taking as given what the other side of the market does. Formally, a *crime equilibrium* is a 7-tuple $(v, \lambda, q, \bar{\ell}, \delta, \alpha, \kappa)$, where $(v, \lambda, q, \bar{\ell})$ are the choice margins, and (δ, α, κ) the derived equilibrium variables, such that:

(i) Given vigilance v, a criminal ℓ who enters chooses (q, λ) to maximize profits (4);

(ii) Given (v, λ, q), the marginal criminal $\bar{\ell}$ obtains zero profits: $\Pi(\lambda, v, q|\bar{\ell}) = 0$;

(iii) Given caliber q and the attempted crime rate α, vigilance v minimizes expected losses (2);

(iv) Given $(v, q, \lambda, \bar{\ell}, \alpha)$, deterrence is $\delta = \Delta(v, q)$, while the crime rate $\kappa = \theta(\delta)\alpha$ is the product of the theft function $\theta(\delta)$ and the attempted crime rate $\alpha = \lambda F(\bar{\ell})$.

As a Nash equilibrium, the actions are inferred but not observed. In particular, a criminal chooses caliber and offenses expecting an equilibrium level of vigilance, but unaware of the variation. Likewise, a potential victim selects his vigilance anticipating the equilibrium level of caliber and attempted crime rate, but unaware of the variation.[23]

Our crime equilibrium is a *long run* notion with all margins optimized. Becker (1968) focuses on the extensive margin captured by the marginal criminal $\bar{\ell}$. Since entry or exit is a major decision, it is the most sticky of our criminal margins, and is fixed in the *medium run*. So, Step (ii) above is omitted, and instead, equilibrium is indexed by a marginal criminal. Finally, criminals can intuitively adjust how many crimes they commit faster than they can change their method. In the *short run*, there are two fixed margins $\bar{\ell}$ and caliber q. In this case, we must remove the caliber maximization from Step (i), and index equilibria by $(\bar{\ell}, q)$.[24]

[22] Indeed, just compare (3) and (6), and recall that $\theta'' \leq 0 < \theta'$ in general.

[23] If instead vigilance is partially observed, then the equilibrium notion fundamentally changes from Nash to a subgame perfect equilibrium in which potential victims move first, and criminals second.

[24] This taxonomy assumes fast adjusting vigilance. If criminals are more attuned to economic changes affecting crime, then vigilance might also be fixed in the short or medium run.

3 Equilibrium Examples

A. A Short Run Equilibrium. We solve the optimization in Step (i) for a fixed caliber $q = \bar{q} > 0$. Since victims directly care about the attempted crime rate α and not per criminal offenses λ, we first solve for the optimal level of offenses λ, faced with vigilance v. When $v \leq D\bar{q}$, the FOC for maximizing (4) in λ yields the first order condition $(1 - T\delta)m = B\lambda$. Altogether, offenses are $\lambda = (1 - T\delta)m/B$, with a corner solution $\lambda = 0$ when $\delta = 1/T$.

Step (ii) does not apply. Next, in Step (iii), potential victims choose vigilance v to minimize $\alpha(1 - T\delta)M + v = \alpha(1 - T\sqrt{v/(D\bar{q})})M + v$. The first order condition yields the optimal vigilance $v = (\alpha TM)^2/(D\bar{q})$. We now invert this and solve for the attempted crime rate. Changing to deterrence space, we get $v = \bar{q}D\delta^2$, and find $\alpha = 2D\bar{q}\delta/(TM)$. This is a "derived demand" for attempted crime by potential victims, justifying their vigilance choice.

Let us highlight the intuitive role of δ as an "implicit price". Since crime is an economic bad, its demand rises in deterrence. Analogously, we construct a falling supply of crime, reflecting optimal criminal behavior. We proceed partially, fixing the marginal criminal $\ell = \bar{\ell} > 0$. Since each criminal contributes λ attempted crimes, step (iv) yields $\alpha = \lambda F(\bar{\ell}) = [(1 - T\delta)m/B](\bar{\ell}/L)$. A unique deterrence δ and attempted crime α clear the market:

$$\delta = \frac{TMm\bar{\ell}}{2D\bar{q}BL + T^2Mm\bar{\ell}} \qquad \text{and} \qquad \alpha = \frac{2D\bar{q}m\bar{\ell}}{2D\bar{q}BL + T^2Mm\bar{\ell}} \tag{8}$$

The equilibrium $(v, \lambda, \delta, \alpha, \kappa)$, with the crime rate $\kappa = (1-\delta)\alpha$, shifts in the cost parameters A, B, D, or T, outside option density L, and the theft values (m, M) as predicted in §5.

B. A Medium Run Example. Step (i) is now a double optimization, since caliber is now flexible:

$$\max_{q,\lambda}(1 - T\sqrt{v/(Dq)})\lambda m - Aq^2/2 - B\lambda^2/2 - \ell \tag{9}$$

The optimal offenses is still $\lambda = (1-T\delta)m/B$. If we substitute this into the FOC for caliber, we discover

$$q(\delta) = m\sqrt{T\delta(1 - T\delta)/(2AB)} \tag{10}$$

Caliber $q(\delta)$ is hump-shaped in δ, vanishing at $\delta = 0, 1$. Finally, the short run equilibrium deterrence at the left in (8) still applies, but now for the optimally chosen caliber (10). For instance, assume that the vigilance parameter D rises after a hurricane knocks out power to a large area. Then in the short run, deterrence falls, but in the medium run this affects caliber too which can magnify or attenuate the fall in deterrence, depending on whether $\delta \gtrless 1/(2T)$, by (10). But changing caliber cannot reverse the fall in deterrence, by Figure 1.

C. A Long Run Example. In the long run, criminals freely enter or exit the market,

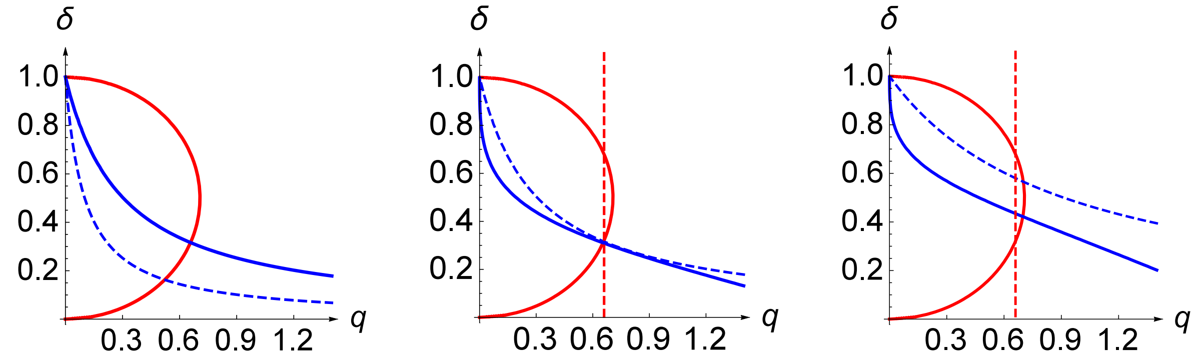

Figure 1: **Short, Medium, and Long run Crime Equilibrium.** All panels assume $T = A = B = 1$, $L = 5$ and $m = M = 2$. Caliber is fixed in the short run; in the medium and long runs, the optimal caliber is the hump-shaped locus (10), vanishing at either end. The decreasing loci are the medium and long run market clearing deterrence in (8). The left panel depicts the medium run effect of a rise in the vigilance parameter from $D = 1$ (thick line) to $D = 3$ (dashed). The middle panel depicts simultaneously a short run equilibrium (vertical dashed line with fixed caliber $\bar{q} = 0.66$ and outside option $\bar{\ell} = 0.76$), medium run equilibrium (dashed curve with variable caliber), and long run equilibrium. The right panel redraws the medium and long run loci after a rise in property losses from $M = 2$ to $M = 6$. Note that deterrence rises more in the medium run than in the long run, after criminals exit.

so that the marginal criminal $\bar{\ell}$ is endogenous. From the objective in (9), the marginal criminal obeys $\bar{\ell} = (1 - T\sqrt{v/(Dq)})\lambda m - Aq^2/2 - B\lambda^2/2$. Since each criminal still attempts $\lambda = (1 - T\delta)m/B$ crimes, $\bar{\ell} = (1 - T\delta)^2 m^2/(2B) - Aq^2/2$. This yields a long run supply of crime $\alpha = \lambda F(\bar{\ell}) = [(1 - T\delta)m/(BL)][(1 - T\delta)^2 m^2/(2B) - Aq^2/2]$, given the caliber q.

The derived demand for crime is still $\alpha = 2Dq\delta/(TM)$. The intersection of supply and demand yields a market clearing deterrence rate δ and attempted crime rate α, now lacking a closed form. In equilibrium, deterrence δ clears the market given the caliber q in (10).

Assume a negative demand shock, such as a greater loss M. Given any caliber q, the market clearing deterrence is greater the shorter the run, as seen in the third panel of Figure 1. For the supply of attempted crime is more elastic as the run lengthens (per *LeChâtelier*).[25] On the other hand, it is unclear what happens to the optimal caliber (10) as the run increases. Figure 1 shows that the medium and long run caliber can coincide after M rises. In general, the answer depends on the slope of $q(\delta)$, i.e., on whether $\delta \gtrless 1/(2T)$, by formula (10).

[25] That $\alpha = \lambda F(\bar{\ell})$ implies $\mathcal{E}_\delta(\alpha) = \mathcal{E}_\delta(\lambda) + \mathcal{E}_{\bar{\ell}}(F)\mathcal{E}_\delta(\bar{\ell})$. In the long run, $\mathcal{E}_{\bar{\ell}}(F) > 0 > \mathcal{E}_\delta(\bar{\ell})$; before $\mathcal{E}_{\bar{\ell}}(F) = 0$.

4 Equilibrium Analysis

A. The Optimal Caliber Locus. We derive general expressions for our key equilibrium curves. We solve the criminal's problem of Step (i). Given caliber $q > 0$ and vigilance $v > 0$, the optimal level of offenses is $\hat{\Lambda}(v,q) = 0$ if $v \geq q\chi(1)$, and otherwise solves:

$$\Pi_\lambda(\hat{\Lambda}(v,q), q, v|\ell) = \Theta(v,q)m - c_\lambda(\hat{\Lambda}(v,q), q) = 0 \tag{11}$$

$\hat{\Lambda}(q,v)$ is continuously differentiable by the Implicit Function Theorem, since $c_{\lambda\lambda} > 0$. The function $\Lambda(q,\delta) \equiv \hat{\Lambda}(q, q\chi(\delta))$ falls in deterrence as each crime becomes less profitable, and falls in caliber:

$$\Lambda_\delta = \theta' m / c_{\lambda\lambda} < 0 \qquad \text{and} \qquad \Lambda_q = -c_{\lambda q}/c_{\lambda\lambda} \leq 0 \tag{12}$$

Caliber maximizes this λ-*optimal criminal profits* $\Pi^*(q,v|\ell) \equiv \Theta(v,q)\hat{\Lambda}(q,v)m - c(\hat{\Lambda}(q,v),q)$:

$$\Pi^*_q(q,v|\ell) = \Theta_q(v,q)\hat{\Lambda}(q,v)m - c_q(\hat{\Lambda}(q,v),q) = 0 \tag{13}$$

By (1) and $\Lambda(q,\delta)$, the *optimal caliber locus* $\mathcal{C}^*$ turns (13) into (q,δ)-space ($\Pi^*_q(q, q\chi(\delta)|\ell) = 0$):

$$MB(q|\delta) \equiv -\theta'(\delta)\frac{\chi(\delta)}{q\chi'(\delta)}\Lambda(q,\delta)m = c_q(\Lambda(q,\delta),q) \equiv MC(q|\delta) \tag{14}$$

Since marginal benefits fall in caliber (as $\Lambda_q < 0$ by (12)), while criminal costs $c(\lambda, q)$ are convex, we have $MB_q(q|\delta) < 0 < MC_q(q|\delta)$. Also, our theft complementarity inequality (6) asserts that $\log(MB)$ increases in δ, for fixed λ. So the $\mathcal{C}^*$ locus slopes up when λ is fixed. But when λ is a choice variable, this key locus is non-monotone in deterrence. For a criminal facing no deterrence has no incentive to invest in caliber and so the caliber locus starts at $(q,\delta) = (0,0)$. Also, with perfect deterrence, he has no incentive to attempt any crimes, and so the caliber locus ends at $(q,\delta) = (0,1)$ since zero caliber is an optimal solution. With endogenous offenses, an indirect effect emerges in the criminal optimization. Assume $\mathcal{E}_\lambda(c_q) < 1$, we make a *profit complements* deduction: $\Pi_{\lambda q} = \Theta_q m - c_{\lambda q} = c_q/\lambda - c_{\lambda q} > 0$. Then greater vigilance (and so greater deterrence) lowers the marginal profits of λ in (11), thereby reducing optimal offenses. But this lowers the marginal profits to caliber along $\mathcal{C}^*$.

Recalling $MB_q < MC_q$, we see that the $\mathcal{C}^*$ locus (14) slopes up iff $MB_\delta > MC_\delta$, namely:

$$\frac{MB_\delta}{MB} - \frac{MC_\delta}{MC} = \left(\frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\theta''}{\theta'}\right) + \frac{\Lambda_\delta}{\Lambda}\left(1 - \frac{\lambda c_{\lambda q}}{c_q}\right) > 0 \tag{15}$$

We have just argued that the first group of terms captures the direct effect with fixed λ,

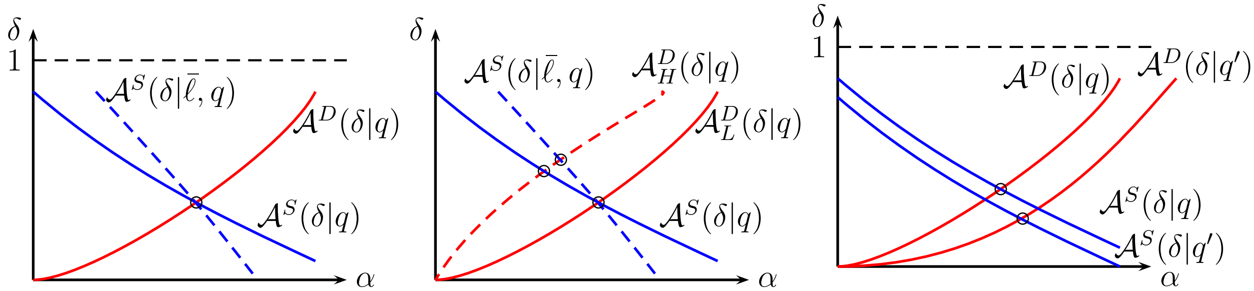

Figure 2: **Supply and Derived Demand for Attempted Crime, and a Rise in Caliber.** Demand slopes upward from the origin. LEFT: Given caliber, supply rises while demand falls in deterrence. MIDDLE: With lower demand, the crime rate falls more in the long or medium run than in the short run. RIGHT: Greater caliber $q' > q$ increases demand from $\mathcal{A}^D(\cdot|q)$ to $\mathcal{A}^D(\cdot|q')$, and supply to the left from $\mathcal{A}^S(\cdot|q)$ to $\mathcal{A}^S(\cdot|q')$. The deterrence rate falls and the attempted crime rate rises, since demand shifts more than supply (Lemma 2).

and is positive by the theft complements assumption (6). The second group subsumes the indirect effect — namely, $\Lambda_\delta \Pi_{\lambda q}$ — and is negative, given (12) and profit complements.

In fact, this logic implies that the $\mathcal{E}_\lambda(c_q) \geq 1$ is incompatible with an interior caliber solution for the criminal optimization. For it would ensure that (15) is positive for all δ, and thus the $\mathcal{C}^*$ locus has infinite slope in q, and so is a line segment joining $(0,0)$ and $(0,1)$.

We henceforth assume $\mathcal{E}_\lambda(c_q) < 1$. Lemma 1 summarizes these insights (proof in §B).

Lemma 1. *In (q,δ)-space, the optimal caliber locus $\mathcal{C}^*$ rises at low deterrence rates, and falls at high deterrence rates. As a function $q(\delta)$, it is perfectly inelastic near $\delta = 0, 1$.*

B. The Derived Demand for Attempted Crime. Consider the victims' optimization in Step (iii). Given caliber $q > 0$ and an attempted crime rate $\alpha > 0$, since any vigilance $v > q\chi(1)$ secures a perfect deterrence $\Delta(v,q) = 1$, we have $v \leq q\chi(1)$ at an optimum. Also, since the loss $\mathcal{L}(v,q,\alpha)$ is convex in v and satisfies $\mathcal{L}_v \downarrow -\infty$ as $v \downarrow 0$, the optimal vigilance is $v = q\chi(1)$ if $\alpha > -q\chi'(1)/(M\theta'(1))$. Otherwise, it obeys the first-order condition for (2):

$$\mathcal{L}_v(v,q,\alpha) = \alpha\Theta_v(v,q)M + 1 = \alpha\theta'(\delta)M/(q\chi'(\delta)) + 1 = 0 \tag{16}$$

Observe how we have used (1) to re-express the FOC in (q,δ)-space. This is the deterrence rate that victims would demand at any attempted crime rate and caliber. But we exploit (16) inversely, securing the per capita derived *demand for attempted crime* by potential victims:

$$\mathcal{A}^D(\delta|q) \equiv \frac{q\chi'(\delta)}{-\theta'(\delta)M} \tag{17}$$

recalling that $\theta' < 0$. Since vigilance optimally rises from zero as the attempted crime rate rises, *the demand for attempted crime slopes upward from the origin* (Figure 2). Also, the demand vanishes as victim's property losses $M \uparrow \infty$. Finally, demand rises in caliber.

From equation (17), the demand for attempted crime $\mathcal{A}^D$ is more elastic with a more elastic marginal vigilance χ' and marginal theft function θ' — because $\mathcal{E}_\delta(\mathcal{A}^D) = \mathcal{E}_\delta(\chi') - \mathcal{E}_\delta(\theta')$. Thus, larger increases in attempted crime leads to the same increase in deterrence when vigilance costs are more elastic, or when the returns to policing diminish faster.

C. The Supply of Attempted Crime. We now derive an analogous supply of crime for fixed caliber q. Given also deterrence δ, if the *marginal criminal* $\bar{\ell}(q,\delta) > 0$, then it must obey:

$$\bar{\ell}(q,\delta) \equiv \max_{\lambda \geq 0} \theta(\delta)\lambda m - c(\lambda, q) = \theta(\delta)\Lambda(q,\delta)m - c(\Lambda(q,\delta), q) \tag{18}$$

We can deduce that the mass of criminals $F(\bar{\ell}(q,\delta))$ is smaller with greater caliber, and smaller with more deterrence, since by the Envelope Theorem in (18):

$$\bar{\ell}_q = -c_q < 0 \qquad \text{and} \qquad \bar{\ell}_\delta = \theta' \Lambda m < 0 \tag{19}$$

Next, since the potential criminals with outside options $\ell \leq \bar{\ell}$ engage in crime, and each attempts $\Lambda(q,\delta)$ crimes before capture, the per capita *supply of attempted crime* (of caliber q) is:

$$\mathcal{A}^S(\delta|q) \equiv \Lambda(q,\delta) F(\bar{\ell}(q,\delta)) \tag{20}$$

Given (12) and (19), *supply falls in deterrence and caliber* (Figure 2). In fact, for all positive calibers $q > 0$, it vanishes at some imperfect deterrence $\delta < 1$ — since $\theta(1) = 0$ and outside options are non-negative in (18). Also, fixing deterrence, a rise in caliber solely raises the marginal costs of attempting more crime. This deters entry and reduces offenses per criminal.

D. The Market Clearing Locus. Combining the insights of §4-B and §4-C, the supply and derived demand for attempted crime move oppositely in deterrence and in caliber.

Lemma 2 (Supply and Demand). *For any caliber $q > 0$, there exists a unique deterrence rate $\delta \in (0,1)$ that clears the market. Also, if caliber rises, then deterrence falls, and when the theft function is complementary in vigilance and caliber, i.e. (6), the attempted crime rises.*

Assume that caliber rises. This reduces the marginal efficacy of vigilance, and thus reduces deterrence for any attempted crime rate — to wit, the derived demand for attempted crime rises. Also, for a fixed deterrence, the rise in caliber increases the cost and marginal cost of each offense, reducing the supply of attempted crime rate. These two shifts reinforce each other, and so the market clearing deterrence falls, as seen in the right panel of Figure 2.

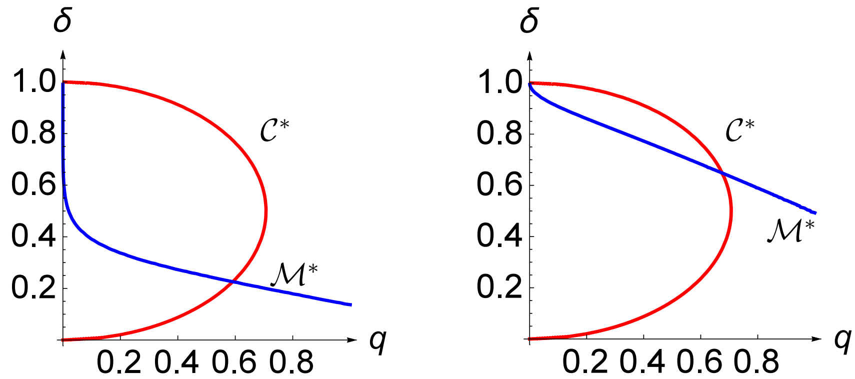

Figure 3: **Optimal Caliber and Market Clearing Loci.** The $\mathcal{C}^*$ locus starts at the origin, first rises and then falls near perfect deterrence, while the $\mathcal{M}^*$ locus monotonically falls. The loci may cross on the rising or falling portions of $\mathcal{C}^*$. Both panels consider the example in §3 with $A = B = T = D = 1$, $L = 5$, $m = 2$ and high property losses $M = 20$. Outside options have cdf $F(\ell) = (\ell/L)^3$ at left, and $G(\ell) = (\ell/L)^{1/2}$ at right (F is higher than G).

Figure 2 depicts this equilibrium in (α, δ)-space. The caliber comparative static follows because demand shifts down more than supply when caliber rises. In (q, δ)-space, the *market clearing locus* $\mathcal{M}^*$ is the curve $\mathcal{A}^D(\delta|q) = \mathcal{A}^S(\delta|q)$. This yields the market clearing deterrence rate at any caliber $\delta(q)$. Lemmas 1 and 3 justify the panels in Figure 3 (proof in §B):

Lemma 3. *The market clearing locus* $\mathcal{M}^*$ *falls from* $(0,1)$ *to some* $(\bar{q}, 0)$*. As a function* $\delta(q)$*, it is perfectly inelastic near* $q=0$*. If* $\mathcal{C}^*$ *slopes down, it is steeper in* q *than* $\mathcal{M}^*$ *in equilibrium.*

Whether the caliber locus $\mathcal{C}^*$ slopes up or down in *equilibrium* is an empirical issue. Structural changes that affect the market clearing locus $\mathcal{M}^*$ but not $\mathcal{C}^*$ can help identify its slope — e.g., a lower outside option distribution F or higher properly losses M shifts $\mathcal{M}^*$ right, but leaves $\mathcal{C}^*$ fixed. As seen in Figure 3, *the* $\mathcal{C}^*$ *locus slopes down in equilibrium for sufficiently low outside options or high enough property losses* M*, and otherwise slopes up.*

E. The Crime Rate. Consider a positive exogenous supply shock shifting $\mathcal{A}^S$ right. When vigilance is fixed, the deterrence δ is constant at each caliber, and since α rises, so too does the crime rate κ. Since the demand $\mathcal{A}^D$ is not perfectly elastic, δ rises. Now, the behavior of $\kappa = \theta(\delta)\alpha$ is ambiguous in general, as α and δ both rise. We wish to understand the slope of the *constant crime rate locus* $\bar{K}$ *locus* in (α, δ)-space with $\kappa = \theta(\delta)\alpha$ fixed.

Observe in Figure 4 that κ increases when supply shifts right iff κ *rises along the demand curve* $\mathcal{A}^D$. In turn, this holds iff the product $\theta(\delta)\mathcal{A}^D(\delta|q)$ rises in δ, namely when:[26]

$$\mathcal{E}_\delta(\theta\mathcal{A}^D) = \mathcal{E}_\delta(\theta) + \mathcal{E}_\delta(\mathcal{A}^D) = \mathcal{E}_\delta(\theta) + \mathcal{E}_\delta(\chi') - \mathcal{E}_\delta(\theta') > 0. \tag{$\bigstar$}$$

[26]By appendix Claim B.2, ($\bigstar$) implies that in equilibrium, criminal costs are more elastic in λ than in q.

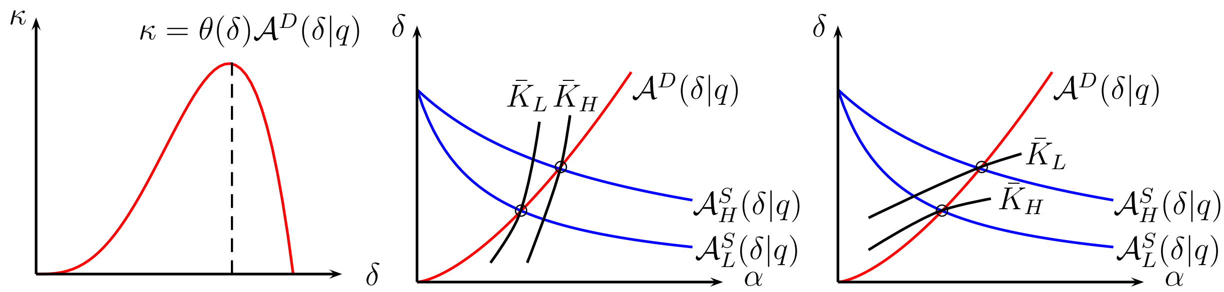

Figure 4: **The Constant Crime Rate Locus $\bar{K}$.** LEFT: The crime rate $\kappa = \theta(\theta)\alpha$ is hump-shaped in δ along the demand curve $\mathcal{A}^D$. MIDDLE AND RIGHT: A right shift in supply from $\mathcal{A}^S_L$ to $\mathcal{A}^S_H$ *raises* the crime rate when the $\bar{K}$ locus is steeper than demand $\mathcal{A}^D$ ($\bar{K}$ shifts to a higher level set $\bar{K}_H$), and *lowers* it otherwise ($\bar{K}$ shifts to a lower level set $\bar{K}_L$).

But as seen in Figure 4, the crime rate $\kappa(\delta) = \theta(\delta)\mathcal{A}^D(\delta|q)$ vanishes at $\delta = 0$, where $\mathcal{A}^D$ vanishes (Figure 2), and at $\delta = 1$, as $\theta(1) = 0$. Inequality ($\bigstar$) intuitively holds for low deterrence:

Lemma 4. *The constant crime rate locus $\bar{K}$ locus is steeper than the demand curve iff inequality ($\bigstar$) holds iff $\kappa M/v > \mathcal{E}_\delta(\chi)/(\mathcal{E}_\delta(\chi') - \mathcal{E}_\delta(\theta'))$. In particular, if the expected losses are at least twice vigilance costs, $\kappa M \geq 2v$, then ($\bigstar$) holds. On the other hand, if the expected losses are less than vigilance costs, $\kappa M \leq v$, then ($\bigstar$) fails.*

Claim E.1 proves this result. Observe that with no police, $\theta(\delta) \equiv 1 - \delta$, and vigilance $\chi(\delta) = D\delta^\gamma$, then ($\bigstar$) holds for all $\delta < 1 - 1/\gamma$, and fails otherwise. For instance, in Figure 3, ($\bigstar$) holds precisely for $\delta < 1/2$, exactly where the locus $\mathcal{C}^*$ is rising. Inequality ($\bigstar$) is easier to satisfy with more convex χ (larger γ).[27] The second test based on observables follows as $\mathcal{E}_\delta(\theta') \leq 0$, and $\mathcal{E}_\delta(\theta') = 0$ absent police, and (6) and (7) imply $\mathcal{E}_\delta(\chi') \leq \mathcal{E}_\delta(\chi) \leq 2\mathcal{E}_\delta(\chi')$. The last claim follows because inequality (6) implies $\mathcal{E}_\delta(\chi)/(\mathcal{E}_\delta(\chi') - \mathcal{E}_\delta(\theta')) \geq 1$.

Consider motor vehicle theft. From the FBI, in 2013 the average property loss from motor vehicle theft was about \$6,000; the theft rate was 221.3 per 100,000 inhabitants, or at least 450 per 100,000 car owners, namely, a theft rate at least $\kappa = 0.004$. So the expected property losses are at least $\kappa M = \$24$ (conservatively assuming stolen cars are a write-off). So ($\bigstar$) holds if $v \leq \kappa M/2 = \$12$. Since a car is on average replaced every seven years, this amounts to a one-shot vigilance investment on car security less than \$84, which is realistic.

For another application, consider Lojack.[28] According to Ayres and Levitt (1998), the property loss of vehicles with Lojack was roughly \$1000, as measured then. They estimate a theft rate in Lojack cities of 0.025, and a \$97 annuity equivalent of the \$600 Lojack

[27]Note that the elasticity $\mathcal{E}_\delta(\theta)$ explodes near perfect deterrence and vanishes at zero deterrence, for $\mathcal{E}_\delta(\theta) \leq \delta\theta'(\delta)/(1-\delta) \downarrow -\infty$ as $\delta \uparrow 1$ and $\theta(1) < 0$, while $\mathcal{E}_\delta(\theta) \uparrow 0$ as $\delta \downarrow 0$, given $\theta'(0) > -\infty$.

[28]The LoJack Stolen Vehicle Recovery System is an aftermarket vehicle tracking system hidden inside the vehicle, that allows vehicles to be tracked by police, with the aim of recovering them in case of theft.

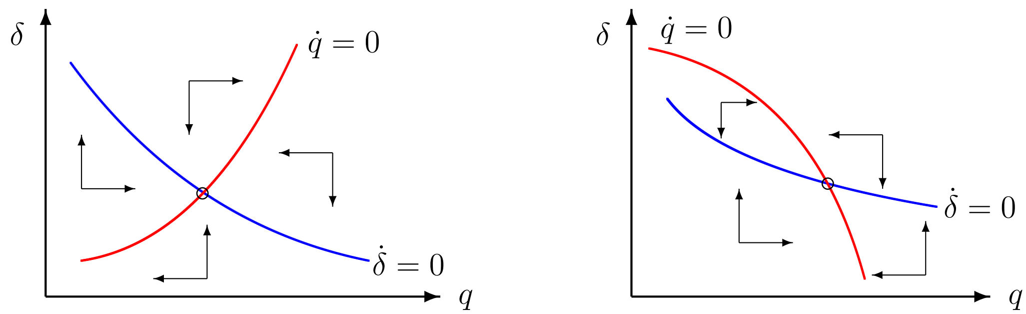

Figure 5: **Theorem 2: An Example of the Tâtonnement Dynamics.** The $\dot{q} = 0$ curve coincides with the optimal caliber $\mathcal{C}^*$ locus, while $\dot{\delta} = 0$ coincides with the *medium-run* market clearing locus, namely, the one that fixes the marginal outside option $\bar{\ell}$. If deterrence is lower (higher) than its equilibrium level, the excess of supply (demand) pushes the deterrence rate up (down). Similarly, if caliber is lower (higher) than its equilibrium level, the marginal profits of caliber are positive (negative) encouraging more (less) caliber.

installation fee. This exceeds the expected property loss of \$25, and so $\kappa M < v$ — thus (★) fails.[29] This makes sense, as these vehicles have a low theft function, and so a high deterrence.

F. Equilibrium. As seen in Figure 3, the $\mathcal{C}^*$ and $\mathcal{M}^*$ loci intersect at $(0, 1)$. Since vigilance and caliber are costly activities, and victims value their property more than criminals do, this is the efficient outcome. But this zero caliber and perfect deterrence outcome cannot arise as an equilibrium in our decentralized random matching model — because $\delta = 1$ requires $v = 0$, and zero vigilance is never a best reply to zero caliber. We argue there is a unique interior crossing of $\mathcal{C}^*$ and $\mathcal{M}^*$ — this ensures the mutual best reply property of $(v, \lambda, q, \bar{\ell})$ for the derived deterrence rate δ, and attempted and actual crime rates α and κ.

Theorem 1 (Equilibrium). *A unique crime equilibrium* $(v^*, \lambda^*, q^*, \bar{\ell}^*, \delta^*, \alpha^*, \kappa^*)$ *exists, in which* $\alpha^* < 1$ *for low enough* F *and high enough* c_λ*, or low enough* $\chi'(1)$*.*

G. Stability. We now show that under a natural tâtonnement adjustment process in $(v, \lambda, q, \bar{\ell})$-space, economic forces drive us toward equilibrium. We venture that *an intensive variable rises iff its marginal benefit exceeds its marginal cost, and an extensive entry variable rises iff profits are positive.* We assume speeds of adjustment proportional to the marginal gains and net profits, with proportionality constants for $(v, q, \lambda, \bar{\ell})$. A crime equilibrium $(v^*, q^*, \lambda^*, \bar{\ell}^*)$ is a steady state of the linear dynamical system (21)–(24). It is *stable* if it is asymptotically stable for the next adjustment process — i.e., every dynamic converges to

[29]This analysis ignores ongoing vigilance costs. According to Anderson (2012), each adult spends one minute and 50 seconds locking and unlocking doors each day.

the equilibrium (Brock and Malliaris, 1989; Arrow and Hurwicz, 1960; Samuelson, 1941):

$$\dot{v} = -k_v \left(\lambda F(\bar{\ell})\Theta_v(v,q)M + 1\right) \quad (21)$$
$$\dot{q} = k_q \left(\Theta_q(v,q)\lambda m - c_q(\lambda,q)\right) \quad (22)$$
$$\dot{\lambda} = k_\lambda \left(\Theta(v,q)m - c_\lambda(\lambda,q)\right) \quad (23)$$
$$\dot{\bar{\ell}} = k_{\bar{\ell}} \left(\Theta(v,q)\lambda m - c(\lambda,q) - \bar{\ell}\right) \quad (24)$$

By a suitable choice of constants, this dynamic can distinguish market stability with respect to different kind of shocks. For instance, a persistent shock affects *all* the variables, whereas a transitory shock — eg. a temporary power outage — need not affect the extensive margin. Such an example can be capture by setting $k_{\bar{\ell}} = 0$ (i.e. the medium run). Figure 5 depicts in (q,δ)-space the dynamics of a transitory shock when the level of offenses adjust very fast, namely, $k_\lambda \uparrow \infty$. Naturally, the short run corresponds to $k_{\bar{\ell}} = k_q = 0$.

Theorem 2 (Stability). *The crime equilibrium is stable in medium run and short run. It is stable in the long-run when the offenses adjust sufficiently fast, or entry occurs sufficiently slow, compared to the other decision margins.*

5 Predictions about Crime

This section uses the following *equilibrium recipe* ($R3$–$R5$ are derived in Appendix §E):

$R1$ The caliber and deterrence rate (q,δ) follow from intersecting the $\mathcal{C}^*$ and $\mathcal{M}^*$ loci;

$R2$ The attempted crime rate α follows from the supply and demand curves $\mathcal{A}^D$ and $\mathcal{A}^S$; these curves also shift indirectly due to caliber q shifts, as governed by Lemma 2;

$R3$ The level of offenses $\lambda = \Lambda(q,\delta)$ falls in q and δ by (12), and thus its level set $\bar{\Lambda}$ slopes down in (q,δ) space. Since $\bar{\Lambda}$ is more elastic than $\mathcal{M}^*$, given inequality (6), and $\mathcal{M}^*$ is more elastic than $\mathcal{C}^*$, offenses λ falls in deterrence δ along $\mathcal{C}^*$ (as seen in Figure 6);

$R4$ Vigilance $v = q\chi(\delta)$ rises in q and δ, and thus its level set $\bar{V}$ slopes down in (q,δ)-space. It is more elastic than $\mathcal{C}^*$, and so vigilance v rises in deterrence δ along $\mathcal{C}^*$ (Figure 6);

$R5$ The crime rate $\kappa = \theta(\delta)\alpha$ rises in α and falls in δ; its level set $\bar{K}$ slopes up in (α,δ)-space.

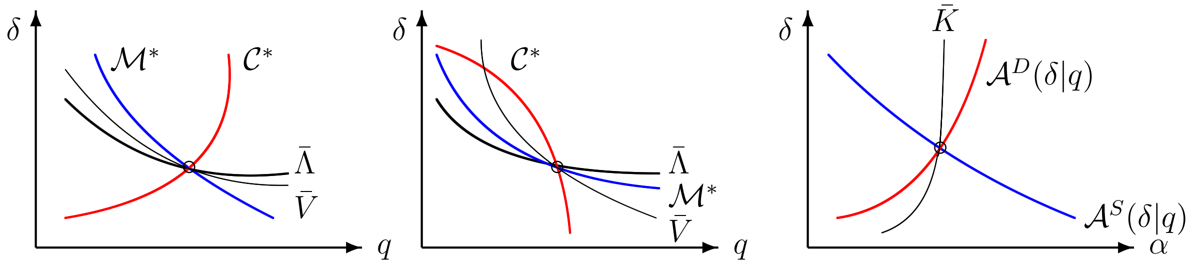

Figure 6: **The $\bar{V}$, $\bar{\Lambda}$ and $\bar{K}$ Loci.** If $\mathcal{C}^*$ slopes up, then $\bar{V}$ is more elastic than the market clearing locus $\mathcal{M}^*$. Otherwise, $\bar{V}$ is more elastic than $\mathcal{C}^*$ but less elastic than $\mathcal{M}^*$. The $\bar{\Lambda}$ locus is more elastic than $\mathcal{M}^*$, given (6); while $\bar{K}$ is more inelastic than $\mathcal{A}^D$, given (4).

5.1 Legal Penalties and Criminal Outside Options

Becker (1968) focused on the deterrence effect of legal penalties, finding that the level of offenses falls with greater anticipated punishment. He argues (on p. 177) that: "a rise in the income available in legal activities would reduce the incentives to enter illegal activities and thus would reduce the level of offenses". We find that in our equilibrium setting this prediction actually fails for individual criminals, but holds at the aggregate level.

Consider an improvement in the distribution of outside options $F(\ell)$ in the sense of first order stochastic dominance, such as a rise in legal penalties, or an improved economy. This deters criminal entry and so depresses the attempted crime rate; vigilance accordingly falls. But then there are fewer criminals, each attempting more offenses λ.

Proposition 1 (Better Outside Options). *If outside options improve, then the deterrence rate δ and vigilance v fall, but the level λ of offenses rises. The attempted crime rate α falls for high enough outside options, and always, given (★). The crime rate κ rises if (★) fails and outside options are low enough; but it falls when outside options are sufficiently high, given (★).*

That the crime rate κ falls in outside options is consistent with the known positive relation between unemployment and property crime (Raphael and Winter-Ebmer, 2001; Levitt, 2004; Chalfin and McCrary, 2014). As Aristotle wrote, "Poverty is the parent of crime". Yet after the Great Recession of 2008–9 the unemployment rate and property crime rate moved inversely, as pointed out by Chalfin and McCrary (2014). The last claim in Proposition 1 identifies a possible non-monotone relation between the crime rate and outside options that would shed light on this. For even though the attempted crime rate rises with worse outside options, absent (★), the theft chance might fall so much that the crime rate ultimately falls.

Figure 7 illustrates the proof of Proposition 1. Better outside options F shifts the supply of attempted crime $\mathcal{A}^S(\cdot|q)$ left, while the derived demand $\mathcal{A}^D(\cdot|q)$ remains constant, for

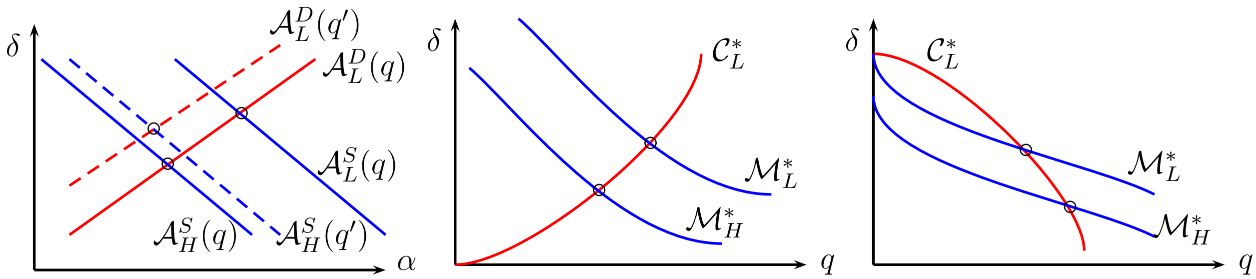

Figure 7: **Proposition 1: Deterrence Rises / Attempted Crime Falls in Penalties.** When the outside option distribution F shifts right to F_H from F_L, supply shifts left from $\mathcal{A}_L^S(q)$ to $\mathcal{A}_H^S(q)$, and the market clearing locus $\mathcal{M}^*$ shifts left from $\mathcal{M}_L^*$ to $\mathcal{M}_H^*$. The deterrence rate falls, and so does caliber iff $\mathcal{C}^*$ slopes up. If caliber falls to $q' < q$, the attempted crime rate falls even more, since demand shifts up more than supply by Lemma 2.

every caliber q in (α, δ)-space.[30] The market clearing deterrence falls at any caliber, and thus the $\mathcal{M}^*$ locus shifts left. The deterrence rate δ falls even if $\mathcal{C}^*$ slopes down, for $\mathcal{C}^*$ is more inelastic than $\mathcal{M}^*$. The criminal caliber q falls iff the $\mathcal{C}^*$ locus slopes up — and so if outside options are high enough. By the equilibrium recipe *R*3 and *R*4, the level of offenses λ rises, and vigilance v falls. Next, by the recipe *R*2, the attempted crime rate falls at any caliber. For high outside options, caliber falls to $q' < q$, further shifting the supply and demand loci to $\mathcal{A}^S(\cdot|q')$ and $\mathcal{A}^D(\cdot|q')$ left (Lemma 2). Finally, by the recipe *R*5, the crime rate κ falls, since the new $\bar{K}$ locus at the intersection of $\mathcal{A}^S(\cdot|q')$ and $\mathcal{A}^D(\cdot|q')$ is left of the original, given (★). Appendix F analyzes the case of low outside options and high property losses M, and so when the caliber locus $\mathcal{C}^*$ slopes down.

5.2 Better Policing

Unlike Becker (1968), improved police manpower or technology no longer has an ambiguous effect on the individual and aggregate level of offenses.[31] Better policing reduces the theft-theft function function, and thus blunts the criminal returns to offenses and caliber. Then both offenses λ and caliber q fall, and entry is deterred. As a result, the attempted crime rate $\alpha = \lambda F(\bar{\ell})$ falls. So policing diminishes the marginal efficacy of vigilance, crowding it out. *The crowding out is more than perfect: deterrence falls when policing improves.*

[30] To ease graphical notation, we refer to $\mathcal{A}^S(\cdot|q)$ and $\mathcal{A}^D(\cdot|q)$ simply as $\mathcal{A}^S(q)$ and $\mathcal{A}^D(q)$.

[31] See p. 188. In Becker's normative model, the authority minimizes convex social losses by choosing the fine f and the apprehension chance p. Social losses are given by the damage per offense $D(O)$ plus apprehension costs $C(O, p)$ plus social costs of punishment. Criminal behavior is subsumed by the function $O = O(p, f)$ with $O_p, O_f \leq 0$. At the optimum, the marginal efficacy of f and p vanish. Since police technology reduces both $C_O \geq 0$ and $C_p \geq 0$, the marginal efficacy of f rises, given p; while the marginal efficacy of p might rise or fall, given f. Thus, the optimal fine f rises but the optimal apprehension chance p can either rise or fall, depending on the magnitude of C_O and C_p. As a result, the offenses O might either rise or fall.

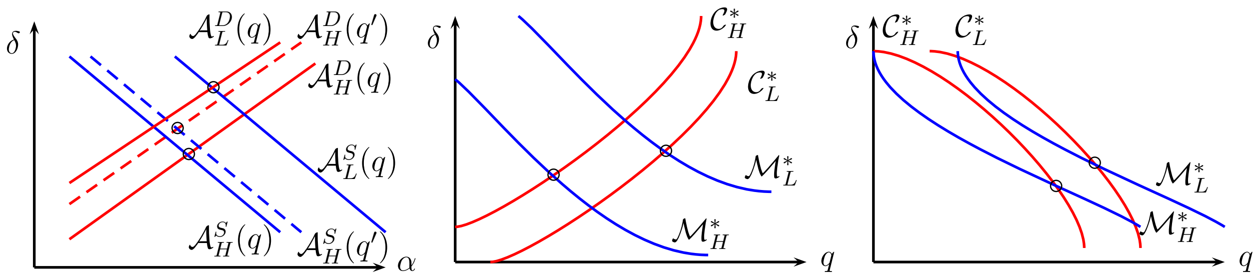

Figure 8: **Proposition 2: Better Policing Lowers Deterrence / Attempted Crime.** A rise in policing from L to H shifts supply to the left (from $\mathcal{A}^S_L(q)$ to $\mathcal{A}^S_H(q)$) and demand to the right (from $\mathcal{A}^D_L(q)$ to $\mathcal{A}^D_H(q)$). The attempted crime rate falls for every caliber q. The market clearing locus $\mathcal{M}^*$ shifts left more than $\mathcal{C}^*$ does, and so deterrence falls. If $\mathcal{C}^*$ slopes down, then caliber falls, since $\mathcal{C}^*$ shifts down more than $\mathcal{M}^*$. By Lemma 2, the attempted crime rate further falls, since caliber falls to $q' < q$.

Proposition 2 (Policing). *If the theft function falls, then the deterrence rate δ, caliber q, offenses λ, vigilance v, attempted crime rate α all fall. The crime rate κ falls, given (★).*

This rationalizes a robust empirical finding that policing reduces crime (Levitt, 1997; McCrary, 2002; Nagin, 2013; Chalfin and McCrary, 2014), and also that better policing displaces private vigilance (Philipson and Posner, 1996; Vollaard and Koning, 2009).

We prove Proposition 2. First, the direct effect of a lower theft function θ is a reduction in criminal offenses (11) and in the marginal criminal (18). So, the supply for attempted crime (20) shifts left from $\mathcal{A}^S_L(\cdot|q)$ to $\mathcal{A}^S_H(\cdot|q)$ in (α, δ)-space (see Figure 8). Also, the marginal efficacy of vigilance falls in (16), raising the demand for attempted crime (17) — so $\mathcal{A}^D_L(\cdot|q)$ shifts right to $\mathcal{A}^D_H(\cdot|q)$. Thus, the market clearing deterrence falls at any caliber q — the $\mathcal{M}^*$ locus shifts left in Figure 8. Policing also reduces the marginal returns to caliber (14). Accordingly, the optimal caliber falls at any deterrence δ, and the caliber locus $\mathcal{C}^*$ shifts left.

Next, consider caliber and deterrence. If $\mathcal{C}^*$ slopes up, then q falls after the left shift in $\mathcal{M}^*$ and $\mathcal{C}^*$. The analysis is trickier if $\mathcal{C}^*$ slopes down. By Claims G.1 and G.2, $\mathcal{M}^*$ shifts down and left more than $\mathcal{C}^*$, and thus, caliber and deterrence both fall below their original levels.

Third, equilibrium vigilance v falls with better policing by equilibrium recipe *R*4.

Fourth, we turn to the attempted crime rate. By Claim G.3, with better policing, supply $\mathcal{A}^S(\cdot|q)$ shifts down more than demand $\mathcal{A}^D(\cdot|q)$ (left panel of Figure 8). Next, consider the indirect effects: Since caliber falls to $q' < q$, the attempted crime rate α is further depressed, by Lemma 2. As seen in Figure 8, supply $\mathcal{A}^D_H(\cdot|q)$ and demand $\mathcal{A}^S_H(\cdot|q)$ both rise as caliber falls, but supply shifts up less than demand — the respective shifts from $\mathcal{A}^D_H(\cdot|q)$ and $\mathcal{A}^S_H(\cdot|q)$ to $\mathcal{A}^D_H(\cdot|q')$ and $\mathcal{A}^S_H(\cdot|q')$. Easily, given (★), the crime rate κ falls as we shift to a lower $\bar{K}$ locus in *R*5, and it is associated to a lower crime rate, since θ has fallen.

The proof for the level of offenses is in §G. It is complicated because the $\bar{\Lambda}$ locus shifts

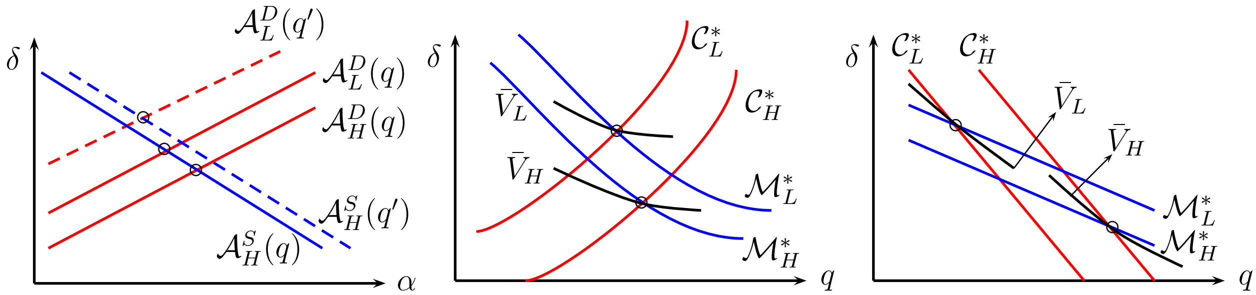

Figure 9: **Proposition 3: Lower Vigilance Cost Raises Deterrence and Reduces the Attempted Crime Rate.** When the vigilance function falls from H to L, supply $\mathcal{A}_H^S$ is not directly affected, while demand shifts left from $\mathcal{A}_H^D(q)$ to $\mathcal{A}_L^D(q)$, lowering the attempted crime rate. The $\mathcal{M}^*$ locus shifts right (from $\mathcal{M}_H$ to $\mathcal{M}_L$), and $\mathcal{C}^*$ shifts left (from $\mathcal{C}_H$ to $\mathcal{C}_L$) more than enough, reducing caliber and raising deterrence. The left panel also shows the reinforcing indirect caliber-effect on the equilibrium α, when caliber falls to $q' < q$.

to a higher level set since caliber and deterrence fall (by the recipe *R3*), but each level set is associated to a lower λ because $\Lambda(q, \delta)$ falls in the theft function θ, by the FOC (11).

5.3 An Improved Vigilance Technology: The Magnification Effect

Consider a better vigilance technology, namely a lower marginal vigilance function χ'. This affects the incentives of both victims and criminals. Victims need less vigilance to secure any deterrence rate, given caliber, while criminals require more caliber to defeat any deterrence, given vigilance. We find a *magnification effect*: If the marginal vigilance function falls, victims relax their guard and reduce their vigilance, but criminals *reduce their caliber so much that deterrence rises.* Thus, the offenses, the attempted crime rate, and crime rate fall.

Proposition 3 (Better Vigilance). *If marginal vigilance costs fall, then deterrence δ rises, and vigilance v, caliber q, offenses λ, the attempted crime α, and the crime rate κ all fall.*

This is consistent with a number of empirical findings. Ayres and Levitt (1998) find that in four years after the introduction of Lojack the auto theft rate fall around 17%. Farrell et al. (2011) find that the dramatic drop in auto theft over the last 20 years in the US, Britain and Australia is negatively correlated with an increase in anti-theft devices for cars. Gonzalez-Navarro (2013) finds a 48% reduction in auto theft for vehicles participating in the Ford Lojack program in Mexico.[32] van Ours and Vollaard (2015) show a huge reduction of car theft in the Netherlands after the EU mandated all new cars to have electronic engine

[32]In Mexico, Lojack was introduced through an exclusivity agreement between Ford Motor and the Lojack company. This together with publicity in the local media revealed which car model were equipped with Lojack, making Lojack an observable theft deterrence device (Gonzalez-Navarro, 2013).

immobilizer; while Vollaard and van Ours (2011) show that burglary greatly fell in the Netherlands after mandate for burglary-proof windows and doors in all new-built homes. Finally, our result also sheds light on the 47% crash in auto theft rates since 2003 in the U.S.,[33] since newer cars have theft-immobilizer devices or part markings.[34,35]

Now we prove Proposition 3. Assume a lower marginal vigilance function χ'.

First, we deduce the direct effects. A lower χ' raises the marginal efficacy of vigilance in (16), shifting the demand for attempted crime up to $\mathcal{A}_L^D(\cdot|q)$ from $\mathcal{A}_H^D(\cdot|q)$, at any caliber q (Figure 9). The supply $\mathcal{A}_H^S(\cdot|q)$ is unchanged as χ does not affect either the level of offenses in (11) or the marginal criminal in (18). As seen in Figure 9, the market clearing deterrence rate thus rises for every caliber, and so $\mathcal{M}^*$ shifts up. Also, the marginal profits in caliber falls in (14), and the caliber locus $\mathcal{C}^*$ shifts left. All told, deterrence rises, for any slope of $\mathcal{C}^*$.

Second, we determine the equilibrium caliber. If $\mathcal{C}^*$ slopes down, caliber q falls after the left and right shift of $\mathcal{C}^*$ and $\mathcal{M}^*$, respectively. Further, caliber falls even if $\mathcal{C}^*$ slopes up, since $\mathcal{M}^*$ shifts down less than $\mathcal{C}^*$ (see the middle panel of Figure 9), by Claim H.1.

Third, the offenses falls as the $\bar{\Lambda}$ locus in our recipe $R3$ shifts up to a lower level set.

Fourth, the attempted crime rate falls, given the two augmenting effects depicted in Figure 9 — namely, an upward demand shift $\mathcal{A}^D$ and a fall in caliber. The crime rate falls because the $\bar{K}$ locus in the equilibrium recipe $R5$ shifts left to a lower level set (not depicted).

Finally, we turn to vigilance. Observe that if $\mathcal{C}^*$ slopes down, then since $\bar{V}$ is steeper than $\mathcal{M}^*$, the respective left and right shift of $\mathcal{C}^*$ and $\mathcal{M}^*$ lead to a left shift of the $\bar{V}$ locus (right panel of Figure 9). Since χ' drops (see $R4$ in the recipe), vigilance falls. But if $\mathcal{C}^*$ slopes up, the analysis is more subtle, since the $\bar{V}$ locus shifts up (middle panel of Figure 9). Claim H.2 argues that the direct effect wins out: vigilance falls.

5.4 Lower Property Losses or Higher Criminal Gains

We now explore how crime is affected by the property losses M, criminal gains m, or more generally, the criminal markdown $1 - m/M$. These variables directly affects one or both sides of the criminal market, but the equilibrium feedback affects both sides. For instance, one might think that higher gains m encourage criminal caliber and thus thwart deterrence. In fact, we next claim that vigilance rises so much in response that the reverse happens. Likewise, if property values fall, then vigilance relaxes, and one might think that criminals reduce their caliber. In fact, this logic is only valid when criminal outside options are high.

[33] We computed this number using Table 1 in the FBI Uniform Crime Report Data 2012.

[34] Fujita and Maxfield (2012) study the relationship between security and the drop in auto theft in the US.

[35] By contrast, larceny and burglary are respectively down 10% and 19% in this same time span.

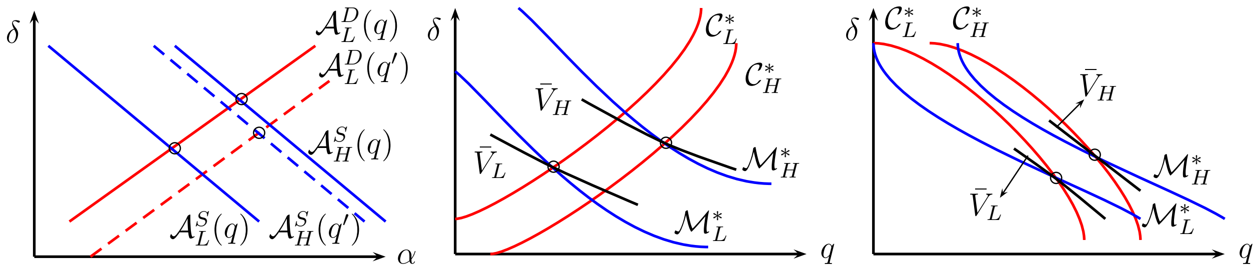

Figure 10: **Proposition 4(*a*): Greater Criminal Gain Raises Deterrence and Attempted Crime Rate.** When the criminal's gain m rises from L to H, the attempted crime rate rises: Supply shifts right from $\mathcal{A}_L^S(q)$ to $\mathcal{A}_H^S(q)$, while demand $\mathcal{A}_L^D(q)$ is unaffected. So the market clearing locus $\mathcal{M}^*$ shifts right and up. But the $\mathcal{C}^*$ locus shifts right more than $\mathcal{M}^*$, and up less than $\mathcal{M}^*$. So deterrence rises, as does the caliber, say to $q' > q$. Supply and demand further shift to $\mathcal{A}_H^S(q')$ and $\mathcal{A}_L^D(q')$, inflating the attempted crime rate more.

Before our next result, we introduce an extra assumption needed here. The marginal rate of transformation of offenses for caliber is $MRT \equiv c_q(\lambda, q)/c_\lambda(\lambda, q)$. We wish to understand what happens to this when the criminal output (λ, q) scales up. We assume that the *scale elasticity of this MRT*, namely, $\sigma \equiv qMRT_q/MRT + \lambda MRT_\lambda/MRT$, is nonnegative.[36,37]

Proposition 4 (Losses / Criminal Gains). (a) *If the gain m rises, then caliber, vigilance, offenses, the attempted crime rate and crime rate all rise. Deterrence, vigilance, and the crime rate rise, provided (★) and a positive scale elasticity of the MRT.*
(b) *If property losses M fall, then deterrence and vigilance both fall, while the offenses, attempted crime rate, and crime rate all rise. Caliber falls for high enough outside options.*
(c) *Fix the markdown $\mu \equiv 1 - m/M$ and assume (★). If (m, M) rise, then caliber, vigilance, deterrence, offenses, attempted crime rate and crime rate all rise.*

This speaks to the variation among the major crime categories. As predicted by parts (a) and (b), the auto theft rate (with a higher markdown) is well below the burglary rate, which in turn is well below the larceny rate.[38] Part (c) assumes that the stakes rise.

Proof of Part (a). Consider a rise in the criminal's gain m. First, this directly raises the offenses in (11) and also the marginal criminal in (18). Thus, the supply for attempted crime (20) shifts right from $\mathcal{A}_L^S(\cdot|q)$ to $\mathcal{A}_H^S(\cdot|q)$ in (α, δ)-space (see Figure 10). The demand for attempted crime $\mathcal{A}_L^D(\cdot|q)$ (17) is unaffected, since property losses are fixed. So the market

[36]The elasticity of scale of a production function yields the response of output when all inputs rise by 1%.
[37]Notice that if costs c are homogeneous or homothetic, then $\sigma = 0$ — for σ would depend only on (λ/q), and so a scale up of (λ, q) leaves the marginal rate of transformation unaffected.
[38]From Table 1 in the FBI Uniform Crime Report Data 2013, the robbery rate is consistently below the burglary rate, which makes sense if we view robbery as having a very high markdown, due to its violence.

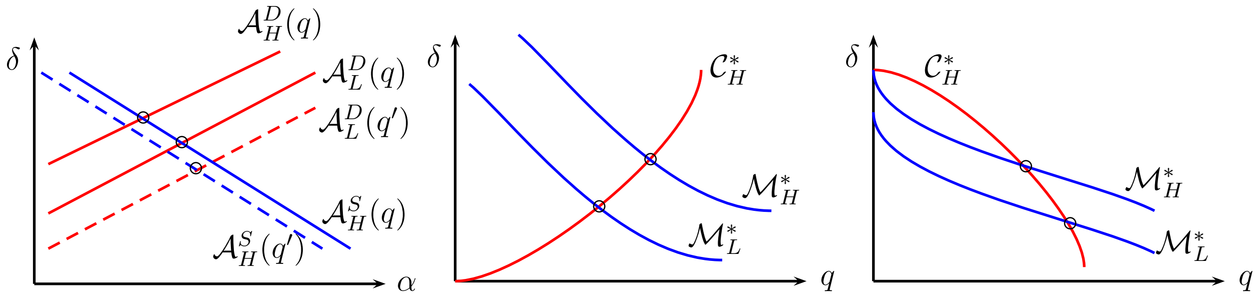

Figure 11: **Proposition 4(*b*): Smaller Property Losses Reduces Deterrence and Raises the Attempted Crime Rate.** If M falls from H to L, demand shifts right to $\mathcal{A}_H^D(q)$ from $\mathcal{A}_L^D(q)$, while the supply $\mathcal{A}_H^S(q)$ is unaffected. Deterrence falls and the attempted crime rate rises for every caliber q. The $\mathcal{M}^*$ locus shifts down and left (from $\mathcal{M}_H^*$ to $\mathcal{M}_L^*$), whereas $\mathcal{C}^*$ is unaffected. If $\mathcal{C}^*$ slopes down, then caliber rises to $q' > q$; and so the attempted crime rate rises even more (Lemma 2), for demand shifts to $\mathcal{A}_L^D(q')$ and supply shifts to $\mathcal{A}_H^S(q')$.

clearing deterrence rises at any caliber, and so the $\mathcal{M}^*$ locus shifts right. Also, the criminal's gain raises the marginal profits to caliber in (14), and thus the caliber locus $\mathcal{C}^*$ shifts right.

Second, we determine the effect on caliber and deterrence. If $\mathcal{C}^*$ slopes up, caliber clearly rises, given the right shifts of $\mathcal{C}^*$ and $\mathcal{M}^*$. If $\mathcal{C}^*$ slopes down, the right shift of $\mathcal{M}^*$ lowers caliber but the right (and up) shift of $\mathcal{C}^*$ raises it. By Claim I.1, $\mathcal{C}^*$ shifts up more than $\mathcal{M}^*$, and so caliber rises. Next, deterrence rises since $\mathcal{M}^*$ shifts right more than $\mathcal{C}^*$ by Claim I.3. These oppositely ordered shifts are possible, because $\mathcal{M}^*$ is more elastic than $\mathcal{C}^*$.

Third, vigilance rises when $\mathcal{C}^*$ is downward sloping since the $\bar{V}$ locus slopes down, and when $\mathcal{C}^*$ is upward sloping, because $\bar{V}$ is more elastic than $\mathcal{C}^*$ (see recipe *R4* and Figure 10).

Fourth, the attempted crime rate rises, since supply shifts right to $\mathcal{A}_H^S(q)$ for every q. Also, the indirect effect of rising caliber reinforces this: For if the new caliber is $q' > q$, then supply shifts down to $\mathcal{A}_H^S(\cdot|q')$, but demand shifts further down to $\mathcal{A}_L^D(\cdot|q')$ by Lemma 2 (see Figure 10). Thus, the attempted crime rate rises even more. The crime rate also rises, since the upward-sloping $\bar{K}$ locus (recipe *R5*) is steeper than the demand $\mathcal{A}_H^D(\cdot|q)$, given (★).

Finally, the effect on offenses is more subtle. The direct effect is that each level set has a higher λ, since $\Lambda(q,\delta)$ in (11) rises in m. But the indirect effect is $\bar{\Lambda}$ shifts to a lower level set, as caliber and deterrence rise. We prove in §I that the direct effect dominates.

Proof of Part (b). First consider the direct effect of a fall in property losses M. This depresses the optimal vigilance in (17), shifting demand right to $\mathcal{A}_L^D(\cdot|q)$ from $\mathcal{A}_H^D(\cdot|q)$ (see Figure 11). The supply $\mathcal{A}_H^S(\cdot|q)$ in (20) is unaffected, since criminal profits do not change. Therefore, the market clearing deterrence falls at any caliber, and so $\mathcal{M}^*$ shifts left; while the $\mathcal{C}^*$ locus is unchanged. As seen in the right two panels of Figure 11, the deterrence rate falls, and so does caliber iff $\mathcal{C}^*$ slopes up — as is true if outside options are high enough.

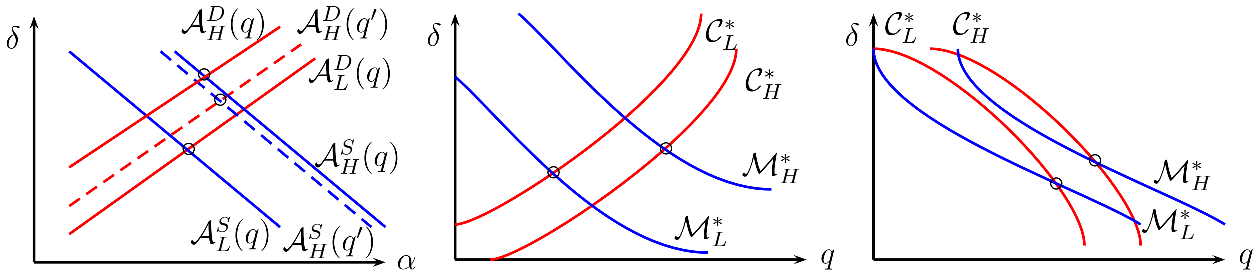

Figure 12: **Proposition 4(*c*): Greater Criminal Gain and Property Losses Raise the Attempted Crime Rate and Deterrence.** When m and M rise from L to H *with a fixed markdown*, supply $\mathcal{A}_L^S(q)$ shifts right to $\mathcal{A}_H^S(q)$, and demand $\mathcal{A}_L^D(q)$ left to $\mathcal{A}_H^D(q)$. The attempted crime rate rises at all q. The $\mathcal{M}^*$ locus shifts right and up more than $\mathcal{C}^*$, and so deterrence and caliber rise. The attempted crime further rises (Lemma 2), as $q' > q$.

Second, vigilance falls and offenses rise, since the former rises and the latter falls in deterrence along $\mathcal{C}^*$, respectively (see the equilibrium recipe *R3* and *R4*).

Finally, if caliber rises to $q' > q$, then supply and demand $\mathcal{A}_L^S(\cdot|q)$ and $\mathcal{A}_L^D(\cdot|q)$ respectively shift down to $\mathcal{A}_L^S(\cdot|q')$ and $\mathcal{A}_L^D(\cdot|q')$. Hence, the attempted crime rate rises by Lemma 2. By the recipe *R5*, the crime rate rises, since the $\bar{K}$ locus shifts right to a higher level set. In §I we analyze the case of high enough outside options (i.e., when $\mathcal{C}^*$ slopes up).

Proof of Part (c). A proportional rise of (m, M) now affects both demand $\mathcal{A}_L^D(\cdot|q)$ and supply $\mathcal{A}_L^S(\cdot|q)$: Demand shifts left to $\mathcal{A}_H^D(\cdot|q)$, and supply shifts right to $\mathcal{A}_H^S(\cdot|q)$ (see Figure 12). Claim I.8 shows that, given (★), supply shifts up more than demand, and so the attempted crime rate rises at any caliber q. The analysis is then similar to part (a) (see §I).

5.5 Lower Criminal Costs

If criminal costs fall but marginal costs are unchanged, then our analysis of changing legal penalties in Proposition 1 applies. In particular, the level of offenses falls! For lower costs leads to more criminal entry, greater vigilance, and so lower criminal offenses per capita.

So assume a marginal cost reduction, occasioned by a technological innovation or lower input cost. We adapt two notions of technological changed commonly used for production functions, and instead use them for our cost function. Parameterize the cost function by t and technology by $\tau(t)$, where $\tau'(t) < 0$. Change is *offense-augmenting* if $c(\lambda, q|t) = c(\tau(t)\lambda, q)$ and *caliber-augmenting* if $c(\lambda, q|t) = c(\lambda, \tau(t)q)$. When $c_{\lambda q} = 0$, this change only affects the λ and q margins, respectively, but in general, both marginal costs fall in t. E.g. daylight and car lock "jammers" respectively affect mainly the marginal costs of offenses and caliber.

While a marginal cost reduction, occasioned by a technological innovation or lower input cost, have many intuitive changes, we discover that there can be distinctly different impacts

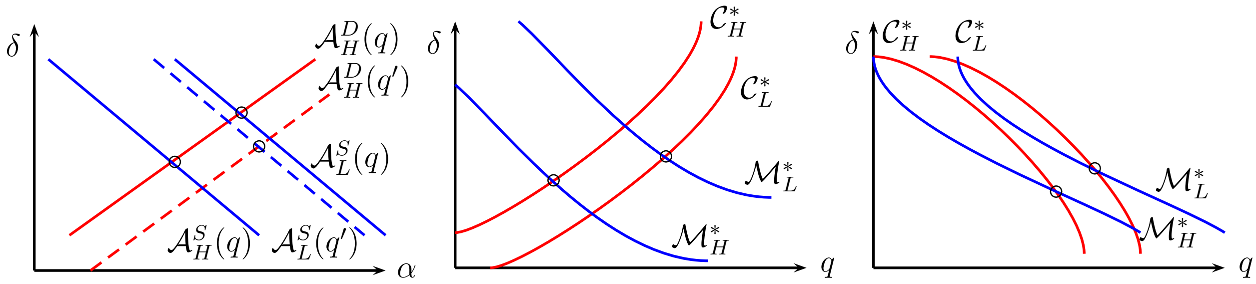

Figure 13: **Proposition 5(*a*): An Offense-Augmenting Change Raises Deterrence.** When this technology shift changes costs from H to L, supply shifts right from $\mathcal{A}_H^S(q)$ to $\mathcal{A}_L^S(q)$, and the attempted crime rate rises for every caliber q. The market clearing locus $\mathcal{M}^*$ and optimal caliber locus $\mathcal{C}^*$ shift right, from $\mathcal{M}_H^*$ to $\mathcal{M}_L^*$ and from $\mathcal{C}_H^*$ to $\mathcal{C}_L^*$. Deterrence rises as the $\mathcal{M}^*$ locus shifts right more than $\mathcal{C}^*$, and caliber rises as the $\mathcal{M}^*$ locus shifts up less than $\mathcal{C}^*$. The attempted crime rate further rises, as caliber rises to $q' > q$ (Lemma 2).

on vigilance and deterrence. A caliber-augmenting change will lessen deterrence and possibly vigilance — opposite to what happens with an offense-augmenting innovation.

Proposition 5 (Criminal Costs). (*a*) *Any offense-augmenting technological change raises deterrence, caliber, vigilance, offenses, the attempted crime rate, and crime rate, given* (★). (*b*) *Any caliber-augmenting technological change raises caliber, offenses, the attempted crime, and crime rate, and lowers deterrence. Vigilance rises for low outside options.*

This result is consistent with the empirical finding that crime rates are higher in big cities than small cities (Glaeser and Sacerdote, 1999). For urban and rural areas have many differences, but a key one is that criminal costs are lower in large cities.[39] Or, consider the fact that the residence burglary crime rate is twice as high during working hours, when a residence is presumably empty, as at night, when it is occupied.[40] For daytime theft is less costly, as it requires passing up fewer occupied houses. As Proposition 5(a) predicts, this elicits more vigilance by home owners in the form of security systems.

Proof of Part (*a*). An offense-augmenting technological change directly raises *each criminal's* supply of offenses solving (11), and the marginal criminal in (18). So the overall supply in (20) shifts right from $\mathcal{A}_H^S(\cdot|q)$ to $\mathcal{A}_L^S(\cdot|q)$, while the demand $\mathcal{A}_H^D(\cdot|q)$ in (17) is unaffected (left panel of Figure 13). So the market clearing deterrence rate rises at any caliber, and thus the $\mathcal{M}^*$ locus shifts right. Next, the rise in offenses raises the marginal benefits of caliber in (14), and so shifts the $\mathcal{C}^*$ locus right (right and middle panel of Figure 13).

[39] Urban density explains about 50% of the variation between crime and city size (Glaeser and Sacerdote, 1999). They point out that greater urban density in cities lowers criminal costs in a variety of ways — eg. criminals need to travel less distance to find a victim; also, they face a lower probability of recognition.

[40] See the offense analysis in Table 23 in the FBI Uniform Crime Report Data 2013.

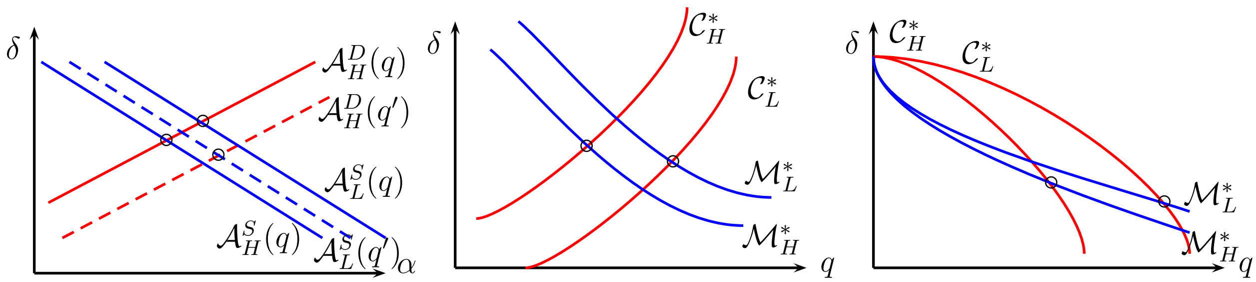

Figure 14: **Proposition 5(*b*): A Caliber-Augmenting Change Lowers Deterrence.** When this technology change lowers costs from H to L, supply shifts right to $\mathcal{A}_L^S(q)$ from $\mathcal{A}_H^S(q)$ at any caliber q. The $\mathcal{C}^*$ and $\mathcal{M}^*$ loci shift right from $\mathcal{C}_H^*$ to $\mathcal{C}_L^*$, and from $\mathcal{M}_H^*$ to $\mathcal{M}_L^*$. Deterrence falls and caliber rises. The attempted crime rate rises even more by Lemma 2.

Second, we determine the equilibrium caliber and deterrence. If $\mathcal{C}^*$ is upward sloping, then right shifts of $\mathcal{C}^*$ and $\mathcal{M}^*$ raise caliber, and have an ambiguous effect on deterrence. If $\mathcal{C}^*$ is downward sloping, then both effects are ambiguous: The right shift of $\mathcal{C}^*$ raises caliber and lowers deterrence, and the right shift of $\mathcal{M}^*$ lowers caliber and raises deterrence. In both cases, the deterrence rate rises, since $\mathcal{M}^*$ shifts right more than $\mathcal{C}^*$ does, by Claim J.1. In the second case, Claim J.2, the $\mathcal{C}^*$ shift overwhelms the $\mathcal{M}^*$ shift, and thus caliber rises.

Third, vigilance rises since caliber and deterrence rise (see the recipe *R*4).

Fourth, we explore the attempted crime rate and crime rate. As argued, both $\mathcal{C}^*$ and $\mathcal{I}^*$ shift right, and so the direct effect on the attempted crime rate is positive (left panel of Figure 13). Next, Lemma 2 addresses the indirect effects, asserting that the attempted crime rate rises when caliber does. Finally, since both the demand and supply loci $\mathcal{A}_L^D(\cdot|q')$ and $\mathcal{A}_L^S(\cdot|q')$ have shifted right, the $\bar{K}$ locus — horizontally sandwiched between them (right panel of Figure 6) — is now higher. All told, the crime rate rises.

The effect on offenses is more subtle, with opposing direct and indirect effects. The direct effect is positive — $\Lambda(q,\delta)$ rises as c_λ falls — and the indirect effect is negative — the $\bar{\Lambda}$ locus in Figure 6 shifts to down. In §J, we show that the direct effect dominates, and offenses rise.

Proof of Part (*b*). In a caliber-augmenting technological change, supply $\mathcal{A}_H^S(\cdot|q)$ in (20) shifts right, since the offenses in (11) and the marginal criminal in (18) rise, for the marginal costs c_λ and cost c fall. Demand $\mathcal{A}_H^D(\cdot|q)$ in (17) is unchanged. So the market clearing deterrence rises at any caliber, and $\mathcal{M}^*$ locus shifts right. Next, the $\mathcal{C}^*$ locus shifts right as marginal costs c_q fall, for this raises marginal profits of caliber in (14), and $\Pi_{qq} < 0$.

Second, we determine the caliber and deterrence. If $\mathcal{C}^*$ is upward sloping, then right shifts of $\mathcal{C}^*$ and $\mathcal{M}^*$ raise caliber, and have an unclear effect on deterrence. If $\mathcal{C}^*$ is downward sloping, then both effects are ambiguous: The right shift of $\mathcal{C}^*$ raises caliber and lowers deterrence, and the right shift of $\mathcal{M}^*$ lowers caliber and raises deterrence. As in Figure 14, in any case, deterrence falls, since $\mathcal{C}^*$ shifts right more than $\mathcal{M}^*$ does, by Claim J.4. In the second case, caliber rises, for $\mathcal{C}^*$ shifts up more than $\mathcal{M}^*$ does, given Claim J.5.

Third, the offenses rise, since $\Lambda(q,\delta)$ rises as c_λ falls, and $\bar{\Lambda}$ in recipe $R3$ shifts right.

Fourth, we explore the equilibrium vigilance. Since $\mathcal{C}^*$ and $\mathcal{M}^*$ sandwich $\bar{V}$ when $\mathcal{C}^*$ slopes down (see Figure 6), the $\bar{V}$ locus shifts right, and thus vigilance rises. Now suppose that caliber does not affect the marginal cost of each offense, i.e., $c_{\lambda q} = 0$. This implies that a caliber-augmenting change leaves c_λ unchanged, and so it has no direct effect on the optimal level of offenses. Also, in the medium run the marginal criminal is fixed, and thus the supply curve $\mathcal{A}^S$ and the market clearing $\mathcal{M}^*$ locus are unaffected. Thus, if $\mathcal{C}^*$ slopes up, vigilance falls since deterrence falls and $\mathcal{M}^*$ is steeper than $\bar{V}$ (Figure 6).

Finally, the attempted crime rate rises, as supply shifts right at any caliber q (Figure 14). Also, by Lemma 2, the attempted crime rate rises even more, as caliber rises to $q' > q$. The crime rate rises, since deterrence falls and the attempted crime rate rises.

6 The Social Cost of Crime

We now explore the welfare loss of crime, owing to criminal costs, vigilance costs, and the *transfer loss of theft*, namely, $\kappa\mu M$, where $\mu = 1 - m/M$ is the criminal markdown. We ignore the unmodeled policing expenditures. Since $c(\lambda,q)+\bar{\ell} = \theta\lambda m$ and $\alpha = \lambda F(\bar{\ell})$, total criminals costs are the sum of direct costs and their foregone outside options $\int_0^{\bar{\ell}}(c(\lambda,q)+\ell)f(\ell)d\ell$, or:

$$F(\bar{\ell})c(\lambda,q) + \int_0^{\bar{\ell}} \ell f(\ell)d\ell = \theta\lambda m F(\bar{\ell}) - \int_0^{\bar{\ell}} F(\ell)d\ell = \alpha\theta m - \int_0^{\bar{\ell}} F(\ell)d\ell$$

Next, the total vigilance costs of crime are $v = q\chi(\delta) = \alpha\theta M(-\theta'/\theta)(\chi/\chi')$, by the victims' optimality equation (16). Given the crime rate $\kappa = \theta\alpha$, the *per capita social costs of crime* are:

$$\kappa\mu M + \kappa(1-\mu)M - \int_0^{\bar{\ell}} F(\ell)d\ell + \kappa M|\mathcal{E}_\delta(\theta)|/\mathcal{E}_\delta(\chi) \tag{25}$$

As seen in Figure 15, the first term is the transfer loss of theft. The next two terms are gross criminal costs. For if all criminals shared a common outside option, then the third term vanishes, while the first term is obviously gross criminal gains, and therefore balances criminal losses, by free entry. Criminal heterogeneity is accounted for by the third term. For a criminal $\ell \in [0,\bar{\ell})$ makes profits $\bar{\ell} - \ell$, and thus total profits across all criminals amount to $\bar{\ell}F(\bar{\ell}) - \int_0^{\bar{\ell}} \ell f(\ell)d\ell = \int_0^{\bar{\ell}} F(\ell)d\ell$. So the third term in (25) subtracts aggregate criminal profits owing to heterogeneity. The last term measures total vigilance costs; these might be at most their expected property losses, for $|\mathcal{E}_\delta(\theta)|/\mathcal{E}_\delta(\chi) \le 1$ by summing (6) and (★).

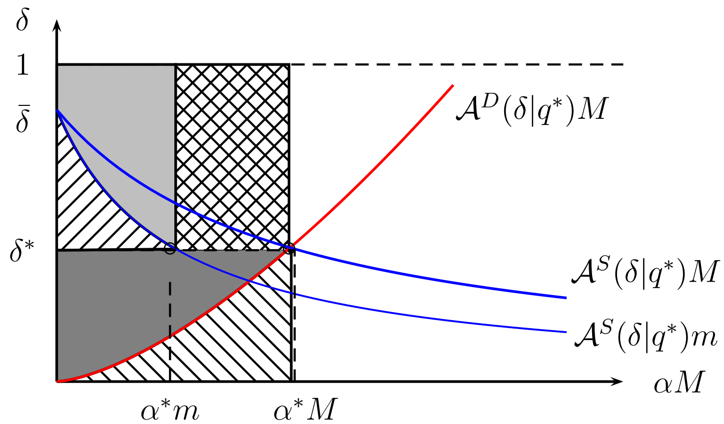

Figure 15: **Per Capita Social Costs of Crime.** The areas are measured in dollars after we scale the units on the horizontal axis by M. Given no police, the crosshatched region represents the transfer loss $\kappa\mu M$. The NW-dashed region is criminal profits, the light-shaded region is criminal costs, and the dark-shaded region vigilance costs v^*. The dashed area *below demand* $\mathcal{A}^D(\cdot|q^*)M$ is the potential victims' surplus, i.e., gains $\alpha^*\delta^*M$ less vigilance costs v^*.

We can suggestively depict the social costs. We assume no police, or $\theta(\delta) \equiv 1-\delta$. In any crime equilibrium $(v^*, \lambda^*, q^*, \bar{\ell}^*, \delta^*, \alpha^*, \kappa^*)$, we argue that *criminal profits* $\int_0^{\bar{\ell}^*} F(\ell)d\ell$ *are captured by the area above equilibrium deterrence* δ^* *and below the value-scaled supply curve* $\mathcal{A}^S(\delta|q^*)m$ in Figure 15. Indeed, consider the marginal criminal $\bar{\ell}(\delta, q^*)$ in (18), and let $\bar{\delta}$ be lowest deterrence that deters all criminals, namely given by $\bar{\ell}(\bar{\delta}, q^*) = 0$. Therefore,

$$\int_0^{\bar{\ell}^*} F(\ell)d\ell = \int_{\bar{\delta}}^{\delta^*} F(\bar{\ell}(\delta, q^*))\bar{\ell}_\delta(\delta, q^*)d\delta = \int_{\bar{\delta}}^{\delta^*} \mathcal{A}^S(\delta|q^*)\frac{\bar{\ell}_\delta(\delta, q^*)}{\Lambda(q^*, \delta)}d\delta = \int_{\delta^*}^{\bar{\delta}} \mathcal{A}^S(\delta|q^*)md\delta$$

for $\mathcal{A}^S(\delta|q) \equiv \Lambda(q,\delta)F(\bar{\ell}(q,\delta))$ by (20), and $\bar{\ell}_\delta = -\Lambda m$ by (19).

Likewise, *vigilance costs* v^* *equal the area below equilibrium deterrence* δ^* *and above the value-scaled demand* $\mathcal{A}^D(\cdot|q^*)M$. For since $\mathcal{A}^D(\delta|q) \equiv q\chi'(\delta)/M$ by (17), and $\chi(0) = 0$, we have:

$$v^* = q^*\chi(\delta^*) - q^*\chi(0) = \int_0^{\delta^*} q^*\chi'(\delta)d\delta = \int_0^{\delta^*} \mathcal{A}^D(\delta|q^*)Md\delta$$

Since the demand curve $\mathcal{A}^D$ depicts a victim's reservation price at any quantity, the area over it and below $\delta = \delta^*$ captures a cost, and his surplus is thus the triangular area *below* his demand. On the other hand, since crime is an economic bad for victims but an economic good for criminals, criminals' profits are the area below supply and above $\delta = \delta^*$.

The social costs of crime (25) do not explicitly depend on criminal costs, the markdown, or the caliber. For policy purposes, one can estimate the social costs of crime from observables and knowledge of the vigilance costs χ, theft function θ, and outside options $F(\cdot)$.

The per capita social costs of crime (25) can be reinterpreted as victim losses (summing

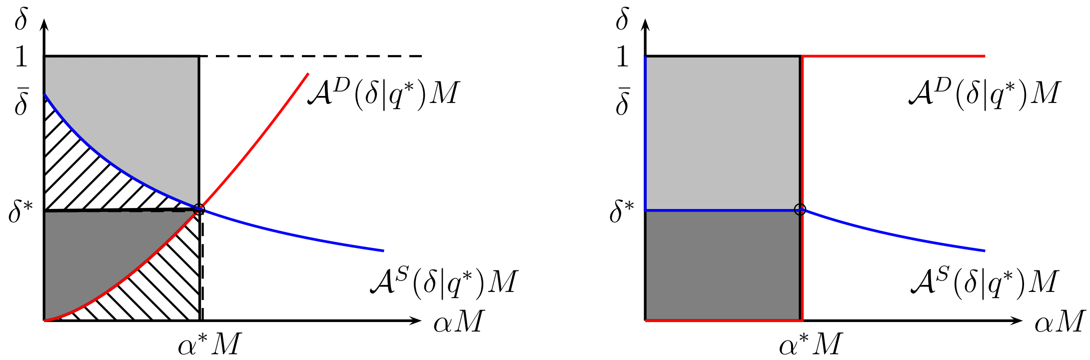

Figure 16: **Gains Dissipate with Linear Vigilance and Homogeneous Criminals.** Both panels assume no markdown $\mu = 0$, and no police $\theta(\delta) \equiv 1 - \delta$. At left, we depict the Tullock paradox. The light shaded region is criminal costs and the dark-shaded area is the vigilance costs, as in Figure 15. This is less than the left rectangle, namely, the value $\alpha^* M$ of potential gains of attempted crime. The difference is the NW-dashed area — profits owing to criminal heterogeneity — and the SE-dashed area below the rising demand curve. The right panel considers the extreme case with homogeneous criminals with outside option $\bar{\ell}^*$ — and so a supply $\mathcal{A}^S(\delta|q^*) = \Lambda(q^*, \delta)$ for $\delta \leq \delta^*$, and 0 otherwise — and linear vigilance $\chi(\delta) = \delta\chi(1)$ — and so a vertical demand. Here, social costs equal potential gains.[41]

the first two and last terms) less criminal profits. These are at most the losses of potential victims, and coincides when criminals are homogeneous (and so their profits vanish). Policy makers might wish to place more weight on victim losses — $\kappa M + v = \kappa M + \kappa M |\mathcal{E}_\delta(\theta)|/\mathcal{E}_\delta(\chi)$. Victim losses unambiguously fall with legal penalties, by Proposition 1, or better policing, by Proposition 2, or a lower vigilance function χ, by Proposition 3 — for each reduces vigilance expenses and the crime rate, given (★). With heterogeneous criminals, it is unclear how social costs vary in the fundamentals — for given the competitive nature of crime, policies that help potential victims tend to hurt criminals, and so the sign of the net effect is ambiguous.

For another lesson, recall that Becker (1968) argues that in a competitive theft market, the total criminal costs should approximate the market value of the property loss (Becker, 1968, page 171, footnote 3). While he ignores vigilance costs, Tullock (1967) had already crystalized a more general insight that rent-seeking behavior by all agents involved might well dissipate the gains — at least absent a transfer loss of theft ($m = M$). On the other hand, his "Tullock paradox" (Tullock, 1980) later observed that rent-seeking expenditures are often swamped by the potential gains. This paradox occurs here, and we see in Figure 16 that it owes precisely to the strict convexity of the vigilance costs, and to criminal heterogeneity.

Finally, the presence of police, namely, $\theta(\delta) < 1 - \delta$, is another reason for the Tullock Paradox. But since we are not modeling police costs, it is not clear how to think of this case.

[41] The right panel strictly steps outside of our model, violating our assumptions on χ to make this point.

7 Discrete Vigilance Expenditures

We now show that our framework is flexible and can be readily adapted to a discrete vigilance context that has been famously explored in Ayres and Levitt (1998) who explored the then new Lojack technology. They argue that partially adoption of Lojack reduced the crime rate, and thereby generated positive spillovers on unequipped cars.[42] To analyze this scenario, we slightly depart from our model, and assume that vigilance expenditures are discrete.[43]

In particular, suppose that the available set of vigilance levels excludes an interval (v_L, v_H) around the original equilibrium level v^*, so that $v_L < v^* < v_H$. Think of high vigilance ($v \geq v_H$) as purchasing Lojack. Our crime equilibrium now entails a mixed strategy in which victims randomize between high and low vigilance — to wit, a fraction p of victims elect v_L, and the rest, v_H. We will see that in equilibrium, spillovers of unobservable vigilance naturally arise: Victims choosing high vigilance exert a positive externality on everyone else.

The mixture chance p is now the potential victims' equilibrium choice variable, and we think of the 8-tuple $(p^*, \lambda^*, q^*, \bar{\ell}^*, \delta_L^*, \delta_H^*, \alpha^*, \kappa^*)$ as a *discrete* crime equilibrium. Criminals may face low or high deterrence, $\delta_L = \Delta(v_L, q)$ or $\delta_H = \Delta(v_H, q)$, given a caliber q. For simplicity, consider the linear theft function $\theta(\delta) \equiv 1 - T\delta$ from §3, so that criminals are only affected by the *expected deterrence* $\underline{\delta}(p) \equiv p\delta_H + (1-p)\delta_L$. Then the marginal criminal in (18)

$$\bar{\ell}(\underline{\delta}, q) \equiv \max_{\lambda \geq 0}(1 - T\underline{\delta})\lambda m - c(\lambda, q)$$

is attained at the offenses $\lambda = \Lambda(\underline{\delta}, q)$. The conditional supply of attempted crime $\mathcal{A}^S(\underline{\delta}|q) \equiv \Lambda(\underline{\delta}, q)F(\bar{\ell}(\underline{\delta}, q))$ falls in the expected deterrence $\underline{\delta}$, for both the benefits and marginal benefits of crime fall in $\underline{\delta}$, and so in p. For simplicity, let's assume a geometric vigilance function $\chi(\delta) = \delta^\gamma$, for then the caliber locus $\mathcal{C}^*$ is also a function of the expected deterrence $\underline{\delta}$.[44]

Assume that v_L is the maximum pre-Lojack vigilance so that in equilibrium, the victim optimization (2) is at a corner solution $v = v_L$. As in Figure 17, the derived demand for attempted crime $\mathcal{A}^D(\underline{\delta}|q)$ initially rises, and then is infinitely elastic at $\delta = \delta_L$. Once Lojack

[42] Ayres and Levitt (1998) estimate an external benefit of Lojack users exceeding \$1300 annually. They find that the auto theft rate fall around 17% four years after Lojack's introduction. By the same logic, people with home security systems confer a positive externality on those with just the stickers for them.

[43] Our model assumes that crimes succeed or fail. Lojack introduces a third possibility of partial success: A car is stolen, but then recovered. Let the *recovery* chance be $r \in [0, 1]$, *damage* fraction $d \in [0, 1]$ and the *criminal fractional gain* given recovery ψ, assumed fixed. Then our original model subsumes this by adjusting the theft function to $\widehat{\Theta}(v, q) \equiv (1 - r + rd)\Theta(v, q)$ for victims and to $\widetilde{\Theta}(v, q) \equiv (r + (1-r)\psi)\Theta(v, q)$ for criminals. According to Lojack, in 2014 the recovery rate exceeded 90% for vehicles equipped with Lojack, compared with the 54.8% for motor vehicles nationwide (Table 24 in the FBI Uniform Crime Report Data 2013). Vehicles with Lojack face less than \$1000 in damage, compared with \$6000 for a typical vehicle.

[44] For instance, in §3 we considered quadratic vigilance and criminal costs, $\chi(\delta) = D\delta^2$ and $c(\lambda, q) = Aq^2 + B\lambda^2$, and found the optimal caliber locus $q(\underline{\delta}) = m\sqrt{T\underline{\delta}(1 - T\underline{\delta})/(2AB)}$, depicted in Figure 3.

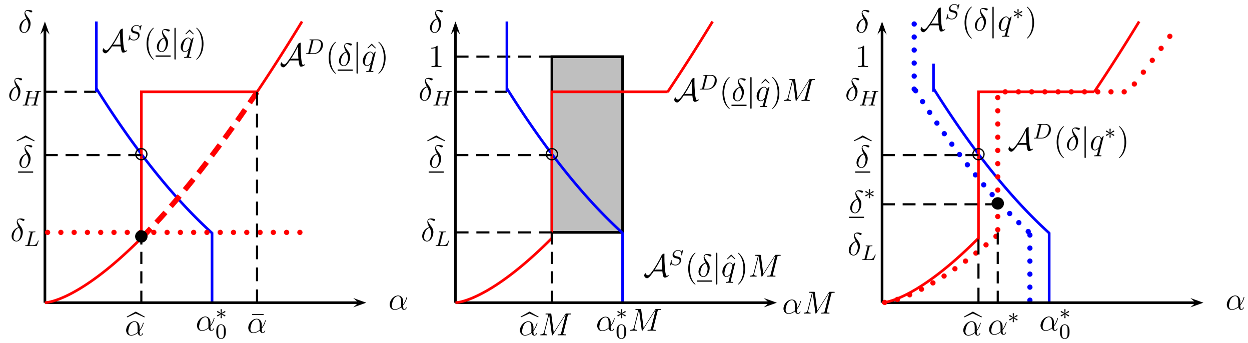

Figure 17: **Supply and Demand with Discrete Vigilance / External Benefits.** All panels assume theft function $\theta(\delta) \equiv 1 - T\delta$. The left panel depicts two scenarios: first where only vigilance $v \leq v_L$ is available (dotted), and next where $v \geq v_H$ is also available. The attempted crime rate drops from α_0^* to $\widehat{\alpha}$. The shaded area in the middle panel equals the external benefits of the change, assuming no markdown ($m = M$), for simplicity. The right panel depicts the (dotted) changes that occur in the medium and long runs, with attenuation.

becomes available (i.e., $v \geq v_H$ is feasible), the demand becomes a step function, where $\delta = \delta_L$ if $\alpha < \widehat{\alpha}$; $\delta = \delta_H$ if $\alpha \in (\widehat{\alpha}, \bar{\alpha})$; and then δ rises for $\alpha \geq \bar{\alpha}$. Here, $\widehat{\alpha}$ reflects the victims' indifference between vigilance $v = v_H$ and $v = v_L$, namely, using $v = q\chi(\delta) = q\delta^\gamma$:

$$\widehat{\alpha}(1 - T\delta_L)M + v_L = \widehat{\alpha}(1 - T\delta_H)M + v_H \quad \Longrightarrow \quad \widehat{\alpha} = \frac{q(\delta_H^\gamma - \delta_L^\gamma)}{MT(\delta_H - \delta_L)} \tag{26}$$

As seen in the left panel of Figure 17, with the introduction of the high vigilance option v_H, the attempted crime rate falls from α_0^* to $\widehat{\alpha}$, and the expected deterrence rises from δ_L to $\widehat{\underline{\delta}}$ in the short run. So when caliber is fixed, the crime rate faced by victims who choose low vigilance falls from $\kappa_0^* = (1 - T\delta_L)\alpha_0^*$ to $\widehat{\kappa} = (1 - T\delta_L)\widehat{\alpha}$. *The shaded area is the external benefit of those choosing high vigilance conferred on those electing low vigilance*, namely, $(\alpha_0^* - \widehat{\alpha})(1 - T\delta_L)M$. In the medium and long runs, criminals re-optimize caliber. Assuming that $\mathcal{C}^*$ is upward sloping, caliber rises from $\widehat{q}$ to some $q^* > \widehat{q}$ (recalling Figure 3). Supply shifts left to $\mathcal{A}^S(\delta|q^*)$, and demand right to $\mathcal{A}^D(\delta|q^*)$, the dotted curves in the right panel of Figure 17. The initial changes attenuate: The attempted crime rate rises to α^*, the expected deterrence falls to $\underline{\delta}^*$, and the external benefit falls to $(\alpha_0^* - \alpha^*)(1 - T\delta_L)M$.

Our framework also affords predictions for this twist on our model. For instance, assume that Lojack had been introduced in better economic circumstances, so that outside options F are higher in both scenarios. Demand would be unaffected, and thus $\widehat{\alpha}$ fixed, by (26). But the supply curve (20) would be shifted left, and so α_0^* would be lower. The external benefits $(\alpha_0^* - \widehat{\alpha})(1 - T\delta_L)M$ would then be lower for a fixed caliber. If, say, $\mathcal{C}^*$ is upward-sloping, then the criminal caliber q would be lower, and thus the external benefits would fall further.

Or assume Lojack had been introduced for cheaper property. With property losses M lower, demand would fall. The attempted crime rate $\widehat{\alpha}$ would fall, by (26), reducing the short-run external benefit. The expected market clearing deterrence would rise. So in the long-run, if $\mathcal{C}^*$ is downward-sloping, criminal caliber would fall and external gains even more.

8 Concluding Remarks

Becker's insight was the maximizing criminal. Ehrlich (1981) added that potential victims likewise respond to incentives. So inspired, we have devised a population game in which: (1) pairwise random matching of criminals and potential victims produces attempted crimes; (2) crime is a repeated choice, or career decision; (3) not all crimes succeed; and (4) deterrence is probabilistic, rising in vigilance and falling in criminal caliber. We have precisely formulated the payoff functions of criminals and victims, and how their actions impact one another.

While expressed as a Nash equilibrium of a game, we reformulate our solution as an equilibrium of an implicit market in which deterrence and the attempted crime rate act as the price and quantity. This is both a tractable and rich economic framework for crime, amenable to comparative statics. By allowing potential victims to optimally respond, we account for the parry and thrust of crime, for victims and criminals both respond when change befalls either party, or when law enforcement adjusts. The strategic nature of the competition is the source of many new and surprising predictions. We analyze a steady state, but show how market responses play out differently in the short- medium- and long-runs.

Every empirical paper on crime explores a change in payoffs or technology about either criminals or potential victims. Our paper now offers a one-stop equilibrium theory for all such criminal queries, that fully accounts for feedback effects. Since our model assumes optimizing marginal analysis by all individuals, and quitting decisions by criminals, we distinguish individual from aggregate behavior. In our paper, these predictions may diverge.

Since the domestic security changes post 9-11, it is more obvious than ever that the costs of crime are not just the actual losses of individuals, but also in their vigilance expenses for crimes that never happen. By modeling such costs, we also have an insight into rent-seeking: The social costs of crime are well below the potential theft gains, given diminishing returns to vigilance and criminal heterogeneity. This offers a new logic for the Tullock paradox.

Since victims in a given submarket are homogeneous, our assumption of unobservable vigilance is without loss of generality. But with heterogeneous victims, one could explain observable vigilance. In such a model, criminals must choose whether to burglar, say, the gated mansion or the ungated one. The model fundamentally changes, and observable vigilance leads criminals to direct their search (Eeckhout and Kircher, 2010), and subgame perfection rather than just Nash equilibrium is needed.[45] This is an intriguing area of future research.

Lastly, our model connects to much empirical work, but also shows the importance of keeping more careful track of other data, like the criminal λ, and the attempted crime rate.

A Supply and Demand: Proof of Lemma 2

Our results exploit some simple math. Consider a vector $(w, x, y, z) \in \mathbb{R}^4$.

Claim A.1. *If $yz > 0$, then $w/y \geq x/z$ iff $w/y \geq (w+x)/(y+z) \geq x/z$.*

Step 1: *For any caliber $q > 0$, there is a unique market-clearing deterrence $\delta^*(q) \in (0,1)$.*
Proof: Define $\nabla\mathcal{A}(\delta|q) \equiv \mathcal{A}^S(\delta|q) - \mathcal{A}^D(\delta|q)$. By (18), we have $\bar{\ell}(q,1) < 0 < \bar{\ell}(q,0)$. So by the Intermediate Value Theorem (IVT), for all $q > 0$, there exists $\bar{\delta} < 1$ yielding marginal criminal $\bar{\ell}(q,\bar{\delta}) = 0$. So supply vanishes: $\mathcal{A}^S(\bar{\delta}|q) = 0$, and $\nabla\mathcal{A}(\bar{\delta}|q) < 0$. Also, $\mathcal{A}^D(0|q) = 0 < \mathcal{A}^S(0|q)$, as seen in Figure 2. Thus, $\nabla\mathcal{A}(\bar{\delta}|q) < 0 < \nabla\mathcal{A}(0|q)$ and there exists $\bar{\delta}(q) > \delta^*(q) > 0$ s.t. $\nabla\mathcal{A}(\delta^*|q) = 0$, by the IVT again. Finally, since supply falls and demand rises in deterrence, $\delta^*(q)$ is unique since $\nabla\mathcal{A}(\delta|q)$ strictly falls in δ. □

Step 2: *If caliber q rises, $\mathcal{A}^D$ shifts down more than $\mathcal{A}^S$, and the attempted crime rate rises.*
Proof: A rise in q shifts $\mathcal{A}^D$ to the right, and $\mathcal{A}^S$ to the left, given (17) and (20). Fix $\alpha > 0$, and log-differentiate $\mathcal{A}^D(\delta|q) \equiv \alpha$ and $\mathcal{A}^S(\delta|q) \equiv \alpha$ with respect to q to get:

$$\delta_q^D \left(\frac{\chi''}{\chi'} - \frac{\theta''}{\theta'} \right) = -\frac{1}{q} \qquad \text{and} \qquad \delta_q^S \left(\frac{\Lambda_\delta}{\Lambda} + \frac{f}{F}\bar{\ell}_\delta \right) = -\left(\frac{\Lambda_q}{\Lambda} + \frac{f}{F}\bar{\ell}_q \right) \tag{27}$$

By (6), $\delta_q^D \leq -\chi/(q\chi')$. Next, $\mathcal{E}_\lambda(c_q) \leq 1$, (12), (14), and (19) imply $-(\Lambda_q/\Lambda)/(\Lambda_\delta/\Lambda) \geq -(f/F)\bar{\ell}_q/[(f/F)\bar{\ell}_\delta] = -\chi/(q\chi')$. Thus, $\delta_q^S \geq -\chi/(q\chi') \geq \delta_q^D$ by (27) and Claim A.1. □

B Caliber $\mathcal{C}^*$ and Market Clearing $\mathcal{M}^*$ Loci Properties

Claim B.1. *If the marginal criminal $\bar{\ell} \geq 0$, then $\mathcal{E}_\lambda(c) \geq 1$.*

Proof: Using (11), $\bar{\ell} = \theta\Lambda m - c(\Lambda, q) = \Lambda c_\lambda(\Lambda, q) - c(\Lambda, q)$. Thus, $\bar{\ell} \geq 0$ implies $\mathcal{E}_\lambda(c) \geq 1$.□

Claim B.2. *Assume (★). Then $\chi'/\chi + \theta'/\theta \geq 0$ and so $\mathcal{E}_\lambda(c) \geq \mathcal{E}_q(c)$; thus $\chi'/\chi + \Lambda_\delta/\Lambda \geq 0$.*

[45] There is a small literature that focuses on how the observability of vigilance might divert crime to less protected targets (Shavell, 1991; Koo and Png, 1994).

Proof: Summing (6) and (★) give $\chi'/\chi + \theta'/\theta \geq 0$. Next, write (13) as $\mathcal{E}_\lambda(c)/\mathcal{E}_q(c) = -(\chi'/\chi)(\theta/\theta')$ using (1) and (11). So $\mathcal{E}_\lambda(c) \geq \mathcal{E}_q(c)$ iff $-(\theta'/\theta)(\chi/\chi') \leq 1$, i.e. $\chi'/\chi + \theta'/\theta \geq 0$. Finally, $\Lambda_\delta/\Lambda \geq \theta'/\theta$ follows from (11), (12) and $\mathcal{E}_\lambda(c_\lambda) \geq 1$. □

THE SLOPE OF $\mathcal{C}^*$. The $\mathcal{C}^*$ locus (14) obeys $MB(q,\delta) = MC(q,\delta)$, where:

$$MB(q,\delta) := -\theta'(\delta)\Lambda(q,\delta)m\chi(\delta)/(q\chi'(\delta)) \equiv c_q(\Lambda(q,\delta),q) =: MC(q,\delta) \tag{28}$$

We'll show that $\mathcal{C}^*$ slopes up iff $MB_\delta > MC_\delta$. For log-differentiate the identity (28) to get:

$$\left.\frac{d\delta}{dq}\right|_{\mathcal{C}^*} = \frac{\frac{MC_q}{MC} - \frac{MB_q}{MB}}{\frac{MB_\delta}{MB} - \frac{MC_\delta}{MC}} = \frac{\frac{c_{qq}+c_{\lambda q}\Lambda_q}{c_q} + \frac{1}{q} - \frac{\Lambda_q}{\Lambda}}{\frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\Lambda_\delta}{\Lambda} + \frac{\theta''}{\theta'} - \frac{c_{\lambda q}\Lambda_\delta}{c_q}} \tag{29}$$

The numerator is positive since $\Lambda_q < 0$ by (12), and as (12) and strict cost-convexity imply:

$$(c_{qq} + c_{\lambda q}\Lambda_q)/c_q = (c_{qq}c_{\lambda\lambda} - c_{\lambda q}^2)/(c_q c_{\lambda\lambda}) > 0 \tag{30}$$

The sign of the slope of $\mathcal{C}^*$ thus coincides with the sign of the denominator of (29).

THE SLOPE OF $\mathcal{M}^*$. This locus solves $\mathcal{A}^D(\delta|q) = \mathcal{A}^S(\delta|q)$, where (17) and (20) imply:

$$\mathcal{A}^S(\delta|q) := \Lambda(q,\delta)F(\bar{\ell}(q,\delta)) \equiv q\chi'(\delta)/(\theta'(\delta)M) =: \mathcal{A}^D(\delta|q) \tag{31}$$

$$\Longrightarrow \qquad \left.\frac{d\delta}{dq}\right|_{\mathcal{M}^*} = \frac{\frac{\mathcal{A}^S_q}{\mathcal{A}^S} - \frac{\mathcal{A}^D_q}{\mathcal{A}^D}}{\frac{\mathcal{A}^D_\delta}{\mathcal{A}^D} - \frac{\mathcal{A}^S_\delta}{\mathcal{A}^S}} = \frac{\frac{f}{F}\bar{\ell}_q + \frac{\Lambda_q}{\Lambda} - \frac{1}{q}}{\frac{\chi''}{\chi'} - \frac{\theta''}{\theta'} - \frac{\Lambda_\delta}{\Lambda} - \frac{f}{F}\bar{\ell}_\delta} \tag{32}$$

Since $\mathcal{A}^S_q < 0 < \mathcal{A}^D_q$ and $\mathcal{A}^S_\delta < 0 < \mathcal{A}^D_\delta$, by (12), (19), and $\chi'', \theta'' > 0$, we have $d\delta/dq|_{\mathcal{M}^*} < 0$.

Claim B.3. *$\mathcal{C}^*$ is perfectly inelastic near $\delta = 0, 1$, and $\mathcal{M}^*$ is perfectly inelastic near $q = 0$.*

Proof: The slope of the $\mathcal{C}^*$ locus $q(\delta)$ in (14) explodes at $(0,0)$, as $\theta'(0) > -\infty$ and $\Lambda(0,0) > 0$ imply $q(\delta)/\delta = -[\chi(\delta)/(\delta\chi'(\delta))][\theta'(\delta)\Lambda m/c_q(\Lambda,q)] \to \infty$ as $\delta \downarrow 0$. Its slope explodes at $(0,1)$:

$$\lim_{\delta\uparrow 1}\frac{q(\delta)}{1-\delta} = \lim_{\delta\uparrow 1}\frac{\Lambda(0,\delta)}{c_q(\Lambda(0,\delta),0)(1-\delta)}\frac{m\theta'(\delta)\chi(\delta)}{-\chi'(\delta)} = \lim_{\delta\uparrow 1}\frac{\Lambda_\delta}{(1-\delta)c_{\lambda q}\Lambda_\delta}\frac{m\theta'(1)\chi(1)}{-\chi'(1)} = \infty$$

by l'Hopital's Rule. Next, the slope of the $\mathcal{M}^*$ locus $\delta(q)$ explodes at $(q,\delta) = (0,1)$:

$$\lim_{q\downarrow 0}\frac{1-\delta(q)}{q} = \lim_{\delta\uparrow 1}\frac{(1-\delta)}{\Lambda(0,\delta)F(\bar{\ell}(0,\delta))}\frac{\chi'(\delta)}{-\theta'(\delta)M} = \lim_{\delta\uparrow 1}\left[\frac{-1}{\Lambda_\delta F(\bar{\ell}) + \Lambda f(\bar{\ell})\bar{\ell}_\delta}\right]\frac{\chi'(1)}{-\theta'(1)M} = \infty$$

since $\Lambda, \bar{\ell} \downarrow 0$ as $\delta \uparrow 1$ (perfect deterrence eliminates crime). We also used l'Hopital's Rule.□

Claim B.4. *At any interior maximum, $\Pi^*_{qq}(q,v) < 0$, or equivalently $\Pi_{qq}\Pi_{\lambda\lambda} - \Pi^2_{\lambda q} > 0$.*

Proof: Twice differentiate Π^* defined before (13) to get: $\Pi^*_{qq} = \Theta_{qq}\hat{\Lambda}m + \Theta_q\hat{\Lambda}_q m - c_{\lambda q}\hat{\Lambda}_q - c_{qq}$. By strict cost convexity, $c_{qq} < 0$. Next, differentiate (11) and use (13) and $\mathcal{E}_\lambda(c_q) \le 1$ to get:

$$\hat{\Lambda}_q = (\Theta_q m - c_{\lambda q})/c_{\lambda\lambda} = (c_q/\lambda - c_{\lambda q})/c_{\lambda\lambda} = (1 - \mathcal{E}_\lambda(c_q))c_q/(\lambda c_{\lambda\lambda}) \ge 0 \tag{33}$$

So $c_{\lambda q}\hat{\Lambda}_q + c_{qq} > 0$. Next, we claim $\Theta_{qq}\hat{\Lambda}m + \Theta_q\hat{\Lambda}_q m \le 0$. Log-differentiate (11) in q to get $\hat{\Lambda}_q/\hat{\Lambda} = \Theta_q/[\Theta\mathcal{E}_\lambda(c_\lambda)] - c_{\lambda q}/[c_\lambda\mathcal{E}_\lambda(c_\lambda)]$. Expressing Θ_{qq} from (5) and $\Theta_q = -\theta'\chi/(q\chi')$:

$$\frac{\Theta_{qq}\hat{\Lambda}m + \Theta_q\hat{\Lambda}_q m}{\Theta_q\hat{\Lambda}m} = \frac{\Theta_{qq}}{\Theta_q} + \frac{\hat{\Lambda}_q}{\hat{\Lambda}} = -\frac{\chi}{q\chi'}\left(\frac{\theta''}{\theta'} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\chi'}{\chi} + \frac{\theta'}{\theta\mathcal{E}_\lambda(c_\lambda)}\right) - \frac{c_{\lambda q}}{c_\lambda\mathcal{E}_\lambda(c_\lambda)} \le 0$$

That $\Pi^*_{qq} < 0$ now follows since $\theta''/\theta' + \chi'/\chi - \chi''/\chi' > 0$ by (6), and $\chi'/\chi + \theta'/[\theta\mathcal{E}_\lambda(c_\lambda)] \ge \chi'/\chi + \theta'/\theta \ge 0$ by $\mathcal{E}_\lambda(c_\lambda) \ge 1$ and Claim B.2, respectively. Finally, since $\Pi^*(q,v) \equiv \max_\lambda \Pi(\lambda,q,v|\ell)$, we have $\Pi^*_q(q,v) = \Pi_q(\hat{\Lambda},q,v|\ell)$ and $\hat{\Lambda}_q = -\Pi_{\lambda q}/\Pi_{\lambda\lambda}$ (by (33)):

$$0 > \Pi^*_{qq}(q,v) = \Pi_{\lambda q}\hat{\Lambda}_q + \Pi_{qq} = -\Pi^2_{\lambda q}/\Pi_{\lambda\lambda} + \Pi_{qq} = \Pi^{-1}_{\lambda\lambda}(\Pi_{qq}\Pi_{\lambda\lambda} - \Pi^2_{\lambda q}) \qquad \square$$

The next claims derive the shape of the caliber and market clearing loci in Figure 3.

Claim B.5. *In (q,δ)-space, the $\mathcal{C}^*$ locus starts rising from $(0,0)$ and falls back to $(0,1)$.*

Proof: Define $\tilde{\Pi}_q(q,\delta) \equiv MB(q,\delta) - MC(q,\delta)$. The slope of $\mathcal{C}^*$ is $d\delta/dq|_{\mathcal{C}^*} = -\tilde{\Pi}_{qq}/\tilde{\Pi}_{q\delta}$. Since $\tilde{\Pi}_{qq} < 0$, we require $\tilde{\Pi}_{q\delta} < 0 (> 0)$ for $\delta \uparrow 1$ ($\delta \downarrow 0$). As deterrence $\delta \downarrow 0$, in the limit $\Pi^*_{q\delta} = (-\theta'\chi/\chi')'\Lambda m/q - c_{\lambda q}\Lambda_\delta > 0$, by (12) and $(-\theta'\chi/\chi')' > 0$ by (6). So $d\delta/dq|_{\mathcal{C}^*} > 0$.

Next, let $\delta \uparrow 1$. Then $\Lambda(\delta,q) \to 0$ for marginal profits in λ are negative near $\delta = 1$. So $\tilde{\Pi}_{q\delta} = (\Theta_q m - c_{\lambda q})\Lambda_\delta \le 0$, since $\Lambda_\delta < 0$ by (12), and $\Theta_q m \ge c_{\lambda q}$ by the FOC (13) and $\mathcal{E}_\lambda(c_q) \le 1$. Altogether, $d\delta/dq|_{\mathcal{C}^*} \le 0$ for $\delta \uparrow 1$. $\square$

Claim B.6. *In (q,δ)-space, the $\mathcal{M}^*$ locus starts at $(0,1)$ and ends at $(\bar{q},0)$ with $\bar{q} > 0$.*

Proof: Let $q = 0$. Then the supply in (20) obeys $\mathcal{A}^S(\delta|0) = \Lambda(0,\delta)F(\bar{\ell}(0,\delta))$, whereas the demand in (17) $\mathcal{A}^D(\delta|0) = 0$. Next, since $c_\lambda(0,0) = \theta(1) = 0$, we have that $\bar{\ell}(0,\delta) = 0$ iff $\Lambda(0,\delta) = 0$ iff $\delta = 1$. Thus, a perfect deterrence $\delta = 1$ clears the market when caliber $q = 0$.

Now, let $\delta = 0$. Then $\mathcal{A}^S(0|q) = \Lambda(q,0)F(\bar{\ell}(q,0))$, while $\mathcal{A}^D(0|q) = 0$ since $\chi'(0) = 0$. Next, $\mathcal{A}^S(0|0) > 0$ since $\mathcal{A}^S_\delta(\delta|0) < 0 = \mathcal{A}^S(1|0)$; and $\bar{\ell}_q(q,0) < 0$ by (19). So as $q \uparrow \infty$, the marginal criminal (18) $\bar{\ell}(q,0) \downarrow -\infty$ since $c(\cdot,q) \uparrow \infty$ and $\Lambda(0,q) \ge 0$; thus, by the Intermediate Value Theorem, $\bar{\ell}(\bar{q},0) = 0$ for some $\bar{q} > 0$. Altogether, $\mathcal{A}^S(0|\bar{q}) = 0 = \mathcal{A}^D(0|\bar{q})$. $\square$

Claim B.7. *If $\mathcal{C}^*$ slopes down in q, then its slope is strictly less than $-\chi(\delta)/(q\chi'(\delta))$.*

Proof: Recall Π^* defined before (13). Since $\tilde{\Pi}_q(q,\delta) \equiv \Pi^*_q(q, q\chi(\delta))$, we get $\tilde{\Pi}_{qq} = \Pi^*_{qq} + \Pi^*_{qv}\chi$ and $\tilde{\Pi}_{q\delta} = \Pi^*_{qv}q\chi'$. So $\tilde{\Pi}_{q\delta} < 0$ iff $\Pi^*_{qv} < 0$, and $\Pi^*_{qq} < 0$ by Claim B.4. Thus, $d\delta/dq|_{\mathcal{C}^*} = -\tilde{\Pi}_{qq}/\tilde{\Pi}_{q\delta} < 0$, and

$$\left.\frac{d\delta}{dq}\right|_{\mathcal{C}^*} = -\frac{\tilde{\Pi}_{qq}}{\tilde{\Pi}_{q\delta}} = -\frac{\Pi^*_{qq}}{\Pi^*_{qv}q\chi'} - \frac{\chi}{q\chi'} < -\frac{\chi}{q\chi'} \qquad \square$$

Claim B.8. *$\mathcal{C}^*$ slopes down in q at an equilibrium iff the slope of $\mathcal{M}^*$ is greater than $-\chi/(q\chi')$.*

Proof: Notice that $-\bar{\ell}_q/\bar{\ell}_\delta = -\chi/(q\chi')$, by (19) and (14). Also, $(q\chi'/\chi)\Lambda_q/\Lambda = \Lambda_\delta c_{\lambda q}/c_q$, by (14) and then (12). Applying Claim A.1 to (32) gives $d\delta/dq|_{\mathcal{M}^*} > -\chi/(q\chi')$ iff:

$$\frac{\frac{\Lambda_q}{\Lambda} - \frac{1}{q}}{\frac{\chi''}{\chi'} - \frac{\Lambda_\delta}{\Lambda} - \frac{\theta''}{\theta'}} > -\frac{\frac{f}{F}\bar{\ell}_q}{\frac{f}{F}\bar{\ell}_\delta} = -\frac{\chi}{q\chi'} \iff \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\Lambda_\delta}{\Lambda} + \frac{\theta''}{\theta'} - \Lambda_\delta\frac{c_{\lambda q}}{c_q} < 0 \tag{34}$$

The left hand side of the last inequality (34) equals the denominator of $d\delta/dq|_{\mathcal{C}^*}$ in (29). Since the numerator in (29) is positive, $\mathcal{C}^*$ slopes down iff $d\delta/dq|_{\mathcal{M}^*} > -\chi/(q\chi')$. $\square$

C Existence and Uniqueness: Proof of Theorem 1

A unique equilibrium $(v^*, q^*, \lambda^*, \delta^*, \bar{\ell}^*, \alpha^*, \kappa^*)$ follows from a unique interior intersection (q^*, δ^*) of $\mathcal{C}^*$ and $\mathcal{M}^*$, via $v^* = q^*\chi(\delta^*)$, $\lambda^* = \Lambda(q^*, \delta^*)$, $\bar{\ell}^* = \bar{\ell}(q^*, \delta^*)$, $\alpha^* = \lambda^* F(\bar{\ell}^*)$, $\kappa^* = \theta(\delta^*)\alpha^*$.

Step 1: Existence of (q^*, δ^*). Applying the Implicit Function Theorem to $\mathcal{A}^D(\delta|q) = \mathcal{A}^S(\delta|q)$, the $\mathcal{M}^*$ locus defines a map $\delta \mapsto q \equiv Q_{\mathcal{M}}(\delta)$, with $Q_{\mathcal{M}}(0) = \bar{q} > 0$ and $Q_{\mathcal{M}}(\bar{\delta}) = 0$. Similarly, the locus $\mathcal{C}^*$ (from (14)) yields a map $\delta \mapsto q \equiv Q_{\mathcal{C}}(\delta)$, with $Q_{\mathcal{C}}(0) = Q_{\mathcal{C}}(1) = 0$. Put $Q(\delta) \equiv Q_{\mathcal{M}}(\delta) - Q_{\mathcal{C}}(\delta)$, i.e. the horizontal gap between $\mathcal{M}^*$ and $\mathcal{C}^*$ in Figure 3. Then $Q(0) = Q_{\mathcal{M}}(0) - Q_{\mathcal{C}}(0) = \bar{q} - 0 > 0$ by Claim B.6. Given the different slopes of $\mathcal{C}^*$ and $\mathcal{M}^*$ near $\delta = 1$ (Claim B.3), there exists $\bar{\delta} < 1$ with $Q(\bar{\delta}) = Q_{\mathcal{M}}(\bar{\delta}) - Q_{\mathcal{C}}(\bar{\delta}) < 0$. So by the Intermediate Value Theorem, there exists $\delta^* \in (0, \bar{\delta})$ with $Q(\delta^*) = 0$, or $Q_{\mathcal{M}}(\delta^*) = Q_{\mathcal{C}}(\delta^*)$. $\square$

Step 2: Uniqueness of (q^*, δ^*). By contradiction, suppose that $(q_1, \delta_1) \neq (q_2, \delta_2)$ belong to $\mathcal{M}^*$ and $\mathcal{C}^*$, with $\delta_1 < \delta_2 < 1$, WLOG. Consider (17) and (20), and call the *excess supply* $\mathcal{S}(q,\delta) \equiv \mathcal{A}^S(\delta|q) - \mathcal{A}^D(\delta|q) = \Lambda(q,\delta)F(\bar{\ell}(q,\delta)) + q\chi'(\delta)/(\theta'(\delta)M)$. Easily, $\mathcal{S}_q, \mathcal{S}_\delta < 0$ by (12) and (19). By the Implicit Function Theorem, $dq/d\delta|_{\mathcal{M}^*} = -\mathcal{S}_\delta/\mathcal{S}_q$. Next, since $\mathcal{S}(q,\delta) = 0$ at any equilibrium:

$$0 - 0 = \mathcal{S}(q_2, \delta_2) - \mathcal{S}(q_1, \delta_1) = \int_\Gamma d\mathcal{S}$$

where Γ is an integration path from (q_1, δ_1) to (q_2, δ_2). On the path $\Gamma = \mathcal{I}^*$, we get:

$$0 = \int_{\mathcal{I}^*} d\mathcal{S} = \int_{\delta_1}^{\delta_2} (\mathcal{S}_q(q(\delta), \delta), \mathcal{S}_\delta(q(\delta), \delta)) \cdot (q'(\delta), 1)\, d\delta = \int_{\delta_1}^{\delta_2} \mathcal{S}_q \left[q'(\delta) + \frac{\mathcal{S}_\delta}{\mathcal{S}_q} \right] d\delta < 0$$

For $q'(\delta) + \mathcal{S}_\delta/\mathcal{S}_q > 0$ when $\mathcal{C}^*$ weakly slopes up ($q' \geq 0$), and so when $\mathcal{C}^*$ slopes down ($q' < 0$), since $q' > -q\chi'/\chi$ by Claim B.7, and $\mathcal{S}_\delta/\mathcal{S}_q = -dq/d\delta|_{\mathcal{M}^*} > q\chi'/\chi$ by Claim B.8. Altogether, $(q_1, \delta_1) = (q_2, \delta_2)$. □

STEP 3: $\alpha^* < 1$. The optimal level λ of "petty crimes" (with caliber $q = 0$) given deterrence δ solves $\theta(\delta)m = c_\lambda(\lambda, 0)$. Define $\zeta^{-1}(\lambda) \equiv c_\lambda(\lambda, 0)$. If *expected criminal gains* are $x = \theta(\delta)m \leq m$, then $\lambda = \zeta(x)$ by (11). So the mass of such crimes is $\rho(x) \equiv \zeta(x)F(\zeta(x)x - c(\zeta(x), 0))$.

Claim C.1. *In any equilibrium, the caliber* $q \leq \bar{q} \equiv -\theta'(1)\zeta(m)m\chi(1)/[c_q(\zeta(m), 0)\chi'(1)]$.

Proof: By the FOC (14), $q = -\theta'\Lambda m\chi/(c_q\chi')$. Next, notice that $-\theta'\chi/\chi'$ rises in δ by (6); also, $c_q(\Lambda, q)/\Lambda$ rises in q and δ, since $\mathcal{E}_\lambda(c_q) \leq 1$ and $\Lambda_q, \Lambda_\delta < 0$. So, caliber $q \leq -\theta'(1)\Lambda(0,0)m\chi(1)/[c_q(\Lambda(0,0), 0)\chi'(1)] = \bar{q}$, as $\Lambda(0,0) = \zeta(\theta(0)m) \leq \zeta(m)$. □

Claim C.2. *If either the mass of petty crimes* $\rho(m) < 1$ *or* $-\theta'(1)M > \bar{q}\chi'(1)$*, then* $\alpha^* < 1$.

Proof: Recall that $\Lambda(q, \delta)$ and $\bar{\ell}(q, \delta)$ fall in (δ, q) by (12) and (19). So $\Lambda(q, \delta) < \Lambda(0, 0) \equiv \zeta(\theta(0)m) \leq \zeta(m)$, since $\theta(0) \leq 1$. Also, using (18), $\bar{\ell}(q, \delta) < \bar{\ell}(0, 0) \equiv \theta(0)m\Lambda(0, 0) - c(\Lambda(0, 0), 0) = x\zeta(x) - c(\zeta(x), 0)$, for $x = \theta(0)m$. This last term has derivative $\zeta(x) > 0$, and so $\bar{\ell}(0, 0) < m\zeta(m) - c(\zeta(m), 0)$. Arguing using supply $\mathcal{A}^S(\delta|q) = \Lambda(\delta, q)F(\bar{\ell}(\delta, q))$, we have:

$$\alpha^* = \mathcal{A}^S(\delta|q) \leq \Lambda(0, 0)F(\bar{\ell}(0, 0)) < \zeta(m)F(m\zeta(m) - c(\zeta(m), 0)) \equiv \rho(m) < 1$$

Now consider demand. First, $\chi'' > 0 > -\theta''$ implies $(-\chi'/\theta')' > 0$, and $q < \bar{q}$ by Claim C.1. So $\alpha^* = \mathcal{A}^D(\delta|q) = -q\chi'/(\theta' M) \leq -\bar{q}\chi'(1)/(\theta'(1)M) < 1$, as $-\theta'(1)M > \bar{q}\chi'(1)$. □

D Stability of the Equilibrium: Proof of Theorem 2

Claim D.1. $\Pi_{\lambda q}\Pi_{qv} - \Pi_{\lambda v}\Pi_{qq} \leq 0$ *always, and* $\Pi_{\lambda v}\Pi_{\lambda q} - \Pi_{qv}\Pi_{\lambda\lambda} \geq 0$ *iff* $\mathcal{C}^*$ *slopes up.*

Proof: First, define the best reply caliber and λ, i.e. $(\tilde{q}(v), \tilde{\lambda}(v)) \equiv \arg\max_{q,\lambda} \Pi(\lambda, q, v|\ell)$. By the FOCs $\Pi_q(\lambda, q, v|\ell) = \Pi_\lambda(\lambda, q, v|\ell) = 0$, the Implicit Function Theorem, and Cramer's Rule:

$$\tilde{q}'(v) = \frac{\Pi_{\lambda v}\Pi_{\lambda q} - \Pi_{qv}\Pi_{\lambda\lambda}}{\Pi_{qq}\Pi_{\lambda\lambda} - \Pi_{\lambda q}^2} \qquad \text{and} \qquad \tilde{\lambda}'(v) = \frac{\Pi_{\lambda q}\Pi_{qv} - \Pi_{\lambda v}\Pi_{qq}}{\Pi_{qq}\Pi_{\lambda\lambda} - \Pi_{\lambda q}^2} \tag{35}$$

The slopes in (35) are well defined, since $\Pi_{qq}\Pi_{\lambda\lambda} - \Pi^2_{\lambda q} > 0$ by Claim B.4. Denote the $\mathcal{C}^*$ locus by $q(\delta)$. First, by the equilibrium recipe *R3*, the optimal offenses $\Lambda(\delta, q(\delta))$ falls in δ. Differentiate $\tilde{\lambda}(v) \equiv \Lambda(\Delta, q(\Delta))$, using $\delta = \Delta(v, q)$, to get $\tilde{\lambda}'(v) \equiv (d\Lambda/d\delta)\Delta_v \leq 0$, since $\Delta_v \geq 0$. So $\Pi_{\lambda q}\Pi_{qv} - \Pi_{\lambda v}\Pi_{qq} \leq 0$, given (35). Next, translate the $\mathcal{C}^*$ locus into (v, q) space, using the identity $v \equiv q(\delta(v))\chi(\delta(v))$ to define a function $\delta(v)$. So $1 = (q'\chi + q\chi')\delta'(v)$, and differentiating $\tilde{q}(v) \equiv q(\delta(v))$ yields $\tilde{q}'(v) = q'(\delta(v))\delta'(v) = q'/(q'\chi + q\chi')$. Finally, if $\mathcal{C}^*$ slopes up, then $q'(v) \geq 0$ and so $\tilde{q}'(v) \geq 0$, whereupon $\Pi_{\lambda v}\Pi_{\lambda q} - \Pi_{qv}\Pi_{\lambda\lambda} \geq 0$, by (35). □

Claim D.2. *If the caliber $\mathcal{C}^*$ locus $q^*(\delta)$ slopes down, then $q'/q \geq -(1/2)(\chi'/\chi)$.*

Proof: Slope (29) yields an equality, and (6), (12), $\mathcal{E}_q(c_q) \geq 1$, and $\Lambda_\delta/\Lambda \geq -\chi'/\chi$ (Claim B.2) give inequalities:

$$\frac{1}{q}\frac{dq}{d\delta}\bigg|_{\mathcal{C}^*} = \frac{\left(\frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\theta''}{\theta'}\right) + \frac{\Lambda_\delta}{\Lambda} - \frac{c_{\lambda q}\Lambda_\delta}{c_q}}{1 + \frac{qc_{qq}}{c_q} + \left(\frac{qc_{\lambda q}\Lambda_q}{c_q} - \frac{q\Lambda_q}{\Lambda}\right)} \geq \frac{\Lambda_\delta}{2\Lambda} \geq -\frac{1}{2}\frac{\chi'}{\chi} \qquad \square$$

Claim D.3. *If $\mathcal{C}^*$ slopes down, then $\mathcal{L}_{vv}(\Pi_{\lambda\lambda}\Pi_{qq} - \Pi^2_{\lambda q}) + \mathcal{L}_{vq}(\Pi_{\lambda v}\Pi_{\lambda q} - \Pi_{\lambda\lambda}\Pi_{qv}) \geq 0$.*

Proof: The desired inequality is equivalent to $\tilde{q}'(v) + \mathcal{L}_{vv}/\mathcal{L}_{vq} \geq 0$ by (35). Next, as shown in the proof of Claim D.1, $\tilde{q}' \equiv q'/(q'\chi + q\chi')$, where $q(\delta)$ is the $\mathcal{C}^*$ locus. Since $2q'\chi \geq -q\chi'$ by Claim D.2, we have $q'\chi \geq -(q\chi' + q'\chi)$. Thus, $\tilde{q}'(v) \geq -1/\chi$, because $q'\chi + q\chi' > 0$ by Claim B.7. Third, $\mathcal{L}_{vv}/\mathcal{L}_{vq} = \Theta_{vv}/\Theta_{vq}$ by (3) and (6). Since $v = q\chi$ and $|\mathcal{E}_v(\Theta_v)| \geq |\mathcal{E}_q(\Theta_v)|$:

$$\tilde{q}'(v) + \frac{\mathcal{L}_{vv}}{\mathcal{L}_{vq}} \geq -\frac{q}{v} + \frac{\Theta_{vv}}{\Theta_{vq}} = \frac{\Theta_v}{v\Theta_{vq}}\left(\frac{v\Theta_{vv}}{\Theta_v} - \frac{q\Theta_{vq}}{\Theta_v}\right) \geq 0 \qquad \square$$

Claim D.4. *The system is stable when $k_\ell = 0$, and thus in the short and medium runs.*

Proof: When $k_\ell = 0$, the marginal criminal is constant: $\bar{\ell}(t) = \bar{\ell}$ for all $t \geq 0$. Linearize the system (21)–(23) around the (steady state) medium-run equilibrium (v^*, q^*, λ^*):

$$\begin{pmatrix} \dot{v} \\ \dot{q} \\ \dot{\lambda} \end{pmatrix} \approx \begin{pmatrix} k_v\mathcal{L}^*_{vv} & k_v\mathcal{L}^*_{vq} & k_v\mathcal{L}^*_{v\lambda} \\ k_q\Pi_{qv} & k_q\Pi_{qq} & k_q\Pi_{q\lambda} \\ k_\lambda\Pi_{\lambda v} & k_\lambda\Pi_{\lambda q} & k_\lambda\Pi_{\lambda\lambda} \end{pmatrix} \begin{pmatrix} v - v^* \\ q - q^* \\ \lambda - \lambda^* \end{pmatrix} \equiv A_0 \begin{pmatrix} v - v^* \\ q - q^* \\ \lambda - \lambda^* \end{pmatrix}$$

where $\Pi_q = \Pi_\lambda = 0$, by optimality of q and λ; and $\Pi_{q\ell} = \Pi_{\lambda\ell} = 0$, since $\Pi_\ell = -1$. This system is stable iff the eigenvalues of the A_0 are either negative, or have negative real parts. The *Routh-Hurwitz Criterion (RHC)* (see e.g. Brock and Malliaris (1989)) allows us to determine whether the system is stable without solving for the eigenvalues. Write the characteristic polynomial $p(e) \equiv Det(A_0 - eI)$ as $p(e) = e^3 + a_1 e^2 + a_2 e^1 + a_3$. Then all roots of $p(e) = 0$ have negative real parts iff $a_1, a_2, a_3 > 0$ and $a_1 a_2 > a_3$, where:

$$a_1 = k_v \mathcal{L}_{vv} - k_q \Pi_{qq} - k_\lambda \Pi_{\lambda\lambda} \tag{36}$$

$$a_2 = k_q k_v \left(\mathcal{L}_{vq}\Pi_{qv} - \mathcal{L}_{vv}\Pi_{qq}\right) + k_\lambda k_v \left(\Pi_{\lambda v}\mathcal{L}_{v\lambda} - \Pi_{\lambda\lambda}\mathcal{L}_{vv}\right) + k_\lambda k_q \left(\Pi_{\lambda\lambda}\Pi_{qq} - \Pi_{\lambda q}^2\right) \tag{37}$$

$$a_3 = k_\lambda k_q k_v \left(\mathcal{L}_{vv}\left[\Pi_{\lambda\lambda}\Pi_{qq} - \Pi_{\lambda q}^2\right] + \mathcal{L}_{v\lambda}\left[\Pi_{\lambda q}\Pi_{qv} - \Pi_{\lambda v}\Pi_{qq}\right] + \mathcal{L}_{vq}\left[\Pi_{\lambda v}\Pi_{\lambda q} - \Pi_{\lambda\lambda}\Pi_{qv}\right]\right) \tag{38}$$

Step 1: $a_1 > 0$. To see this, note that the second order conditions imply $\Pi_{qq}, \Pi_{\lambda\lambda} < 0 < \mathcal{L}_{vv}$.

Step 2: $a_2 > 0$. One can verify that $-\mathcal{L}_{vq}, \mathcal{L}_{v\lambda} < 0$, and $-\Pi_v, -\Pi_{\lambda v}, \Pi_{vq} > 0$, and $\Pi_{qq}\Pi_{\lambda\lambda} - \Pi_{\lambda q}^2 > 0$ (Claim B.4). Since every parenthesized term in (37) is positive, $a_2 > 0$.

Step 3: $a_3 > 0$. By Claims D.1–D.3 and Steps 1–2, every factor of the speed of adjustment constant $k_\lambda k_q k_v$ in (38) is positive, and so $a_3 > 0$.

Step 4: $a_1 a_2 > a_3$. We consider two cases depending on the slope of the caliber locus $\mathcal{C}^*$.

Case 1: $\mathcal{C}^*$ slopes down. Since every term in a_1 and a_2 is positive by Steps 1–2, we have:

$$\frac{a_1 a_2}{k_v k_\lambda k_q} > \mathcal{L}_{vv}\left(\Pi_{\lambda\lambda}\Pi_{qq} - \Pi_{\lambda q}^2\right) - \Pi_{qq}\left(\Pi_{\lambda v}\mathcal{L}_{v\lambda} - \Pi_{\lambda\lambda}\mathcal{L}_{vv}\right) - \Pi_{\lambda\lambda}\left(\mathcal{L}_{vq}\Pi_{qv} - \mathcal{L}_{vv}\Pi_{qq}\right)$$

Now subtract $a_3/(k_v k_\lambda k_q)$ in both sides, and do some basic algebra to get:

$$\frac{a_1 a_2 - a_3}{k_v k_\lambda k_q} > \Pi_{qq}\Pi_{\lambda\lambda}\mathcal{L}_{vv} - \mathcal{L}_{v\lambda}\Pi_{\lambda q}\Pi_{qv} - \Pi_{\lambda\lambda}\left(\mathcal{L}_{vq}\Pi_{qv} - \mathcal{L}_{vv}\Pi_{qq}\right) - \mathcal{L}_{vq}\left(\Pi_{\lambda v}\Pi_{\lambda q} - \Pi_{\lambda\lambda}\Pi_{qv}\right) \tag{39}$$

The first two terms are positive, since $-\mathcal{L}_{vv}, \Pi_{qq}, \Pi_{\lambda\lambda}, \mathcal{L}_{v\lambda} < 0 < \Pi_{qv}, \Pi_{\lambda q}$. The third term is positive, since $\mathcal{L}_{vq} > 0$. The last term is positive, for $\Pi_{\lambda v}\Pi_{\lambda q} < \Pi_{\lambda\lambda}\Pi_{qv}$ by Claim D.1.

Case 2: $\mathcal{C}^*$ slopes up. Now the last term in (39) is negative, for $\Pi_{\lambda v}\Pi_{\lambda q} > \Pi_{\lambda\lambda}\Pi_{qv}$ by Claim D.1. Next, that own effects dominate cross effects, namely, $|\mathcal{E}_v(\Theta_v)| \geq |\mathcal{E}_q(\Theta_v)|$ by (7), imply $v\mathcal{L}_{vv} > q\mathcal{L}_{vq}$. Next, sum the first and last term in the right side of (39):

$$\Pi_{qq}\Pi_{\lambda\lambda}\mathcal{L}_{vv} + \mathcal{L}_{vq}\Pi_{\lambda\lambda}\Pi_{qv} > (\mathcal{L}_{vq}\Pi_{\lambda\lambda}/v)(\Pi_{qq}q + v\Pi_{qv}) > 0$$

For $-q\Pi_{qq} > v\Pi_{qv}$, given (5) and (6). Thus, the right side of (39) is positive. □

Claim D.5. *For large $k_\lambda \uparrow \infty$, the system is stable in the long-run.*

Proof: In the limit $k_\lambda \uparrow \infty$, offenses λ adjusts instantaneously. For any $(v(t), q(t))$, the offenses $\lambda(t) = \hat{\Lambda}(v(t), q(t))$, recalling §4-A. So caliber obeys $\dot{q} = k_q \Pi_q^*$, and the marginal criminal, $\dot{\bar{\ell}} = k_\ell \Pi^*$, where $\Pi^* \equiv \max_\lambda \Pi(\lambda, q, v)$, as defined in §4-A. Vigilance obeys $\dot{v} =$

$-k_v\mathcal{L}_v^*$, where $\mathcal{L}_v^* \equiv \hat{\Lambda}(v,q)F(\ell)\Theta_v(v,q)M+1$. Linearizing the induced system near the steady state:

$$\begin{pmatrix} \dot{v} \\ \dot{q} \\ \dot{\ell} \end{pmatrix} \approx \begin{pmatrix} -k_v\mathcal{L}_{vv}^* & -k_v\mathcal{L}_{vq}^* & -k_v\mathcal{L}_{v\ell}^* \\ k_q\Pi_{qv}^* & k_q\Pi_{qq}^* & k_q\Pi_{q\ell}^* \\ k_\ell\Pi_v^* & k_\ell\Pi_q^* & k_\ell\Pi_\ell^* \end{pmatrix} \begin{pmatrix} v-v^* \\ q-q^* \\ \ell-\ell^* \end{pmatrix}$$

where $\Pi_{q\ell}^*=0$, and $\Pi_q^*=0$ by (13). Write the characteristic polynomial $p(e)\equiv Det(A_0-eI)$ as $p(e)=e^3+a_1e^2+a_2e^1+a_3$. By RHC, stability follows iff $a_1,a_2,a_3>0$ and $a_1a_2>a_3$:

$$\begin{aligned} a_1 &= k_v\mathcal{L}_{vv}^* - k_q\Pi_{qq}^* - k_\ell\Pi_\ell^* \\ a_2 &= k_\ell k_v\left(\mathcal{L}_{v\ell}^*\Pi_v^* - \Pi_\ell^*\mathcal{L}_{vv}^*\right) + k_\ell k_q\Pi_\ell^*\Pi_{qq}^* + k_q k_v\left(\mathcal{L}_{vq}^*\Pi_{qv}^* - \mathcal{L}_{vv}^*\Pi_{qq}^*\right) \\ a_3 &= k_\ell k_q k_v\left(\Pi_\ell\mathcal{L}_{vv}^*\Pi_{qq}^* - \Pi_\ell\mathcal{L}_{vq}^*\Pi_{qv}^* - \mathcal{L}_{v\ell}^*\Pi_{qq}^*\Pi_v^*\right) \end{aligned}$$

Step 1: $a_1>0$. First, $\Pi_{qq}^*<0$ by the second order conditions; second (Claim B.4), $\Pi_\ell^*=-1<0$; third, $\mathcal{L}_{vv}^*=(\hat{\Lambda}F\Theta_{vv}+\hat{\Lambda}_vF\Theta_v)M>0$, for $\hat{\Lambda}_v<0$ by (11).

Step 2: $a_2>0$. Every factor of a speed of adjustment constant k is positive, since one can easily verify that $\mathcal{L}_{v\ell}^*,\Pi_v^*,\Pi_\ell^*<0<\Pi_{qv}^*,\mathcal{L}_{vq}^*$.

Step 3: $a_3>0$. This holds by the reasons given in Steps 1–2.

Step 4: $a_1a_2>a_3$. Since each term in a_1 and a_2 is positive by Steps 1–2, this follows from:

$$\frac{a_1a_2}{k_vk_\ell k_q} > \mathcal{L}_{vv}^*\Pi_\ell\Pi_{qq}^* - \Pi_{qq}^*\left(\mathcal{L}_{v\ell}^*\Pi_v - \Pi_\ell\mathcal{L}_{vv}^*\right) - \Pi_\ell^*\left(\mathcal{L}_{vq}^*\Pi_{qv}^* - \mathcal{L}_{vv}^*\Pi_{qq}^*\right) > \frac{a_3}{k_vk_\lambda k_q} + 2\Pi_{qq}^*\Pi_\ell^*\mathcal{L}_{vv}^*\square$$

Finally, given adjustment constants $k\equiv(k_v,k_\lambda,k_q,k_\ell)$ and a (steady state) crime equilibrium $x^*\equiv(v^*,\lambda^*,q^*,\ell^*)$, our dynamical system $\dot{x}\equiv(\dot{v},\dot{\lambda},\dot{q},\dot{\ell})$ near the crime equilibrium obeys: $\dot{x}=A(k)(x-x^*)\equiv F(k,x)$, where the matrix A depends on k. Next, F is continuous in (k,x) and Lipschitz in x, since it is linear in (k,x). Thus, by Theorem 7.1 in Brock and Malliaris (1989), the solution $x(\cdot)$ to the initial value problem $\dot{x}=F(k,x)$, $x(t_0)=x_0$, is continuous in (t,t_0,x_0,k). Altogether, $x(t)\to x^*$ for large enough k_λ or low enough k_ℓ.

E Loci of Crime Rate $\bar{K}$, Offenses $\bar{\Lambda}$, Vigilance $\bar{V}$

The $\bar{K}$ Locus. The slope in δ of the $\bar{K}$ locus is $d\alpha/d\delta|_{\bar{K}}=-\alpha\theta'/\theta$. Recalling $\mathcal{A}^D$ in (17):

Claim E.1. *$\mathcal{A}^D$ is steeper than $\bar{K}$ in δ iff (★) holds iff $\kappa M/v\geq\mathcal{E}_\delta(\chi)/(\mathcal{E}_\delta(\chi')-\mathcal{E}_\delta(\theta'))$.*

Proof: Differentiating $\mathcal{A}^D(\delta|q)$ in δ yields $\mathcal{A}_\delta^D=\alpha(\chi''/\chi'-\theta''/\theta')$; thus, $\mathcal{A}_\delta^D\geq d\alpha/d\delta|_{\bar{K}}$ iff (★).

For the second assertion, recall that the equilibrium attempted crime rate obeys $\alpha = \mathcal{A}^D(\delta|q) = q\chi'/(-\theta' M)$, by (17). Next, multiply the left and right hand side by the theft function θ, and use that vigilance $v = q\chi$, and the crime rate $\kappa = \theta\alpha$ to get:

$$\kappa = \theta\alpha = \frac{q\chi}{M} \cdot \frac{\theta\chi'}{-\theta'\chi} = \frac{v}{M}\frac{\mathcal{E}_\delta(\chi)}{-\mathcal{E}_\delta(\theta)}$$

So $\mathcal{E}_\delta(\theta) = -\mathcal{E}_\delta(\chi)v/(\kappa M)$. On the other hand, (★) holds iff $\mathcal{E}_\delta(\theta) \geq \mathcal{E}_\delta(\theta') - \mathcal{E}_\delta(\chi')$. Thus, (★) holds iff $-\mathcal{E}_\delta(\chi)v/(\kappa M) \geq \mathcal{E}_\delta(\theta') - \mathcal{E}_\delta(\chi')$, or $\kappa M/v \geq \mathcal{E}_\delta(\chi)/(\mathcal{E}_\delta(\chi') - \mathcal{E}_\delta(\theta'))$. □

The $\bar{\Lambda}$ Locus. The slope of the $\bar{\Lambda}$ locus is $d\delta/dq|_{\bar{\Lambda}} = -\Lambda_q/\Lambda_\delta = c_{\lambda q}/(\theta' m) < 0$ by (12).

Claim E.2. *The slope of $\bar{\Lambda}$ is greater than $\mathcal{M}^*$. If $\mathcal{C}^*$ slopes down, its slope is lower than $\bar{\Lambda}$.*

Proof: First, $d\delta/dq|_{\bar{\Lambda}} = -\mathcal{E}_\lambda(c_q)\chi/(q\chi') \geq -\chi/(q\chi')$ by (14) and so $\mathcal{E}_\lambda(c_q) \leq 1$. But if $\mathcal{C}^*$ slopes up, then $d\delta/dq|_{\mathcal{M}^*} < -\chi/(q\chi')$, by Claim B.8. So the slope of $\bar{\Lambda}$ is greater than $\mathcal{M}^*$.

If $\mathcal{C}^*$ slopes down, rewrite (32) using the left side of (34) and Claim A.1 to get the first inequality:

$$\left.\frac{d\delta}{dq}\right|_{\mathcal{M}^*} < \frac{\Lambda_q/\Lambda - 1/q}{\chi''/\chi' - \Lambda_\delta/\Lambda - \theta''/\theta'} < -\frac{\Lambda_q}{\Lambda_\delta} = \left.\frac{d\delta}{dq}\right|_{\bar{\Lambda}} \tag{40}$$

The second inequality follows from using Claim A.1 and inequality (6). Hence, $\mathcal{C}^*$ is steeper than $\mathcal{M}^*$ (Lemma 3), and thus its slope is lower than $\mathcal{M}^*$'s, and so lower than $\bar{\Lambda}$, by (40).□

Since $\Lambda_\delta < 0$ and $\Lambda_q \leq 0$ by (12), offenses Λ weakly falls moving up and right. Since $\mathcal{C}^*$ is steeper than $\bar{\Lambda}$ in Figure 6, the offenses Λ falls in δ. This yields recipe claim *R*3.

The $\bar{V}$ Locus. The slope of $\bar{V}$ (see *R*4) is $d\delta/dq|_{\bar{V}} = -\chi/(q\chi')$. By Claims B.7 and B.8:

Claim E.3. *If $\mathcal{C}^*$ slopes down, then the slope of $\bar{V}$ exceeds the slope of $\mathcal{C}^*$ and is lower than the slope of $\mathcal{M}^*$. If $\mathcal{C}^*$ slopes up, then the slope of $\bar{V}$ is greater than the slope of $\mathcal{M}^*$.*

F Greater Legal Penalties: Proof of Proposition 1

Index the outside option distribution as $F(\ell|\eta)$, where $\eta \in \mathbb{R}$, so that the derivative $F_\eta(\ell|\eta)$ exists and is negative: $F_\eta < 0$. So greater η corresponds to higher outside options. By the discussion after Lemma 3, the caliber locus $\mathcal{C}^*$ slopes down for low outside options.

Claim F.1. *Assume (★), and let $\mathcal{C}^*$ slope down. If η rises, the attempted crime rate α falls.*

Proof: As seen in the right panel of Figure 7, $q_\eta > 0 > \delta_\eta$. Also, $q_\eta = (dq/d\delta|_{\mathcal{C}^*})\delta_\eta$, since the $\mathcal{C}^*$ locus in (14) is fixed in η. Thus, log-differentiating $\mathcal{A}^D(\delta|q) = -q\chi'/(\theta' M)$ in (17) yields:

$$\frac{1}{\mathcal{A}^D}\frac{d\mathcal{A}^D}{d\eta} = \frac{q_\eta}{q} + \left(\frac{\chi''}{\chi'} - \frac{\theta''}{\theta'}\right)\delta_\eta = \left(\frac{1}{q}\frac{dq}{d\delta}\bigg|_{\mathcal{C}^*} + \frac{\chi''}{\chi'} - \frac{\theta''}{\theta'}\right)\delta_\eta \tag{41}$$

Thus, the attempted crime rate falls when the parenthesized term in the right hand side of (41) is positive. Substituting the formula (29) for $dq/d\delta|_{\mathcal{C}^*}$ into (41), we obtain:

$$\frac{1}{q}\frac{dq}{d\delta}\bigg|_{\mathcal{C}^*} + \frac{\chi''}{\chi'} - \frac{\theta''}{\theta'} > 0 \iff \frac{\chi'}{\chi} + \frac{\Lambda_\delta}{\Lambda}\left(1 - \frac{\Lambda c_{\lambda q}}{c_q}\right) + \left(\frac{\chi''}{\chi'} - \frac{\theta''}{\theta'}\right)\left(\frac{qc_{qq} + qc_{\lambda q}\Lambda_q}{c_q} - \frac{q\Lambda_q}{\Lambda}\right) > 0$$

On the right side, the first two terms are $\chi'/\chi + \Lambda_\delta(1 - \mathcal{E}_\lambda(c_q))/\Lambda \geq 0$, by $\mathcal{E}_\lambda(c_q) \leq 1$ and Claim B.2. The last term is positive: $c_{qq} + c_{\lambda q}\Lambda_q > 0$ by (30) and $\Lambda_q \leq 0$ by (12). □

Claim F.2. *Suppose (★) fails, and let $\mathcal{C}^*$ slope down. If η rises, then the crime rate κ rises.*

Proof: The crime rate obeys $\kappa \equiv \theta(\delta)\mathcal{A}^D(\delta|q)$. Paralleling the proof of Claim F.1, κ falls in η iff:

$$\frac{1}{\kappa}\frac{d\kappa}{d\eta} = \frac{q_\eta}{q} + \left(\frac{\chi''}{\chi'} - \frac{\theta''}{\theta'} + \frac{\theta'}{\theta}\right)\delta_\eta < 0$$

which holds since $\delta_\eta < 0 < q_\eta$, and the parenthesized term is negative since (★) fails. □

G Better Policing: Proof of Proposition 2

Index the theft function as $\theta(\delta|\tau)$, where $\tau \in \mathbb{R}$ is a policing parameter, with $\theta(\delta|0) = \theta(\delta)$ and $\theta_\tau < 0$ when $\delta < 1$. We change variables, implicitly defining the transformed deterrence function $\mathcal{D}(\delta|\tau) \in [0,1]$ via $\theta(\delta|\tau) \equiv \theta(\mathcal{D}(\delta|\tau))$. So $\mathcal{D}(\delta,0) \equiv \delta$ and $\mathcal{D}(1,\tau) \equiv 1$ and $\mathcal{D}_\delta, \mathcal{D}_\tau > 0$.

(◇) $\Lambda_\tau = \theta'\mathcal{D}_\tau m/c_{\lambda\lambda} = \Lambda_\delta \mathcal{D}_\tau$, by differentiating $\theta(\delta|\tau)m = c_\lambda(\Lambda, q)$ in (11), and using (12)

(△) $\bar{\ell}_\tau = \theta'\mathcal{D}_\tau \Lambda m = \bar{\ell}_\delta \mathcal{D}_\tau$, by differentiating (18), and using (19).

Claim G.1. *If policing τ rises, then $\mathcal{M}^*$ shifts left more than $\mathcal{C}^*$, and deterrence δ falls.*

Proof: Log-differentiate the equations for $\mathcal{C}^*$ and $\mathcal{M}^*$ in τ, namely, (28) and (31), and evaluate at $\tau = 0$. Recalling §B, $(MC_q/MC - MB_q/MB)q_\tau = (MB_\tau/MB - MC_\tau/MC)$ and $(\mathcal{A}^S_q/\mathcal{A}^S - \mathcal{A}^D_q/\mathcal{A}^D)q_\tau = (\mathcal{A}^D_\tau/\mathcal{A}^D - \mathcal{A}^S_\tau/\mathcal{A}^S)$. As with (29) and (32), (◇) and (△) imply:

$$\frac{dq}{d\tau}\bigg|_{\mathcal{C}^*} = \frac{\left(\frac{\theta''}{\theta'} + \frac{\Lambda_\delta(1-\mathcal{E}_\lambda(c_q))}{\Lambda}\right)\mathcal{D}_\tau}{\frac{c_{qq}+c_{\lambda q}\Lambda_q}{c_q} + \frac{1}{q} - \frac{\Lambda_q}{\Lambda}} \qquad \text{and} \qquad \frac{dq}{d\tau}\bigg|_{\mathcal{M}^*} = \frac{\left(\frac{\theta''}{\theta'} + \frac{\Lambda_\delta}{\Lambda} + \frac{f}{F}\bar{\ell}_\delta\right)\mathcal{D}_\tau}{-\frac{f}{F}\bar{\ell}_q - \frac{\Lambda_q}{\Lambda} + \frac{1}{q}} \tag{42}$$

Case 1: $\mathcal{C}^*$ slopes up. First, by equations (12), (28), and then (12) again:

$$\mathcal{E}_\lambda(c_q)\frac{\Lambda_\delta}{\Lambda} = \frac{c_{\lambda q}}{c_q}\frac{\theta' m}{c_{\lambda\lambda}} = -\frac{q\chi'}{\chi\Lambda}\frac{c_{\lambda q}}{c_{\lambda\lambda}} = \frac{q\chi'}{\chi}\frac{\Lambda_q}{\Lambda} \tag{43}$$

Also, $\chi'/\chi = -\theta'\Lambda m/(qc_q) = \bar{\ell}_\delta/(q\bar{\ell}_q)$ by (28) and (19). Next, as argued in §B, the denominator in (29) is positive. We simplify this using $-\chi''/\chi' < 0$ and (43), and deduce:

$$\frac{\theta''}{\theta'} + \frac{\Lambda_\delta}{\Lambda} > -\frac{\chi'}{\chi}\left(1 - \frac{q\Lambda_q}{\Lambda}\right) = -\frac{\bar{\ell}_\delta}{q\bar{\ell}_q}\left(1 - \frac{q\Lambda_q}{\Lambda}\right) \tag{44}$$

Bound the left expression in (42) using (30) and $\mathcal{E}_\lambda(c_q) \geq 0$, and the right expression in (42) using (44) and Claim A.1, to deduce $q_\tau|_{\mathcal{M}^*} < q_\tau|_{\mathcal{C}^*}$, so $\mathcal{M}^*$ shifts left more than $\mathcal{C}^*$:

$$\left.\frac{dq}{d\tau}\right|_{\mathcal{M}^*} < \frac{(\theta''/\theta' + \Lambda_\delta/\Lambda)\mathcal{D}_\tau}{1/q - \Lambda_q/\Lambda} < \left.\frac{dq}{d\tau}\right|_{\mathcal{C}^*}$$

Case 2: $\mathcal{C}^*$ slopes down. The denominator in (29) is now negative. Thus two possibilities might arise: $\chi'/\chi + \theta''/\theta' + (1-\mathcal{E}_\lambda(c_q))\Lambda_\delta/\Lambda \gtrless 0$. The first inequality uses (43) to rephrase (44) and thus $q_\tau|_{\mathcal{M}^*} < q_\tau|_{\mathcal{C}^*}$ by Case 1. Consider the second inequality. Naturally, inequality (44) reverses. Using Claim A.1, this inequality yields $(q_\tau/q)|_{\mathcal{M}^*} < -\mathcal{D}_\tau\bar{\ell}_\delta/(q\bar{\ell}_q) = -\mathcal{D}_\tau\chi'/\chi$, from the right expression in (42), and then simplifying using (28) and (19). Finally, we deduce $q_\tau|_{\mathcal{M}^*} < q_\tau|_{\mathcal{C}^*}$ by noting that $(q_\tau/q)|_{\mathcal{C}^*} > -\mathcal{D}_\tau\chi'/\chi$ follows from (42), using (43) and (6):

$$\frac{\chi'}{\chi}\left(1 + \frac{qc_{qq}}{c_q} - \frac{q\Lambda_q(1-\mathcal{E}_\lambda(c_q))}{\Lambda}\right) + \frac{\theta''}{\theta'} + \frac{\Lambda_\delta(1-\mathcal{E}_\lambda(c_q))}{\Lambda} = \frac{\chi'}{\chi}\left(1 + \frac{qc_{qq}}{c_q}\right) + \frac{\theta''}{\theta'} \geq 0 \quad \square$$

Claim G.2. *If τ rises and $\mathcal{C}^*$ slopes down, $\mathcal{C}^*$ shifts down more than $\mathcal{M}^*$; so caliber q falls.*

Proof: As with (42), log-differentiate equations (28) and (31) in τ at $\tau = 0$:

$$\left.\frac{d\delta}{d\tau}\right|_{\mathcal{C}^*} = \frac{\left(\frac{\theta''}{\theta'} + \frac{\Lambda_\delta(1-\mathcal{E}_\lambda(c_q))}{\Lambda}\right)\mathcal{D}_\tau}{-\frac{\chi'}{\chi} + \frac{\chi''}{\chi'} - \frac{\theta''}{\theta'} - \frac{\Lambda_\delta}{\Lambda} + \frac{c_{\lambda q}\Lambda_\delta}{c_q}} \quad \text{and} \quad \left.\frac{d\delta}{d\tau}\right|_{\mathcal{M}^*} = \frac{\left(\frac{\theta''}{\theta'} + \frac{\Lambda_\delta}{\Lambda} + \frac{f}{F}\bar{\ell}_\delta\right)\mathcal{D}_\tau}{-\frac{\theta''}{\theta'} + \frac{\chi''}{\chi'} - \frac{\Lambda_\delta}{\Lambda} - \frac{f}{F}\bar{\ell}_\delta}$$

where the numerators follows as justified after (42). Now notice that $\delta_\tau|_{\mathcal{C}^*} < -\mathcal{D}_\tau$, since $\chi'/\chi > \chi''/\chi'$ by (6); and $\delta_\tau|_{\mathcal{M}^*} > -\mathcal{D}_\tau$ since $\chi''/\chi' > 0$. Altogether, $\delta_\tau|_{\mathcal{C}^*} < -\mathcal{D}_\tau < \delta_\tau|_{\mathcal{M}^*}$. $\square$

Claim G.3. *For any caliber $q > 0$, if policing τ rises, then the attempted crime rate α falls.*

Proof: In §5.2 we saw that supply $\mathcal{A}^S$ and demand $\mathcal{A}^D$, inversely written as functions δ^S and δ^D, shift down in τ for any q. It suffices that δ^S shifts down more than δ^D. Log-differentiate

$\mathcal{A}^D(\delta^D|q,\tau) \equiv \alpha$ and $\mathcal{A}^S(\delta^S|q,\tau) \equiv \alpha$ in τ using (17) and (20), and substitute $\tau = 0$: So $\delta_\tau^D = (\mathcal{D}_\tau\theta''/\theta')/(\chi''/\chi' - \theta''/\theta')$ and $\delta_\tau^S = -\mathcal{D}_\tau$. Easily, $\delta_\tau^D > -\mathcal{D}_\tau = \delta_\tau^S$, as $\chi'' > 0$. □

Claim G.4. *If policing τ rises, then the level of offenses λ falls.*

Proof: Define $\tilde{\Lambda}(\tau) \equiv \Lambda(q(\tau),\delta(\tau),\tau)$, and suppose for a contradiction that $\tilde{\Lambda}'(\tau) > 0$. Define $\tilde{\ell}(\tau) \equiv \bar{\ell}(q(\tau),\delta(\tau),\tau)$. Since the attempted crime rate $\alpha = \tilde{\Lambda}F(\tilde{\ell})$ falls by Claim G.3, we must have $\tilde{\ell}'(\tau) < 0$. So using (12) and ($\diamondsuit$), $\mathcal{E}_\lambda(c_q) \le 1$ (i.e. $c_{\lambda q} \le c_q/\lambda$), and (19) and ($\triangle$),

$$\tilde{\Lambda}'(\tau) = \Lambda_q q_\tau + \Lambda_\delta \delta_\tau + \Lambda_\tau = \frac{-c_{\lambda q}q_\tau + \theta' m \delta_\tau + \theta'\mathcal{D}_\tau m}{c_{\lambda\lambda}} \le \frac{-\frac{c_q}{\Lambda}q_\tau + \theta' m\delta_\tau + \theta'\mathcal{D}_\tau m}{c_{\lambda\lambda}} = \frac{\tilde{\ell}'(\tau)}{\Lambda c_{\lambda\lambda}} < 0$$

H Lower Vigilance Function: Proof of Proposition 3

Let $\tau \in \mathbb{R}$ index technology. Write the vigilance function as $\chi(\delta|\tau)$, with $\chi(\delta|0) \equiv \chi(\delta)$ and $\chi_\tau < 0$. As in §G, let $\chi(\delta|\tau) \equiv \chi(\mathcal{D}(\delta,\tau))$, where $\mathcal{D}(\delta,\tau) \in [0,1]$, $\mathcal{D}(\delta|0) = \delta$, and $\mathcal{D}_\delta > 0 > \mathcal{D}_\tau$.

Claim H.1. *Let $\mathcal{C}^*$ slope up. If τ rises, then $\mathcal{C}^*$ shifts up more than $\mathcal{M}^*$, and caliber q falls.*

Proof: Fix q and log-differentiate (28) and (31) in τ at $\tau = 0$. This yields:

$$\left.\frac{d\delta}{d\tau}\right|_{\mathcal{C}^*} = \frac{-\left(\frac{\chi'}{\chi} - \frac{\chi''}{\chi'}\right)\mathcal{D}_\tau}{\frac{\theta''}{\theta'} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\Lambda_\delta}{\Lambda} - \frac{c_{\lambda q}\Lambda_\delta}{c_q}} \quad \text{and} \quad \left.\frac{d\delta}{d\tau}\right|_{\mathcal{M}^*} = \frac{\frac{\chi''}{\chi'}\mathcal{D}_\tau}{\frac{\theta''}{\theta'} - \frac{\chi''}{\chi'} + \frac{\Lambda_\delta}{\Lambda} + \frac{f}{F}\bar{\ell}_\delta}$$

The left denominator is positive as $\mathcal{C}^*$ slopes up (§B), and the right denominator negative. So $\delta_\tau|_{\mathcal{C}^*} > \delta_\tau|_{\mathcal{M}^*}$ iff:

$$\frac{f}{F}\bar{\ell}_\delta\left(\frac{\chi''}{\chi'} - \frac{\chi'}{\chi}\right) > \frac{\chi'}{\chi}\left(\frac{\theta''}{\theta'} + \frac{\Lambda_\delta}{\Lambda}\right) - \frac{\chi''}{\chi'}\mathcal{E}_\lambda(c_q)$$

which holds, since $\chi'/\chi > \chi''/\chi'$ by (6), and $\bar{\ell}_\delta, \Lambda_\delta < 0$ by (12) and (19), and $\mathcal{E}_\lambda(c_q) > 0$. □

Claim H.2. *Let $\mathcal{C}^*$ slope up. If technology τ rises, then the vigilance v falls.*

Proof: Differentiating $v \equiv q\chi(\mathcal{D})$ in τ at $\tau = 0$ yields: $dv/d\tau = dq/d\tau + q\chi'(d\delta/d\tau + \mathcal{D}_\tau)$. Since $dq/d\tau < 0$ by Claim H.1, it suffices to show that $d\delta/d\tau < -\mathcal{D}_\tau$. Log-differentiate the $\mathcal{C}^*$ locus (28), or $MB(q,\delta|\tau) \equiv MC(q,\delta)$, and the $\mathcal{M}^*$ locus, or $\mathcal{A}^D(\delta|q,\tau) \equiv \mathcal{A}^S(\delta|q)$, in τ:

$$\begin{bmatrix} \frac{MB_q}{MB} - \frac{MC_q}{MC} & \frac{MB_\delta}{MB} - \frac{MC_\delta}{MC} \\ \frac{\mathcal{A}_q^D}{\mathcal{A}^D} - \frac{\mathcal{A}_q^S}{\mathcal{A}^S} & \frac{\mathcal{A}_\delta^D}{\mathcal{A}^D} - \frac{\mathcal{A}_\delta^S}{\mathcal{A}^S} \end{bmatrix} \begin{bmatrix} \frac{dq}{d\tau} \\ \frac{d\delta}{d\tau} \end{bmatrix} = -\begin{bmatrix} \frac{MB_\tau}{MB} \\ \frac{\mathcal{A}_\tau^D}{\mathcal{A}^D} \end{bmatrix} = -\begin{bmatrix} \left(\frac{\chi'}{\chi} - \frac{\chi''}{\chi'}\right)\mathcal{D}_\tau \\ \frac{\chi''}{\chi'}\mathcal{D}_\tau \end{bmatrix}$$

hereby simplifying using (28) and (17), respectively. Solve for $d\delta/d\tau$ using Cramer's Rule:

$$\frac{d\delta}{d\tau}=\frac{\left[\;-\frac{\mathcal{A}_q^D}{\mathcal{A}^D}+\frac{\mathcal{A}_q^S}{\mathcal{A}^S}\quad,\frac{MB_q}{MB}-\frac{MC_q}{MC}\;\right]\cdot\left[\;-\left(\frac{\chi'}{\chi}-\frac{\chi''}{\chi'}\right)\mathcal{D}_\tau\quad,-\frac{\chi''}{\chi'}\mathcal{D}_\tau\;\right]}{\left(\frac{MB_q}{MB}-\frac{MC_q}{MC}\right)\left(\frac{\mathcal{A}_\delta^D}{\mathcal{A}^D}-\frac{\mathcal{A}_\delta^S}{\mathcal{A}^S}\right)-\left(\frac{MB_\delta}{MB}-\frac{MC_\delta}{MC}\right)\left(\frac{\mathcal{A}_q^D}{\mathcal{A}^D}-\frac{\mathcal{A}_q^S}{\mathcal{A}^S}\right)}$$

The denominator is negative, as $MB_\delta > MC_\delta$ if $\mathcal{C}^*$ slopes up (§B). So $d\delta/d\tau < -\mathcal{D}_\tau$ iff:

$$\left(\frac{MB_q}{MB}-\frac{MC_q}{MC}\right)\left(\frac{\chi''}{\chi'}-\frac{\mathcal{A}_\delta^D}{\mathcal{A}^D}+\frac{\mathcal{A}_\delta^S}{\mathcal{A}^S}\right)>\left(\frac{\chi'}{\chi}-\frac{\chi''}{\chi'}-\frac{MB_\delta}{MB}+\frac{MC_\delta}{MC}\right)\left(\frac{\mathcal{A}_q^D}{\mathcal{A}^D}-\frac{\mathcal{A}_q^S}{\mathcal{A}^S}\right)$$

Using our expressions for $(MB_q, MB_\delta, \mathcal{A}_q^S, \mathcal{A}_\delta^S, \mathcal{A}_q^D, \mathcal{A}_\delta^D)$ in (29) and (32), this reduces to:

$$\left(\frac{c_{qq}+c_{\lambda q}\Lambda_q}{c_q}+\frac{1}{q}-\frac{\Lambda_q}{\Lambda}\right)\left(\frac{\theta''}{\theta'}+\frac{\Lambda_\delta}{\Lambda}+\frac{f}{F}\bar{\ell}_\delta\right)<\left(\frac{\theta''}{\theta'}+\frac{\Lambda_\delta}{\Lambda}-\frac{c_{\lambda q}\Lambda_\delta}{c_q}\right)\left(\frac{1}{q}-\frac{\Lambda_q}{\Lambda}-\frac{f}{F}\bar{\ell}_q\right)$$

We finally prove this inequality. Since the first and last parenthesized terms are positive:

$$\frac{\frac{\theta''}{\theta'}+\frac{\Lambda_\delta}{\Lambda}+\frac{f}{F}\bar{\ell}_\delta}{\frac{1}{q}-\frac{\Lambda_q}{\Lambda}-\frac{f}{F}\bar{\ell}_q}<\frac{-\frac{\chi''}{\chi'}+\frac{\theta''}{\theta'}+\frac{\Lambda_\delta}{\Lambda}}{\frac{1}{q}-\frac{\Lambda_q}{\Lambda}}<\frac{\frac{\theta''}{\theta'}+\frac{\Lambda_\delta}{\Lambda}}{\frac{1}{q}-\frac{\Lambda_q}{\Lambda}}<\frac{\frac{\theta''}{\theta'}+\frac{\Lambda_\delta}{\Lambda}-\frac{c_{\lambda q}\Lambda_\delta}{c_q}}{\frac{c_{qq}+c_{\lambda q}\Lambda_q}{c_q}+\frac{1}{q}-\frac{\Lambda_q}{\Lambda}}$$

The first inequality ensues from (34) (left inequality reversed) and Claim A.1; the second one owes to $\chi''>0$; the third one follows from $c_{qq}+c_{\lambda q}\Lambda_q>0$ by (30), and $\Lambda_\delta<0$ by (12). □

I Losses and Criminal Gains: Proof of Proposition 4

PART (*a*): *m* RISES. Differentiating (11) and (18) in m yields: (▽) $\Lambda_m=\theta/c_{\lambda\lambda}$ and $\bar{\ell}_m=\theta\Lambda$.

Claim I.1. *Let $\mathcal{C}^*$ slope down. If m rises, $\mathcal{C}^*$ shifts up more than $\mathcal{M}^*$ and so caliber rises.*

Proof: Fix caliber q, and log-differentiate (28) and (31) in m. Respectively:

$$\left.\frac{d\delta}{dm}\right|_{\mathcal{C}^*}=\frac{\frac{1}{m}+\frac{\Lambda_m}{\Lambda}-\frac{c_{\lambda q}\Lambda_m}{c_q}}{\frac{\chi''}{\chi'}-\frac{\chi'}{\chi}-\frac{\Lambda_\delta}{\Lambda}-\frac{\theta''}{\theta'}+\frac{c_{\lambda q}\Lambda_\delta}{c_q}}\quad\text{and}\quad\left.\frac{d\delta}{dm}\right|_{\mathcal{M}^*}=\frac{\frac{\Lambda_m}{\Lambda}+\frac{f}{F}\bar{\ell}_m}{\frac{\chi''}{\chi'}-\frac{\theta''}{\theta'}-\frac{\Lambda_\delta}{\Lambda}-\frac{f}{F}\bar{\ell}_\delta}\tag{45}$$

Next,

$$\frac{\Lambda_m}{\Lambda_\delta}=\frac{\theta}{\theta' m}=\frac{\bar{\ell}_m}{\bar{\ell}_\delta}\quad\text{and}\quad\frac{\Lambda_m}{\Lambda}\cdot\frac{\bar{\ell}_\delta}{\bar{\ell}_m}=\frac{\theta' m}{\Lambda c_{\lambda\lambda}}=\frac{\Lambda_\delta}{\Lambda}\tag{46}$$

The left expression in (46) uses (▽), (12) and (19), and the right owes to (19), (▽) and (12). Now

$$\frac{1/m + \Lambda_m/\Lambda}{\chi''/\chi' - \chi'/\chi - \Lambda_\delta/\Lambda - \theta''/\theta'} \geq -\frac{\Lambda_m}{\Lambda_\delta} = -\frac{\bar{\ell}_m}{\bar{\ell}_\delta} \geq \frac{\Lambda_m/\Lambda}{\chi''/\chi' - \theta''/\theta' - \Lambda_\delta/\Lambda} \tag{47}$$

For since the left denominator is positive, as $\mathcal{C}^*$ slopes down (recall §B), the first inequality owes to (6) and the left side of (46). Next, the equality is the left half of (46), and the right inequality holds given (46) and $\chi''/\chi' \geq \theta''/\theta'$. Now, given the left inequality in (47), Claim A.1 applied to the left expression of (45) yields $\delta_m|_{\mathcal{C}^*} \geq -\Lambda_m/\Lambda_\delta$. Likewise, given the right inequality in (47), Claim A.1 applied to the right expression of (45) yields $\delta_m|_{\mathcal{M}^*} \leq -\bar{\ell}_m/\bar{\ell}_\delta$. Finally, given the left side of (46), we get $\delta_m|_{\mathcal{C}^*} \geq -\Lambda_m/\Lambda_\delta = -\bar{\ell}_m/\bar{\ell}_\delta \geq \delta_m|_{\mathcal{M}^*}$. □

Claim I.2. *The equilibrium level of offenses λ rises in the criminal gain m.*

Proof: Define $\tilde{\Lambda}(m) \equiv \Lambda(q(m), \delta(m), m)$, and suppose for a contradiction that $\tilde{\Lambda}'(m) < 0$. Define $\tilde{\ell}(m) \equiv \bar{\ell}(q(m), \delta(m), m)$. Since the attempted crime rate $\alpha = \tilde{\Lambda}F(\tilde{\ell})$ rises in m, we must have $\tilde{\ell}'(m) > 0$. Differentiating $\tilde{\Lambda}$ and using (12), (19), and $\mathcal{E}_\lambda(c_q) \leq 1$:

$$\tilde{\Lambda}' = \Lambda_q q_m + \Lambda_\delta \delta_m + \Lambda_m = \frac{-c_{\lambda q} q_m + \theta' m \delta_m + \theta}{c_{\lambda\lambda}} \geq \frac{-c_q q_m + \theta' \Lambda m \delta_m + \theta\Lambda}{\Lambda c_{\lambda\lambda}} = \frac{\tilde{\ell}'}{\Lambda c_{\lambda\lambda}} > 0 \quad \square$$

With an extra assumption, we can also predict how deterrence changes. Assume that the ratio of the marginal cost of caliber c_q to offenses c_λ is more responsive to caliber q than to the level of offenses λ, namely: $\mathcal{E}_q(c_q/c_\lambda) + \mathcal{E}_\lambda(c_q/c_\lambda) \geq 0$ (‡). This inequality, for instance, holds if costs are homogeneous, or homothetic such as $c(\lambda, q) \equiv \tilde{c}(\lambda + q)$ with $\tilde{c}', \tilde{c}'' > 0$.

Claim I.3. *Assume (‡) and (★). If m rises, then $\mathcal{M}^*$ shifts more than $\mathcal{C}^*$, thus δ rises.*

Proof: We'll prove that $q_m|_{\mathcal{M}^*} \geq q_m|_{\mathcal{C}^*}$. Fix δ, and log-differentiate (28) and (31) in m to get:

$$\frac{m}{q} \cdot \frac{dq}{dm}\bigg|_{\mathcal{C}^*} = \frac{1 + \frac{m\Lambda_m}{\Lambda} - \frac{m\Lambda_m c_{\lambda q}}{c_q}}{1 + \frac{qc_{qq}}{c_q} + \frac{qc_{\lambda q}\Lambda_q}{c_q} - \frac{q\Lambda_q}{\Lambda}} \quad \text{and} \quad \frac{m}{q} \cdot \frac{dq}{dm}\bigg|_{\mathcal{M}^*} = \frac{\frac{m\Lambda_m}{\Lambda} + m\frac{f}{F}\bar{\ell}_m}{1 - \frac{q\Lambda_q}{\Lambda} - \frac{f}{F}\bar{\ell}_q q} \tag{48}$$

Next, (▽), (19), (28), and then (12) and Claim B.2 (i.e. $\Lambda_\delta/\Lambda \geq -\chi'/\chi$) imply:

$$\frac{m\Lambda_m}{\Lambda} \cdot \frac{-q\bar{\ell}_q}{m\bar{\ell}_m} = \frac{m\theta}{\Lambda c_{\lambda\lambda}} \cdot \frac{qc_q}{\theta\Lambda m} = \frac{m\theta}{\Lambda c_{\lambda\lambda}} \cdot \frac{-\chi}{\chi'} \cdot \frac{\theta'}{\theta} = \frac{\Lambda_\delta}{\Lambda} \cdot \frac{-\chi}{\chi'} \leq 1 \tag{49}$$

So since $\Lambda_q \leq 0$ by (12), we deduce $(m\Lambda_m/\Lambda)/(1 - q\Lambda_q/\Lambda) \leq m\Lambda_m/\Lambda \leq -m\bar{\ell}_m/(q\bar{\ell}_q)$ by (49). Thus, applying Claim A.1 to (48) yields $mq_m/q|_{\mathcal{M}^*} \geq (m\Lambda_m/\Lambda)/(1 - q\Lambda_q/\Lambda)$.

Next, by Claim A.1 and (48), we know that $mq_m/q|_{\mathcal{C}^*} \leq (m\Lambda_m/\Lambda)/(1-q\Lambda_q/\Lambda)$ iff

$$\frac{1-\frac{m\Lambda_m c_{\lambda q}}{c_q}}{\frac{qc_{qq}}{c_q}+\frac{qc_{\lambda q}\Lambda_q}{c_q}} \leq \frac{\frac{m\Lambda_m}{\Lambda}}{1-\frac{q\Lambda_q}{\Lambda}} \iff \left(\frac{\Lambda}{m\Lambda_m}-\frac{\Lambda c_{\lambda q}}{c_q}\right)\left(1-\frac{q\Lambda_q}{\Lambda}\right) \leq \frac{qc_{qq}}{c_q}+\frac{qc_{\lambda q}\Lambda_q}{c_q}$$

cross-multiplying. Using (∇), (12), and $\theta m = c_\lambda$ from (11), this inequality reduces to:

$$\frac{\Lambda c_{\lambda\lambda}}{c_\lambda}+\frac{qc_{\lambda q}}{c_\lambda}-\frac{\Lambda c_{\lambda q}}{c_q} \leq \frac{qc_{qq}}{c_q} \iff \frac{\Lambda c_{\lambda\lambda}}{c_\lambda}-\frac{\Lambda c_{\lambda q}}{c_q} \leq \frac{qc_{qq}}{c_q}-\frac{qc_{\lambda q}}{c_\lambda} \iff (\ddagger) \qquad \square$$

Proof of Part (b): Property Loss M Rises.

Claim I.4. *The marginal criminal $\bar{\ell}(q,\delta)$ in (18) falls as deterrence δ rises along $\mathcal{C}^*$.*

Proof: By (18), $d\bar{\ell}/d\delta = \bar{\ell}_q dq/d\delta|_{\mathcal{C}^*} + \bar{\ell}_\delta$. If $\mathcal{C}^*$ slopes up, then $\bar{\ell}$ falls, given (19). If $\mathcal{C}^*$ slopes down, then $dq/d\delta|_{\mathcal{C}^*} \geq -q\chi'/\chi = -\bar{\ell}_\delta/\bar{\ell}_q$ by Claim B.7, (14), and (19). So, $d\bar{\ell}/d\delta < 0$. $\square$

Claim I.5. *If M rises, then the attempted crime rate α and crime rate κ fall.*

Proof: Let M rise. Then δ rises as $\mathcal{M}^*$ shifts right, and $\mathcal{C}^*$ is fixed. Since Λ and $\bar{\ell}$ fall in δ along $\mathcal{C}^*$, by recipe *R3* and Claim I.4, the attempted crime rate $\mathcal{A}^S(\delta(M)|q(M)) = \Lambda(q,\delta)F(\bar{\ell}(q,\delta))$ falls. Finally, the crime rate $\theta(\delta)\mathcal{A}^S(\delta|q)$ falls, since θ falls when M rises. $\square$

Part (c): Fixed Markdown μ. We now change m, M along a constant markdown $\mu = 1-m/M$. Parameterize $M = t, m = (1-\mu)t$. Differentiating (11) and (18) yields ($\triangleright$): $\Lambda_t = \theta(1-\mu)/c_{\lambda\lambda}$ and $\bar{\ell}_t = \theta(1-\mu)\Lambda$. We assume ($\bigstar$) in Claims I.6–I.9.

Claim I.6. *If t rises, then $\mathcal{M}^*$ shifts right more than $\mathcal{C}^*$, and so deterrence δ rises.*

Proof: Fixing δ, and log-differentiating (28) and (31) in t yields $(tq_tq)|_{\mathcal{C}^*}$ and $(tq_t/q)|_{\mathcal{M}^*}$ which coincide with the respective left and right expression in (48), with the exception that now we need to add 1 to the numerator of the right expression in (48).
Case 1: $(1+t\Lambda_t/\Lambda)/(1-q\Lambda_q/\Lambda) \leq t\bar{\ell}_t/(-q\bar{\ell}_q)$. Then $(tq_t/q)|_{\mathcal{M}^*} \geq (1+t\Lambda_t/\Lambda)/(1-q\Lambda_q/\Lambda)$ by Claim A.1, whereas $(tq_t/q)|_{\mathcal{C}^*} < (1+t\Lambda_t/\Lambda)/(1-q\Lambda_q/\Lambda)$ by cost-convexity (30).
Case 2: $(1+t\Lambda_t/\Lambda)/(1-q\Lambda_q/\Lambda) > t\bar{\ell}_t/(-q\bar{\ell}_q)$. Then $(tq_t/q)|_{\mathcal{M}^*} \geq t\bar{\ell}_t/(-q\bar{\ell}_q) = \Lambda c_\lambda/(qc_q)$, where the inequality follows by Claim A.1, and the equality owes to (11), (19) and ($\triangleright$). But since $t\Lambda_t/\Lambda = c_\lambda/(\Lambda c_{\lambda\lambda})$, we have $(tq_t/q)|_{\mathcal{C}^*} < (1+c_\lambda/(\Lambda c_{\lambda\lambda}))/(1+qc_{qq}/c_q) \leq 1$, for $\mathcal{E}_\lambda(c_\lambda), \mathcal{E}_q(c_q) \geq 1$. Thus $(tq_t/q)|_{\mathcal{C}^*} < 1 \leq (tq_t/q)|_{\mathcal{M}^*}$, since $\mathcal{E}_\lambda(c) \geq \mathcal{E}_q(c)$ by Claim B.2. $\square$

Claim I.7. *Let $\mathcal{C}^*$ slope down. If M, m rise subject to a constant markdown μ, then the $\mathcal{C}^*$ locus shifts up more than $\mathcal{M}^*$, and therefore caliber q rises.*

Proof: Fix q and log-differentiate equations (28) and (31) in t, emulating (45):

$$\left.\frac{d\delta}{dt}\right|_{\mathcal{C}^*} = \frac{\frac{1}{t} + \frac{\Lambda_t}{\Lambda} - \frac{\Lambda_t c_{\lambda q}}{c_q}}{\frac{\chi''}{\chi'} - \frac{\chi'}{\chi} - \frac{\Lambda_\delta}{\Lambda} - \frac{\theta''}{\theta'} + \frac{c_{\lambda q}\Lambda_\delta}{c_q}} \qquad \text{and} \qquad \left.\frac{d\delta}{dt}\right|_{\mathcal{M}^*} = \frac{\frac{1}{t} + \frac{\Lambda_t}{\Lambda} + \frac{f}{F}\bar{\ell}_t}{\frac{\chi''}{\chi'} - \frac{\theta''}{\theta'} - \frac{\Lambda_\delta}{\Lambda} - \frac{f}{F}\bar{\ell}_\delta} \tag{50}$$

First, $\delta_t|_{\mathcal{C}^*} \geq -\Lambda_t/\Lambda_\delta$, as in the proof of Claim I.1. Next, by equations (12), (19) and (★), $(1/t + \Lambda_t/\Lambda)/(\chi''/\chi' - \theta''/\theta' - \Lambda_\delta/\Lambda) \leq -\bar{\ell}_t/\bar{\ell}_\delta$. Applying Claim A.1 to (50) yields $\delta_t|_{\mathcal{M}^*} \leq -\bar{\ell}_t/\bar{\ell}_\delta$. Finally, $\Lambda_t/\Lambda_\delta = -\theta/(t\theta') = \bar{\ell}_t/\bar{\ell}_\delta$ by (12) and (19). Altogether, $\delta_t|_{\mathcal{C}^*} \geq \delta_t|_{\mathcal{M}^*}$. □

Claim I.8. *If t rises, then supply $\mathcal{A}^S$ shifts up more than demand $\mathcal{A}^D$ iff (★) holds.*

Proof: Log-differentiate $\mathcal{A}^S(\delta^S|q) \equiv \alpha$ in t using (20), and simplify using (12), (19) and (▷):

$$-\frac{d\delta^S}{dt}\left(\frac{\Lambda_\delta}{\Lambda} + \frac{f}{F}\bar{\ell}_\delta\right) = \frac{\Lambda_t}{\Lambda} + \frac{f}{F}\bar{\ell}_t = \frac{1}{t}\cdot\frac{\theta}{\theta'}\left(\frac{\Lambda_\delta}{\Lambda} + \frac{f}{F}\bar{\ell}_\delta\right) \quad\Longrightarrow\quad \frac{d\delta^S}{dt}\cdot\frac{\theta'}{\theta} = -\frac{1}{t}$$

Next, log-differentiate $\mathcal{A}^D(\delta^D|q) \equiv \alpha$, using the demand expression (17), to obtain:

$$\frac{d\delta^D}{dt}\left(\frac{\theta''}{\theta'} - \frac{\chi''}{\chi'}\right) = -\frac{1}{t}$$

Finally, comparing expressions for $d\delta^S/dt$ and $d\delta^D/dt$ yields $d\delta^S/dt > d\delta^D/dt$ iff (★). □

Claim I.9. *The level of offenses λ rises in M, m, along a constant markdown μ.*

Proof: Define $\tilde{\Lambda}(t) \equiv \Lambda(q(t), \delta(t), t)$, and suppose for a contradiction that $\tilde{\Lambda}'(t) \leq 0$. Define $\tilde{\ell}(t) \equiv \bar{\ell}(q(t), \delta(t), t)$. Since the attempted crime rate $\alpha = \tilde{\Lambda}F(\tilde{\ell})$ rises in t by Claim I.8, we must have $\tilde{\ell}'(t) > 0$. But then (▷) and(12), and then $\mathcal{E}_\lambda(c_q) \leq 1$, imply:

$$\tilde{\Lambda}'(t) = \Lambda_t + \Lambda_\delta\delta_t + \Lambda_q q_t = \frac{(1-\mu)(\theta + \theta' t\delta_t) - c_{\lambda q}q_t}{c_{\lambda\lambda}} \geq \frac{(1-\mu)(\theta\Lambda + \theta'\Lambda t\delta_t) - c_q q_t}{\Lambda c_{\lambda\lambda}}$$

Using (▷) and (19), this last numerator equals $\tilde{\ell}_t + \tilde{\ell}_\delta\delta_t + \tilde{\ell}_q q_t = \tilde{\ell}'(t)$, which is positive. □

J Lower Criminal Cost: Proof of Proposition 5

PROOF OF PART (a). OFFENSES-AUGMENTING MARGINAL COST REDUCTION. Smoothly index the cost function by $t \in \mathbb{R}$ as $c(\lambda, q|t) \equiv c(\tau(t)\lambda, q)$, where $\tau(1) = 1$ and $\tau' < 0$. Next, differentiate (11) and (18) and this function in t at $t = 1$, to get: (◁) $\bar{\ell}_t = -c_t = -\lambda c_\lambda \tau'$, $\lambda c_{\lambda t}/c_t = \lambda c_{\lambda\lambda}/c_\lambda + 1$, and $\Lambda_t = -c_{\lambda t}/c_{\lambda\lambda}$.

Claim J.1. *If t rises, then $\mathcal{M}^*$ shifts right more than $\mathcal{C}^*$, and so deterrence δ rises.*

Proof: Fix deterrence δ and log-differentiate equations (28) and (31) in t, at $t = 1$:

$$\left.\frac{q_t}{q}\right|_{\mathcal{C}^*} = \frac{\frac{\Lambda_t}{\Lambda} - \frac{\Lambda_t c_{\lambda q}}{c_q} - \frac{c_{qt}}{c_q}}{1 + \frac{q c_{qq}}{c_q} + \frac{q c_{\lambda q}\Lambda_q}{c_q} - \frac{q\Lambda_q}{\Lambda}} \quad \text{and} \quad \left.\frac{q_t}{q}\right|_{\mathcal{M}^*} = \frac{\frac{\Lambda_t}{\Lambda} + \frac{f}{F}\bar{\ell}_t}{1 - \frac{q\Lambda_q}{\Lambda} - \frac{f}{F}\bar{\ell}_q q} \tag{51}$$

To understand the first term, observe that $\mathcal{E}_\lambda(c_q) \leq 1$, and $\Lambda_q \leq 0$ by (12), and ($\lhd$) imply:

$$-\frac{\Lambda_t c_{\lambda q}}{c_q} - \frac{c_{qt}}{c_q} = \frac{c_{\lambda t}}{c_{\lambda\lambda}}\frac{c_{\lambda q}}{c_q} - \frac{c_{qt}}{c_q} = \frac{(\lambda c_{\lambda\lambda} + c_\lambda)\tau'}{c_{\lambda\lambda}}\frac{c_{\lambda q}}{c_q} - \frac{\lambda c_{\lambda q}\tau'}{c_q} = \frac{c_{\lambda q}}{c_q}\frac{c_\lambda}{c_{\lambda\lambda}}\tau' < 0$$

The last two terms in the denominator of the left side of (51) share the sign of $\Lambda_q(\mathcal{E}_\lambda(c_q) - 1)$, for each factor is non-positive by (12), and assumption. So $(q_t/q)|_{\mathcal{C}^*} \leq (\Lambda_t/\Lambda)/(1 + qc_{qq}/c_q)$. Also, $(q_t/q)|_{\mathcal{C}^*} \leq (\Lambda_t/\Lambda)/(1 - q\Lambda_q/\Lambda)$ since $qc_{qq} + qc_{\lambda q}\Lambda_q > 0$ by (30). So we have two cases.

Case 1: $(\Lambda_t/\Lambda)/(1 - q\Lambda_q/\Lambda) \leq \bar{\ell}_t/(-q\bar{\ell}_q)$. Here, $q_t/q|_{\mathcal{M}^*} \geq (\Lambda_t/\Lambda)/(1 - q\Lambda_q/\Lambda) \geq q_t/q|_{\mathcal{C}^*}$ by Claim A.1 and (51).

Case 2: $(\Lambda_t/\Lambda)/(1 - q\Lambda_q/\Lambda) > \bar{\ell}_t/(-q\bar{\ell}_q)$. Now, $q_t/q|_{\mathcal{M}^*} > \bar{\ell}_t/(-q\bar{\ell}_q)$ by Claim A.1 and (51). Applying ($\lhd$) and (19), and then Claim B.2 (namely, $\mathcal{E}_q(c) \leq \mathcal{E}_\lambda(c)$) and ($\lhd$):

$$\frac{\Lambda_t}{\Lambda} \cdot \frac{-q\bar{\ell}_q}{\bar{\ell}_t} = \frac{-c_{\lambda t}}{\Lambda c_{\lambda\lambda}} \cdot \frac{qc_q}{-c_t} \leq \frac{c_{\lambda t}}{c_t} \cdot \frac{c_\lambda}{c_{\lambda\lambda}} = \left(\frac{\Lambda c_{\lambda\lambda}}{c_\lambda} + 1\right)\frac{c_\lambda}{\Lambda c_{\lambda\lambda}}$$

Using this inequality, and $\mathcal{E}_\lambda(c_\lambda), \mathcal{E}_q(c_q) \geq 1$, we obtain $q_t/q|_{\mathcal{M}^*} > \bar{\ell}_t/(-q\bar{\ell}_q) \geq q_t/q|_{\mathcal{C}^*}$, since:

$$-\frac{\bar{\ell}_t}{q\bar{\ell}_q} \geq \frac{(\Lambda_t/\Lambda)\Lambda c_{\lambda\lambda}/c_\lambda}{1 + \Lambda c_{\lambda\lambda}/c_\lambda} \geq \frac{\Lambda_t/\Lambda}{1 + qc_{qq}/c_q} = \left.\frac{q_t}{q}\right|_{\mathcal{C}^*} \iff \frac{\Lambda c_{\lambda\lambda}}{c_\lambda} \cdot \frac{qc_{qq}}{c_q} \geq 1 \qquad \square$$

Claim J.2. *Let $\mathcal{C}^*$ slope down. If t rises, $\mathcal{C}^*$ shifts up more than $\mathcal{M}^*$, and caliber q rises.*

Proof: Fix caliber q and log-differentiate equations (28) and (31) in t at $t = 1$ to get:

$$\left.\frac{d\delta}{dt}\right|_{\mathcal{C}^*} = \frac{-\frac{\Lambda_t}{\Lambda} + \frac{\Lambda_t c_{\lambda q}}{c_q} + \frac{c_{qt}}{c_q}}{\frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\theta''}{\theta'} + \frac{\Lambda_\delta}{\Lambda} - \frac{c_{\lambda q}\Lambda_\delta}{c_q}} \quad \text{and} \quad \left.\frac{d\delta}{dt}\right|_{\mathcal{M}^*} = \frac{-\frac{\Lambda_t}{\Lambda} - \frac{f}{F}\bar{\ell}_t}{\frac{\theta''}{\theta'} - \frac{\chi''}{\chi'} + \frac{\Lambda_\delta}{\Lambda} + \frac{f}{F}\bar{\ell}_\delta} \tag{52}$$

By ($\lhd$) and (19), and (11) and (12):

$$\frac{\Lambda_t}{\Lambda} \cdot \frac{\bar{\ell}_\delta}{\bar{\ell}_t} = \frac{c_{\lambda t}}{c_t} \cdot \frac{\theta' m}{c_{\lambda\lambda}} = \frac{\Lambda c_{\lambda t}}{c_t} \cdot \frac{\Lambda_\delta}{\Lambda} \quad \text{and} \quad \frac{\Lambda_\delta}{\Lambda} \cdot \frac{\Lambda c_{\lambda\lambda}}{c_\lambda} = \frac{\theta'}{\theta} \tag{53}$$

We can now conclude that $\delta_t|_{\mathcal{M}^*} \leq -\bar{\ell}_t/\bar{\ell}_\delta$ using Claim A.1 and the right side of (52), since:

$$\frac{\Lambda_t/\Lambda}{\theta''/\theta' - \chi''/\chi' + \Lambda_\delta/\Lambda} \geq \frac{\bar{\ell}_t}{\bar{\ell}_\delta} \iff \frac{\chi''}{\chi'} - \frac{\theta''}{\theta'} - \frac{\Lambda_\delta}{\Lambda}\left(1 - \frac{\Lambda c_{\lambda t}}{c_t}\right) = \frac{\chi''}{\chi'} - \frac{\theta''}{\theta'} + \frac{\theta'}{\theta} \geq 0$$

by (★). Next, we prove that $\delta_t|_{\mathcal{C}^*} \geq -\Lambda_t/\Lambda_\delta$ using Claim A.1 and (52), since:

$$\frac{-\Lambda_t/\Lambda + c_{qt}/c_q}{\chi'/\chi - \chi''/\chi' + \theta''/\theta' + \Lambda_\delta/\Lambda} \geq -\frac{\Lambda_t}{\Lambda_\delta} \iff \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\theta''}{\theta'} \geq -\frac{\Lambda_\delta}{\Lambda_t}(c_{qt}/c_q)$$

because $\mathcal{C}^*$ slopes down. This second inequality is true by (6). Finally, ($\lhd$) implies $\mathcal{E}_\lambda(c_t) > 1$. So $-\Lambda_t/\Lambda_\delta = c_{\lambda t}/(\theta' m) > c_t/(\theta'\Lambda m) = -\bar{\ell}_t/\bar{\ell}_\delta$, by ($\lhd$), (12) and (19), i.e. $\delta_t|_{\mathcal{C}^*} \geq -\bar{\ell}_t/\bar{\ell}_\delta$. □

Claim J.3. *If t rises, then the level of offenses λ rises.*

Proof: As in the proof of Claim I.9, let $\tilde{\Lambda}(t) \equiv \Lambda(q(t), \delta(t), t)$, and suppose for a contradiction that $\tilde{\Lambda}'(t) < 0$. In §5.5 we deduced that $\alpha = \tilde{\Lambda} F(\tilde{\ell})$ rises in t. So we must have $\tilde{\ell}'(t) > 0$, where $\tilde{\ell}(t) \equiv \bar{\ell}(q(t), \delta(t), t)$. Using (12), (19), and ($\lhd$), we get $\mathcal{E}_\lambda(c_t) > 1 \geq \mathcal{E}_\lambda(c_q)$, and:

$$\tilde{\Lambda}'(t) = \Lambda_q q_t + \Lambda_\delta \delta_t + \Lambda_t = \frac{-c_{\lambda q} q_t + \theta' m \delta_t - c_{\lambda t}}{c_{\lambda\lambda}} \geq \frac{-c_q q_t + \theta' \Lambda m \delta_t - c_t}{\Lambda c_{\lambda\lambda}} = \frac{\tilde{\ell}'(t)}{\Lambda c_{\lambda\lambda}} > 0 \quad \square$$

Proof of Part (*b*). Caliber-Augmenting Marginal Cost Reduction. Now, $c(\lambda, q|t) \equiv c(\lambda, \tau(t) q)$, where $\tau(1) = 1$ and $\tau' < 0$. Differentiating costs in t at $t = 1$ yields: (†) $c_t = q c_q \tau'$, $c_{qt} = q c_{qq} \tau' + c_q \tau'$, and $c_{\lambda t} = q c_{\lambda q} \tau'$.

Claim J.4. *If t rises, then $\mathcal{C}^*$ shifts right more than $\mathcal{M}^*$, and so deterrence δ falls.*

Proof: Since ($\lhd$), (†) and (12) imply $\Lambda_t = -c_{\lambda t}/c_{\lambda\lambda} = -\tau' q c_{\lambda q}/c_{\lambda\lambda} = q\Lambda_q \tau'$, we may simplify the left expression in (51) using (†):

$$\left.\frac{q_t}{q}\right|_{\mathcal{C}^*} = \frac{\frac{\Lambda_t}{\Lambda} - \frac{\Lambda_t c_{\lambda q}}{c_q} - \frac{c_{qt}}{c_q}}{1 + \frac{q c_{qq}}{c_q} + \frac{q c_{\lambda q}\Lambda_q}{c_q} - \frac{q\Lambda_q}{\Lambda}} = \frac{\frac{q\Lambda_q\tau'}{\Lambda} - \frac{q c_{\lambda q}\Lambda_q \tau'}{c_q} - \frac{(q c_{qq} + c_q)\tau'}{c_q}}{1 + \frac{q c_{qq}}{c_q} + \frac{q c_{\lambda q}\Lambda_q}{c_q} - \frac{q\Lambda_q}{\Lambda}} = -\tau' > 0$$

We bound $q_t/q|_{\mathcal{M}^*}$ in (51). First, $(\Lambda_t/\Lambda)/(1 - q\Lambda_q/\Lambda) \leq -\tau'$. Next, ($\lhd$), (†), and (19) imply $\bar{\ell}_t = -c_t = -q c_q \tau' = q\bar{\ell}_q \tau'$. So $\bar{\ell}_t/(-q\bar{\ell}_q) = -\tau'$, and $q_t/q|_{\mathcal{M}^*} \leq -\tau' = q_t/q|_{\mathcal{C}^*}$ by Claim A.1. □

Claim J.5. *Let $\mathcal{C}^*$ slope down. If t rises, $\mathcal{C}^*$ shifts up more than $\mathcal{M}^*$, and caliber rises.*

Proof: First, consider (52). Since $\mathcal{C}^*$ slopes down, (29) and Claim B.7 imply:

$$\left.\frac{d\delta}{dt}\right|_{\mathcal{C}^*} = \frac{-\frac{\Lambda_t}{\Lambda} + \frac{\Lambda_t c_{\lambda q}}{c_q} + \frac{c_{qt}}{c_q}}{\frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\theta''}{\theta'} + \frac{\Lambda_\delta}{\Lambda} - \frac{c_{\lambda q}\Lambda_\delta}{c_q}} = \frac{\tau'\left(1 + \frac{q c_{qq}}{c_q} + \frac{q c_{\lambda q}\Lambda_q}{c_q} - \frac{q\Lambda_q}{\Lambda}\right)}{\frac{\chi'}{\chi} - \frac{\chi''}{\chi'} + \frac{\theta''}{\theta'} + \frac{\Lambda_\delta}{\Lambda} - \frac{c_{\lambda q}\Lambda_\delta}{c_q}} > -\frac{\chi}{\chi'}\tau'$$

Since $\mathcal{E}_\lambda(c_q) \leq 1$ and $\Lambda_\delta < 0$ by (12), the next premise holds:

$$\frac{\chi''}{\chi'} - \frac{\theta''}{\theta'} - \frac{\Lambda_\delta}{\Lambda}\left(1 - \frac{\Lambda c_{\lambda q}}{c_q}\right) \geq 0 \Longrightarrow \frac{\Lambda_t/\Lambda}{\theta''/\theta' - \chi''/\chi' + \Lambda_\delta/\Lambda} \geq \frac{\bar{\ell}_t}{\bar{\ell}_\delta}$$

The implication owes to $(\Lambda_t/\Lambda)(\bar{\ell}_\delta/\bar{\ell}_t) = (\Lambda c_{\lambda q}/c_q)(\Lambda_\delta/\Lambda)$, by the left side of (53) and (†). Next:

$$\frac{\bar{\ell}_t}{\bar{\ell}_\delta} = \frac{-c_t}{\theta'\Lambda m} = \frac{-q c_q \tau'}{\theta'\Lambda m} = \frac{\chi}{\chi'}\tau'$$

by (◁), (†), (19) and (28). By Claim A.1 and (52), $\delta_t|_{\mathcal{M}^*} \leq -\bar{\ell}_t/\bar{\ell}_\delta = -(\chi/\chi')\tau' < \delta_t|_{\mathcal{C}^*}$. □

References

Anderson, D. A. (2012): "The Cost of Crime," *Foundations and Trends in Microeconomics*, 7, 209–265.

Arrow, K. J. and L. Hurwicz (1960): "Stability of the gradient process in n-person games," *Journal of the Society for Industrial and Applied Mathematics*, 8, 280–294.

Ayres, I. and S. D. Levitt (1998): "Measuring Positive Externalities from Unobservable Victim Precaution: An Empirical Analysis of Lojack," *The Quarterly Journal of Economics*, 113, 43–77.

Becker, G. S. (1968): "Crime and Punishment: An Economic Approach," *The Journal of Political Economy*, 76, 169–217.

Brock, W. A. and A. G. Malliaris (1989): *Differential equations, stability and chaos in dynamic economics*, North-Holland Amsterdam.

Chalfin, A. and J. McCrary (2014): "Criminal Deterrence: A Review of the Literature," *mimeo*.

Clotfelter, C. T. (1978): "Private security and the public safety," *Journal of Urban Economics*, 5, 388–402.

Di Tella, R. and E. Schargrodsky (2004): "Do police reduce crime? Estimates using the allocation of police forces after a terrorist attack," *American Economic Review*, 115–133.

Eeckhout, J. and P. Kircher (2010): "Sorting and decentralized price competition," *Econometrica*, 78, 539–574.

Ehrlich, I. (1981): "On the usefulness of controlling individuals: an economic analysis of rehabilitation, incapacitation and deterrence," *The American Economic Review*, 71, 307–322.

——— (2010): "The market model of crime: a short review and new directions," *Handbook on the Economics of Crime*, 3, 3–23.

Ehrlich, I. and G. S. Becker (1972): "Market insurance, self-insurance, and self-protection," *The Journal of Political Economy*, 623–648.

Farrell, G., A. Tseloni, J. Mailley, and N. Tilley (2011): "The Crime Drop and The Security Hypothesis," *Journal of Research in Crime and Delinquency*, 48, 147–175.

Fujita, S. and M. Maxfield (2012): "Security and the drop in car theft in the United States," *The International Crime Drop: New Directions in Research*, 231–249.

Glaeser, E. L. and B. Sacerdote (1999): "Why Is There More Crime in Cities?" *Journal of Political Economy*, 107.

Gonzalez-Navarro, M. (2013): "Deterrence and geographical externalities in auto theft," *American Economic Journal: Applied Economics*, 5, 92–110.

Hotte, L., F. Valognes, and T. van Ypersele (2003): "Property Crime with Private Protection: A Market-for-Offenses Approach," *mimeo*.

Knowles, J., N. Persico, and P. Todd (2001): "Racial Bias in Motor Vehicle Searches: Theory and Evidence," *The Journal of Political Economy*, 109, 203–229.

Koo, H.-W. and I. Png (1994): "Private security: Deterrent or diversion?" *International Review of Law and Economics*, 14, 87–101.

Lazear, E. P. (2015): "Gary Becker's Impact on Economics and Policy," *American Economic Review*, 105, 80–84.

Levitt, S. D. (1997): "Using Electoral Cycles in Police Hiring to Estimate the Effect of Police on Crime," *The American Economic Review*.

——— (2004): "Understanding why crime fell in the 1990s: Four factors that explain the decline and six that do not," *The Journal of Economic Perspectives*, 18, 163–190.

McCrary, J. (2002): "Using electoral cycles in police hiring to estimate the effect of police on crime: Comment," *American Economic Review*, 1236–1243.

Nagin, D. S. (2013): "Deterrence: A Review of the Evidence by a Criminologist for Economists," *Annual Review of Economics*, 5, 83–105.

Persico, N. (2002): "Racial profiling, fairness, and effectiveness of policing," *The American Economic Review*, 92, 1472–1497.

Philipson, T. J. and R. A. Posner (1996): "The Economic Epidemiology of Crime," *Journal of Law and Economics*, 39, 405–33.

Piquero, A. R., D. P. Farrington, and A. Blumstein (2003): "The criminal career paradigm," *Crime and Justice*, 359–506.

Polinsky, A. M. and S. Shavell (2000): "The Economic Theory of Public Enforcement of Law," *Journal of Economic Literature*, 38, 45–76.

Quercioli, E. and L. Smith (2015): "The Economics of Counterfeiting," *Econometrica*, 83, 1211–1236.

Raphael, S. and R. Winter-Ebmer (2001): "Identifying the Effect of Unemployment on Crime," *Journal of Law and Economics*, 44, 259–283.

Samuelson, P. A. (1941): "The stability of equilibrium: comparative statics and dynamics," *Econometrica*, 97–120.

Shavell, S. (1991): "Individual Precautions To Prevent Theft: Private Versus Socially Optimal Behavior," *International Review of Law and Economics*, 11, 123–132.

Tullock, G. (1967): "The welfare costs of tariffs, monopolies, and theft," *Economic Inquiry*, 5, 224–232.

——— (1980): "Efficient rent seeking," in *Toward a theory of the rent-seeking society*, ed. by J. M. Buchanan, R. D. Tollison, and G. Tullock, Texas A&M University Press, 97–112.

van Ours, J. C. and B. Vollaard (2015): "The engine immobilizer: A non-starter for car thieves," *The Economic Journal*, forthcoming.

Vollaard, B. and P. Koning (2009): "The effect of police on crime, disorder and victim precaution. Evidence from a Dutch victimization survey," *International Review of Law and Economics*, 29, 336–348.

Vollaard, B. and J. C. van Ours (2011): "Does Regulation of Built-in Security Reduce Crime? Evidence from a Natural Experiment," *The Economic Journal*, 121, 485–504.